Comparison of control strategies for a 2-DOF helicopter

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Abstract

The two degrees of freedom (2-DOF) helicopter is an openloop unstable multi-variable process. Various control strategies can be applied to stabilize the system for tracking and regulation problems but not all control methods show equal capabilities for stabilizing the system. This paper compares the implementation of a classical PID controller, a linear quadratic regulator with integral action (LQR+I) and a model predictive controller (MPC) for stabilizing the system. It has been hypothesized that for such an unstable MIMO (multi input multi output) process showing cross coupling behavior, the model based controllers produces smoother control inputs than the classical controller. The paper also discusses the necessity of including the derivative part of the PID controller for stabilization and its influence to the measurement noises. A Kalman filter used for estimating unmeasured states may produce bias due to model mismatch. The implementation and comparison is based on a 2-DOF experimental helicopter prototype.

Keywords: 2-DOF helicopter, MPC, LQR, PID, Kalman filter, qpOASES

1 Introduction

In many publications where various control strategies are implemented and compared, there is a lack of verification of the implementation and experimentation with a real process. Most of the studies are simply based on simulation results and there are no experimental data to back-up their results. In this paper, we have tried to bridge the gap between simulation results and the real world implementation.

This paper is based on a 2-DOF helicopter unit. Various studies about this process can be found in literature with respect to tracking and regulation problem, see (Su et al., 2002; Lopez-Martinez et al., 2004; Yu, 2007; M. et al., 2010; Barbosa et al., 2016; Neto et al., 2016). The results of these studies look very promising, however, many of these studies are solely based on simulation results. At the university college of Southeast Norway (USN), a prototype of a two degrees of freedom helicopter model has been built from the scratch. All the control and estimation strategies discussed in this paper are actually implemented to the real unit.

Figure 1 shows the schematic of a 2 DOF helicopter unit at USN with the side view and the top view. It consists of two propellers (pitch and yaw) driven by motors.

The unit has two inputs: (a) voltage to the front or pitch motor/propeller system, and (b) voltage to the back or yaw motor/propeller system. When voltage is applied to the pitch motor, the pitch propeller rotates and it generates thrust, and the helicopter lifts up. Thus voltage to the pitch motor/propeller control the elevation (or pitch) of the helicopter nose about the pitch axis. When voltage is applied to the yaw motor, the yaw propeller rotates and it generates torque in anti-clockwise direction, and the helicopter rotates about the yaw axis. The angle between the pitch axis and the helicopter body axis is called the pitch angle. The angle between the yaw axis and the helicopter body axis is called the yaw angle. The pitch and the yaw angles are measured by using the angle sensors as shown in Figure 1. Thus, these are the two outputs of the system which are measurable.

When designing a controller for this process, the goal is to stabilize the system and keep track of the pitch angle and the yaw angle. Initially this task seems straight forward and simple. In reality, it is not so due to the presence of the cross coupling nature and the dead band feature in the process making the control task very challenging.

In this paper, three different control strategies are implemented for stabilizing the system. The response shown by the real unit for these control strategies are compared and discussed in detail. The paper is organized as follows: In section 2, a brief description of the mathematical model of the process is given. Section 3 describes the implementation of the three control strategies. Experimental results are presented in section 4. A detailed discussion on the experimental results and their comparison is provided in section 5. Finally, conclusion are drawn in section 6.

2 Model of the 2-DOF helicopter unit

Let us define that V_{mp} := voltage applied to the pitch motor, V_{my} := voltage applied to the yaw motor, θ := pitch angle and ψ := yaw angle.

The process is a cross-coupled MIMO system. When sufficient voltage is applied to the front motor, the helicopter not only pitches up but it also starts to rotate at the same time i.e. the input V_{mp} affects both outputs θ and ψ. Similarly, when sufficient voltage is applied to the back motor, the helicopter rotates in the anti-clockwise direction and at the same time, it also changes its pitch a little i.e. the input V_{my} affects both outputs θ and ψ. The effect of V_{mp} on ψ is very strong denoted by strong cross-

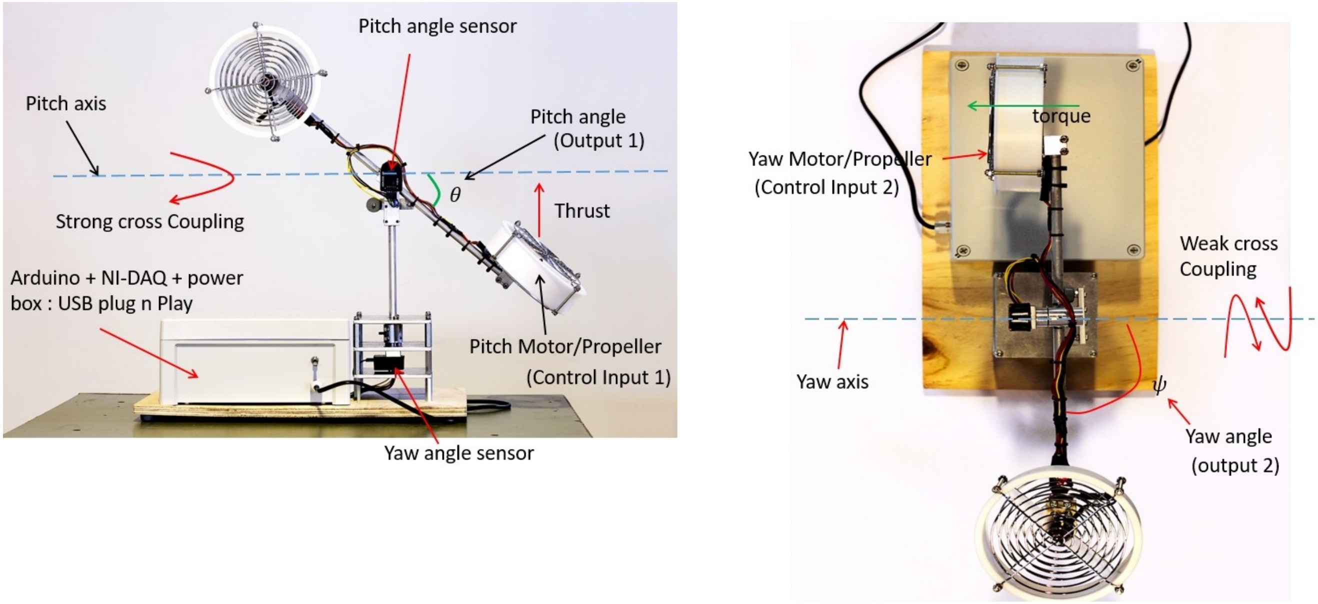

Figure 1. 2-DOF helicopter unit at USN (side view and top view)

coupling in Figure 1, while the effect of V_{my} on θ is weak denoted by weak cross-coupling.

The system can be described with four states: θ, ψ, pitch angular velocity (ω_θ) and yaw angular velocity (ω_ψ). The dynamics of the 2-DOF helicopter unit is modelled using Newton′s laws of motion and Euler-Lagrange equations of energy. The model is described by a set of four ordinary differential equations (ODEs) developed by Quansar Inc. (Qunasar Inc., 2011).

$$\frac{d\theta}{dt} = \omega_\theta, \quad (1)$$

$$\frac{d\psi}{dt} = \omega_\psi, \quad (2)$$

$$\begin{aligned}\frac{d\omega_\theta}{dt} = {} & \frac{K_{pp}V_{mp}}{J_{eq,p}+m_{heli}l_{cm}^2} - \frac{K_{py}V_{my}}{J_{eq,p}+m_{heli}l_{cm}^2} \\ & - \frac{m_{heli}\omega_\psi^2 sin(\theta) l_{cm}^2 cos(\theta) + m_{heli} g cos(\theta) l_{cm}}{J_{eq,p}+m_{heli}l_{cm}^2} \\ & - \frac{B_p \omega_\theta}{J_{eq,p}+m_{heli}l_{cm}^2},\end{aligned} \quad (3)$$

$$\begin{aligned}\frac{d\omega_\psi}{dt} = {} & \frac{K_{yp}V_{mp}}{J_{eq,y}+m_{heli}cos^2(\theta)l_{cm}^2} - \frac{K_{yy}V_{my}}{J_{eq,y}+m_{heli}cos^2(\theta)l_{cm}^2} \\ & - \frac{2m_{heli}\omega_\psi sin(\theta) l_{cm}^2 cos(\theta)\omega_\theta}{J_{eq,y}+m_{heli}cos^2(\theta)l_{cm}^2} \\ & - \frac{B_y \omega_\psi}{J_{eq,y}+m_{heli}cos^2(\theta)l_{cm}^2}.\end{aligned} \quad (4)$$

The parameters of the system with the description is listed in Table 1.

2.1 Dead band

The real process features dead band dynamics. There exists a minimum positive voltage on the pitch motor that is required to just lift up the helicopter, here defined as V_{mp}^{dead}. Application of pitch voltage lower than V_{mp}^{dead} produces no effect on the unit. Similarly, there exists a range of positive voltages to the yaw motor for which the system does not react at all (including the case when $V_{mp} > V_{mp}^{dead}$). This voltage range, here defined as V_{my}^{dead} is the minimum voltage on the yaw motor required to just rotate the helicopter. In the helicopter units at USN, V_{my}^{dead} is not a constant parameter but varies slightly with how much V_{mp} is being applied to the system. However, V_{my}^{dead} which is found experimentally is assumed to be constant in this paper.

To incorporate the effect of dead band, the process model should be adjusted. If the yaw voltage is less than V_{my}^{dead}, then $V_{my} = 0$ in Equations 3 and 4, i.e. for $0 \leq V_{my} \leq V_{my}^{dead}$, $V_{my} = 0$. Further more, if the pitch voltage is less than V_{mp}^{dead}, then it has no effect on the yaw angle ψ. Thus, the first term of Equation 4 is set to zero, i.e. for $0 \leq V_{mp} \leq V_{mp}^{dead}$,

$$\begin{aligned}\frac{d\omega_\psi}{dt} = {} & -\frac{K_{yy}V_{my}}{J_{eq,y}+m_{heli}cos^2(\theta)l_{cm}^2} \\ & - \frac{2m_{heli}\omega_\psi sin(\theta) l_{cm}^2 cos(\theta)\omega_\theta}{J_{eq,y}+m_{heli}cos^2(\theta)l_{cm}^2} \\ & - \frac{B_y \omega_\psi}{J_{eq,y}+m_{heli}cos^2(\theta)l_{cm}^2}.\end{aligned} \quad (5)$$

2.2 Linear Model

To implement a linear MPC and LQR, a linearized version of the nonlinear model was developed. Linearization was carried out as the truncated Taylor series expan-

Table 1. Parameters of the system.

Parameter	*Description*	*Value*
l_{cm}	Distance between the pivot point and the center of mass of helicopter	0.015 $[m]$
m_{heli}	Total moving mass of the helicopter	0.479 $[kg]$
$J_{eq,p}$	Moment of inertia about the pitch axis	0.0172 $[kg-m^2]$
$J_{eq,y}$	Moment of inertia about the yaw axis	0.0210 $[kg-m^2]$
g	Acceleration due to gravity on planet earth	9.81 $[m-s^2]$
K_{pp}	Torque constant on pitch axis from pitch motor/propeller	0.0556 $[Nm/V]$
K_{yy}	Torque constant on yaw axis from yaw motor/propeller	0.21084 $[Nm/V]$
K_{py}	Torque constant on pitch axis from yaw motor/propeller	0.005 $[Nm/V]$
K_{yp}	Torque constant on yaw axis from pitch motor/propeller	0.15 $[Nm/V]$
B_p	Damping friction factor about pitch axis	0.01 $[N/V]$
B_y	Damping friction factor about yaw axis	0.08 $[N/V]$

sion and the details have not been shown in this paper. Let $x = [\theta, \psi, \omega_\theta, \omega_\psi]^T$ represent the states of the system, $u = [V_{mp}, V_{my}]^T$ are the control inputs to the system and $y = [\theta, \psi]^T$ are the measured outputs from the system. If x_{op}, u_{op} and y_{op} denotes the operating points for the states, control inputs and outputs, and $dt = 0.1$ is the sampling time, then the linearized model in the deviation form in discrete time domain is written as,

$$\delta x_{k+1} = A\delta x_k + B\delta u_k \tag{6}$$

$$\delta y_k = C\delta x_k \tag{7}$$

Here, A, B, C and D are the system matrices such that $A \in \mathbb{R}^{n_x \times n_x}$ $B \in \mathbb{R}^{n_x \times n_u}$ and $C \in \mathbb{R}^{n_y \times n_x}$ with $n_x = 4$ is the number of states, $n_u = 2$ is the number of control inputs and $n_y = 2$ is the number of outputs. $\delta x_k = x_k - x_{op}$ is the deviation of the states from the operating point. Similarly, $\delta u_k = u_k - u_{op}$ and $\delta y_k = y_k - y_{op}$.

3 Control of the process

The primary use of controllers for this process is to stabilize the helicopter such that the pitch angle (θ) and the yaw angle (ψ) are kept at their given setpoints. The voltages to the pitch and yaw motors are the control inputs. Although in real life, a helicopter remains horizontal during normal flight i.e with a setpoint of $\theta_{SP} = 0^0$, for this helicopter unit at USN, the performance of the controller will be studied for different setpoint changes for both angles. Three control structures are utilized to control the process. Each of them will be discussed briefly in this section.

3.1 PID controller

The classical PID control algorithm is described by,

$$u(t) = K_p\Big(e(t) + \frac{1}{T_i}\int_0^t e(\tau)d\tau + T_d\frac{de(t)}{dt}\Big) \tag{8}$$

where e is the control error given by $e = y_{ref} - y$ with y_{ref} being the reference variable or the setpoint. K_p is the proportional gain, T_i is the integral time and T_d is the derivative time; the three parameters of the control algorithm. The PID controller was implemented in Simulink along with the anti-windup feature.

Two independent PID controllers were used: one to control the pitch angle by manipulating the pitch voltage, and the other to control the yaw angle by manipulating the yaw voltage. The parameters of the PID controllers were obtained by using the auto tuning feature (which utilizes the model of the process) available in the simulink PID block. The values of the PID parameters obtained from the auto tuning was manually refined slightly. For this particular process, it is worth mentioning that manual tuning of the PID parameters is not trivial and can be difficult even with an expert knowledge on both the process and the controller.

3.2 LQR with integral action

The linear quadratic regulator is a well established model based control algorithm. The theory behind LQR is not the aim of the paper and hence is not included here. Interested readers can refer to Bertsekas (2017) for theoretical details. Due to the presence of dead band and large uncertainty in the model parameters, it is necessary to add integrators to the two controlled outputs (the pitch angle and the yaw angle) for obtaining the integral action for zero steady state offset. The four states of the system denoted by x are,

$$x = \big[\theta, \psi, \omega_\theta, \omega_\psi\big]^T \tag{9}$$

For the unmeasured states(ω_θ and ω_ψ), a standard Kalman filter is used to estimate them. Let $\tilde{x}$ denote the estimated states. With an infinite horizon cost function defined as

$$\min\ J = \int_0^\infty \big(\tilde{x}^T Q\tilde{x} + u^T Ru\big)dt, \tag{10}$$

the feedback control law is given by,

$$u = -K\big(\tilde{x} - \tilde{x}_{SP}\big) + u_{op} \tag{11}$$

Here, K is the state feedback gain and is calculated using MATLAB function lqr. Q and R are the weighting matrices for the outputs and the inputs respectively. The control signal produced by the output integrators are added to u to generate the control signal applied to the actual process,

$$u_{app} = u + K_i \int \left(y - y_{SP}\right) dt \tag{12}$$

Here, y are the controlled outputs (pitch and yaw angles) with their setpoints y_{SP} and K_i are the output integrators gain vector. The estimates of the states are obtained as,

$$\delta\tilde{x}_{k+1} = A\delta\tilde{x}_k + B\delta u_k + L(\delta y_k - \delta\tilde{y}_k) \tag{13}$$

$$\tilde{x}_k = \delta\tilde{x}_k + x_{op} \tag{14}$$

Here, L is the Kalman filter gain and is calculated using the MATLAB function *kalman* for appropriately chosen covariance matrices for states and measurements.

3.3 MPC with integral action

MPC is an advanced model based control where process constraints can be systematically included in the optimal control problem formulation. Details about the theory of MPC is discarded in this paper as it is not the main aim. Interested readers can refer to Rawlings and Mayne (2009) for theoretical details. To achieve offset-free performance and disturbance rejection, the system state is augmented with integrating constant nonzero disturbance model as,

$$\underbrace{\begin{bmatrix} \delta x_{k+1} \\ d_{k+1} \end{bmatrix}}_{\delta\tilde{x}_{k+1}} = \underbrace{\begin{bmatrix} A & B_d \\ 0 & I \end{bmatrix}}_{\tilde{A}} \underbrace{\begin{bmatrix} \delta x_k \\ d_k \end{bmatrix}}_{\delta\tilde{x}_k} + \underbrace{\begin{bmatrix} B \\ 0 \end{bmatrix}}_{\tilde{B}} \delta u_k \tag{15}$$

$$\delta y_k = \underbrace{\begin{bmatrix} C & C_d \end{bmatrix}}_{\tilde{C}} \underbrace{\begin{bmatrix} \delta x_k \\ d_k \end{bmatrix}}_{\tilde{x}_k} \tag{16}$$

where, $d_k \in \mathbb{R}^{n_d}$ with $n_d = n_y$ being the number of unmeasured disturbance variables and equal to the number of measurements. The matrices $B_d \in \mathbb{R}^{n_x \times n_d}$ and $C_d \in \mathbb{R}^{n_y \times n_d}$ are chosen appropriately such that the following condition holds true for detectability (Pannocchia and Rawlings, 2003).

$$rank \begin{bmatrix} I - A & -B_d \\ C & C_d \end{bmatrix} = n_x + n_y \tag{17}$$

With a prediction horizon of N, the MPC problem is formulated as a tracking problem with the disturbance augmented model as,

$$\begin{aligned} \min \quad & J = \frac{1}{2} \sum_{k=1}^{N} \delta e_k^T Q \delta e_k + \delta u_{k-1}^T P \delta u_{k-1} \\ \text{subject to,} \quad & \\ & \delta\tilde{x}_{k+1} = \tilde{A}\delta\tilde{x}_k + \tilde{B}\delta u_k, \ \text{ with } \delta\tilde{x}_0 \text{ known} \\ & \delta y_k = \tilde{C}\delta\tilde{x}_k \\ & \delta e_k = \delta r_k - \delta y_k \\ & u_L \leq u_k \leq u_H \end{aligned} \tag{18}$$

Here, δr_k is the reference, $Q \in \mathbb{R}^{n_y \times n_y}$ and $P \in \mathbb{R}^{n_u \times n_u}$ are the weighting matrices, $u_L = 0[V]$ and $u_H = 3[V]$ are the lower and the upper values of the voltage applied to the motors and $\delta\tilde{x}_0$ is the known initial values of the augmented state. Let the choice of the vector of unknowns be $z^T = \left[\delta u^T, \delta x^T, \delta e^T, \delta y^T\right]$ with n_z number of variables. The MPC problem of Equation 18 is formulated as a standard quadratic programming (QP) problem as,

$$\begin{aligned} \min_z \quad & \frac{1}{2} z^T H z + c^T z \\ \text{subject to,} \quad & \\ & b_{\varepsilon,L} \leq A_\varepsilon z = b_{\varepsilon,H} \\ & z_L \leq z \leq z_H \end{aligned} \tag{19}$$

Without going into details of the problem formulation, only the final matrices and vectors are listed here.

$$H = \begin{bmatrix} I_N \otimes P & 0 & 0 & 0 \\ 0 & I_N \otimes 0_{n_x} & 0 & 0 \\ 0 & 0 & I_N \otimes Q & 0 \\ 0 & 0 & 0 & I_N \otimes 0_{n_y} \end{bmatrix} \tag{20}$$

$$c = 0_{n_z \times 1} \tag{21}$$

$$A_\varepsilon = \begin{bmatrix} -I_N \otimes \tilde{B} & I_{Nn_x} - (I_{N,-1} \otimes \tilde{A}) & 0_{Nn_x \times Nn_y} & 0_{Nn_x \times Nn_y} \\ 0_{Nn_y \times Nn_u} & -I_N \otimes \tilde{C} & 0_{Nn_y \times Nn_y} & I_{Nn_y} \\ 0_{Nn_y \times Nn_u} & 0_{Nn_y \times Nn_x} & I_{Nn_y} & I_{Nn_y} \end{bmatrix} \tag{22}$$

Here, $\otimes$ denotes the Kronecker product. For this particular problem there are no inequality constraints other than the bounds on the decision variables. So we have,

$$b_{\varepsilon,L} = b_{\varepsilon,H} = \begin{bmatrix} \tilde{A}\delta\tilde{x}_0 \\ 0_{(N-1)n_x \times 1} \\ 0_{Nn_y \times 1} \\ \delta r_1 \\ \vdots \\ \delta r_N \end{bmatrix} \tag{23}$$

The optimal control problem given by Equation 19 is implemented in Simulink and an opensource solver *qpOASES* (Ferreau et al., 2014; Potschka and Kirches, 2007–2017) is used to solved the QP problems. Out of Nn_u number of optimal values of the control inputs, only the first n_u control signals are applied to the process and the process is repeated at each sampling time (receeding horizon strategy).

The unmeasured states ω_θ and ω_ψ and the unknown disturbances d_k are estimated using a standard Kalman filter algorithm. The details about Kalman filter algorithm can be found at Simon (2006).

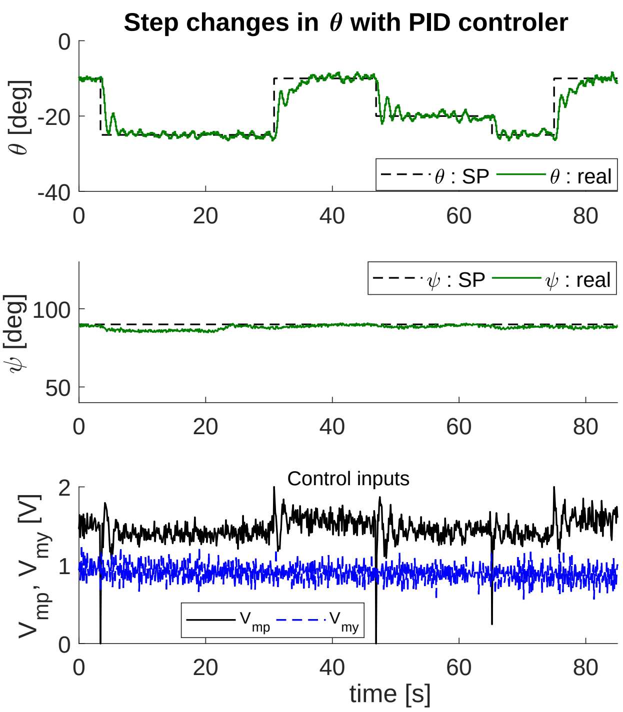

Figure 2. Step changes in θ with PID

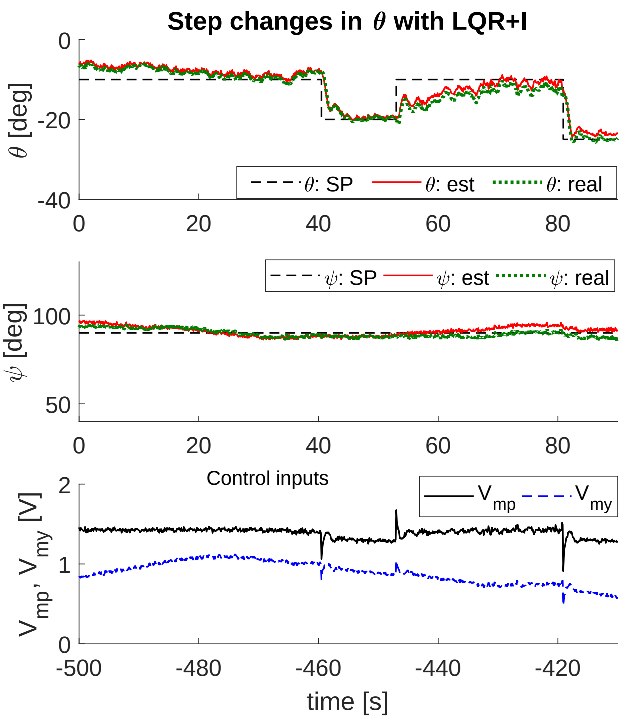

Figure 3. Step changes in θ with LQR

4 Experimental results

4.1 Tracking: step changes

In this section, the performance of the three control strategies for tracking the setpoint is compared. For easy comparison, the setpoints for the pitch angles and the yaw angles are changed separately for all three controllers. Figures 2, 3 and 4 show the results from the PID, LQR+I and MPC controllers respectively for tracking the pitch angle of the 2-DOF experimental helicopter unit with step changes.

Similarly, Figures 5, 6 and 7 show the results from the PID, LQR+I and MPC controllers respectively for tracking the yaw angle of the experimental helicopter unit with step changes.

4.2 Tracking: Ramping setpoint

In reality, the helicopters do not change angles (both pitch and yaw angles) in sharp steps. To turn the helicopter, the yaw angle setpoints should be instead ramped up and/or down. Similarly, to change the altitude of flight, the pitch angle setpoints should be ramped up and/or down. To illustrate the capability of controlling the helicopter unit when the setpoints are ramped up and down, the model predictive controller is used with ramped setpoint changes. For the sake of brevity and to save space, the other two controllers are not discussed. Figure 8 shows the results of ramping both the pitch angle and the yaw angle with MPC.

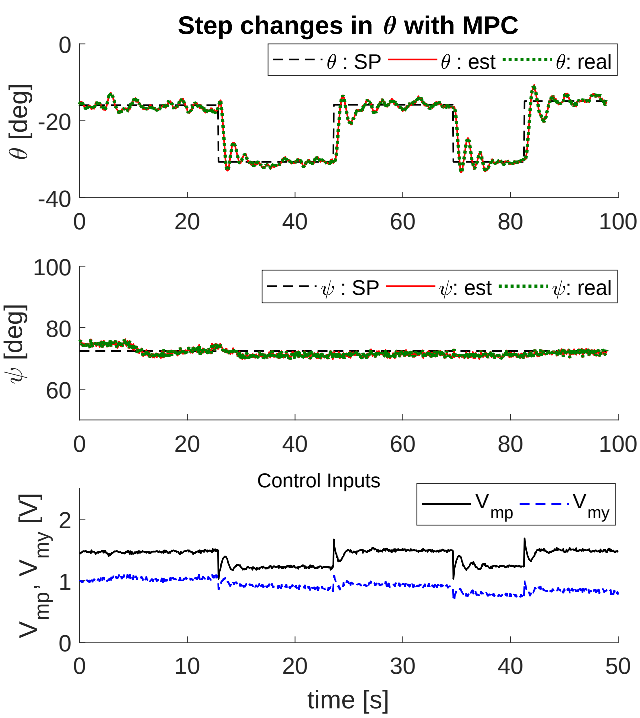

Figure 4. Step changes in θ with MPC

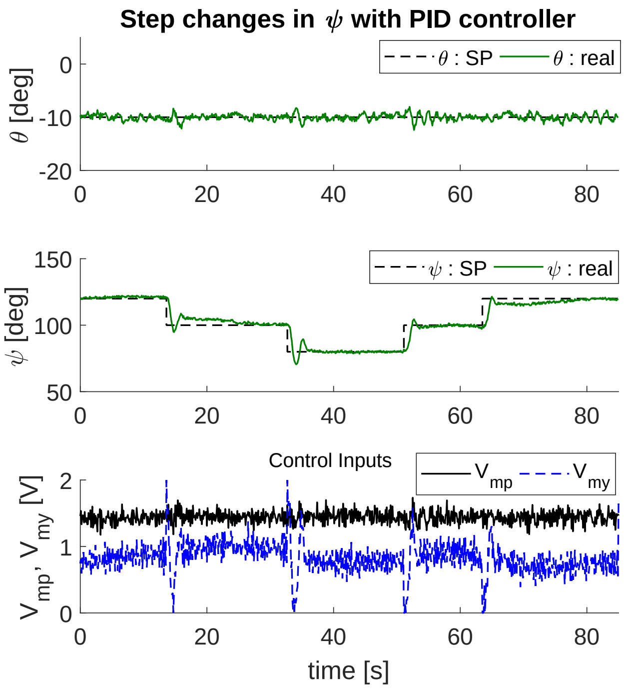

Figure 5. Step changes in ψ with PID

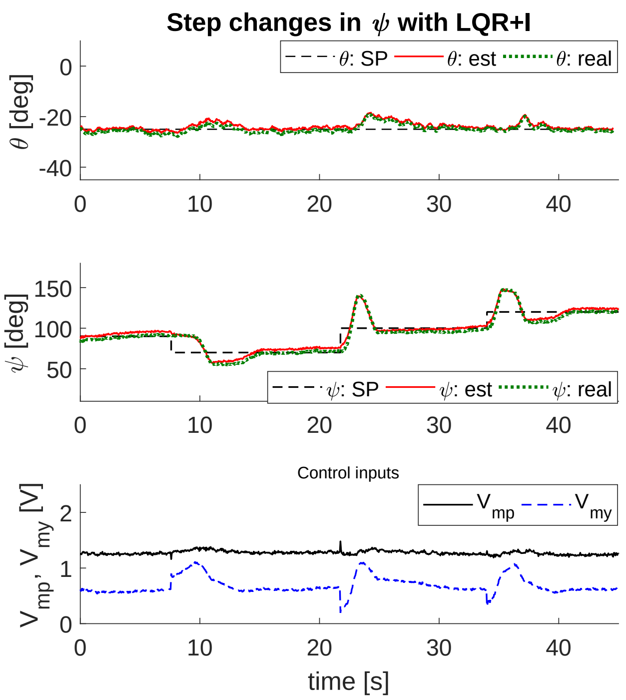

Figure 6. Step changes in ψ with LQR

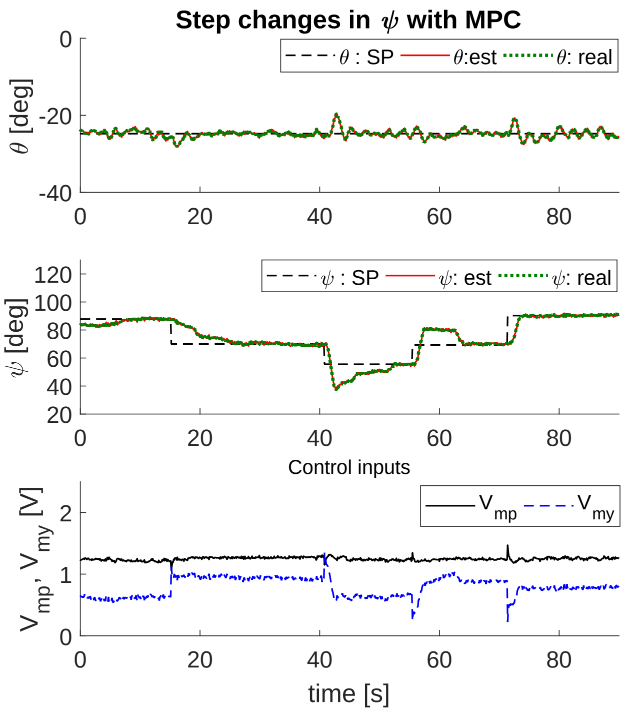

Figure 7. Step changes in ψ with MPC

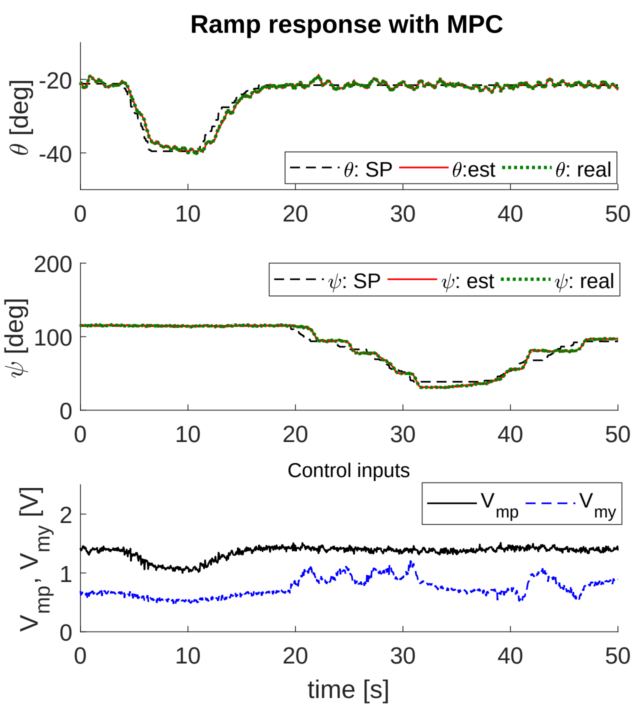

Figure 8. Ramp response with MPC

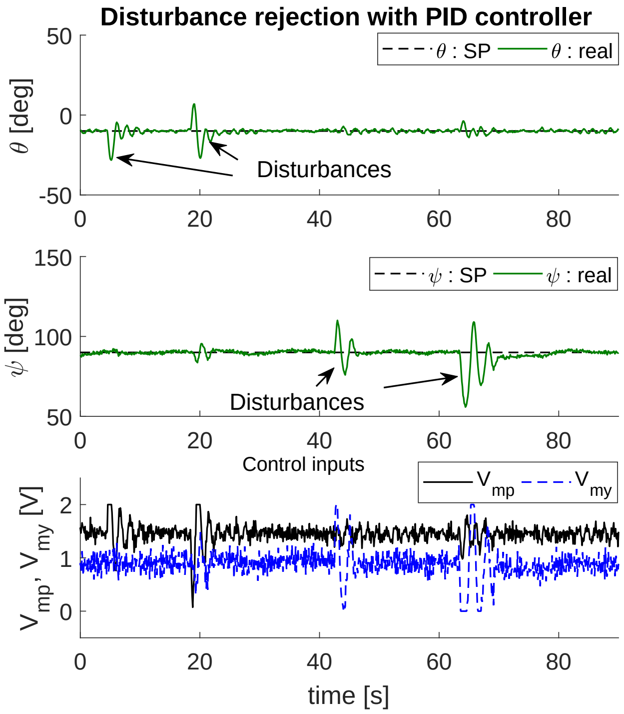

Figure 9. Disturbance rejection with PID

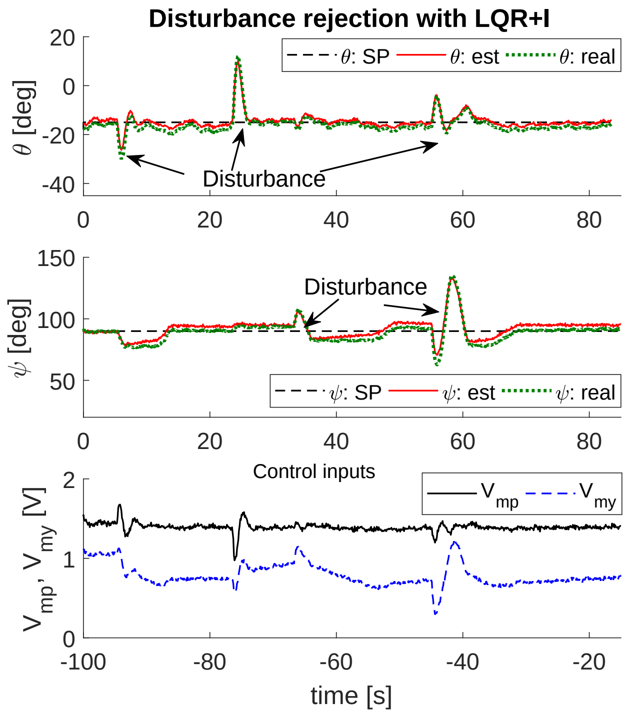

Figure 10. Disturbance rejection with LQR

4.3 Disturbance rejection

In real case scenarios, several disturbances can influence the flight path of a helicopter. A strong gust of wind, sudden drop in air pressure and density can all affect the pitch and the yaw angle of the helicopter. Such disturbances should be compensated by the controllers in order to stabilize the system. For the helicopter prototype at USN, to supply an external disturbance, the body of the helicopter was moved by applying force manually by hand. In other words, to mimic the presence of disturbance, external perturbation on the pitch and yaw angles were provided manually by the user. Figures 9, 10 and 11 show the performance of PID, LQR+I and MPC controller respectively under the presence of disturbances.

5 Discussion

5.1 Tracking performance

All the three control strategies could satisfactorily stabilize the system for different setpoint changes of both the pitch and the yaw angles. With the PID controller, the derivative term (D) is utmost important and should be used for stabilizing the system. A pure proportional and integral (PI) controller was not able to stabilize the system. However, the inclusion of D term was strongly influenced by the measurement noises and the resulting control inputs became noisy. There was chattering of the voltages applied to the motors. In the experimental helicopter unit, the chattering of the voltages resulted in unpleasant vibrational sounds from the propellers and mechanical vibra-

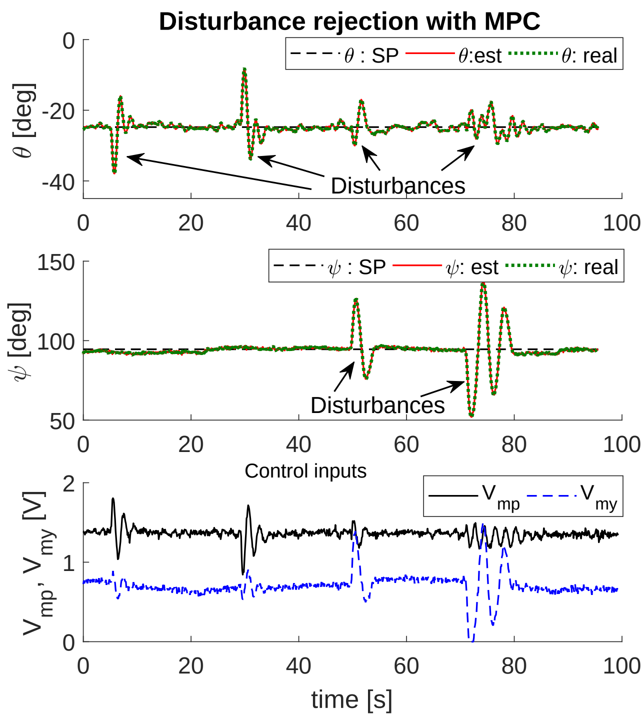

Figure 11. Disturbance rejection with MPC

tions of the whole rotor propeller system. On the other hand, the model based controllers (LQR+I and MPC) produced relatively cleaner control inputs without any chattering. In addition, proper tuning of the PID controllers was not straight forward owing to the cross coupling nature of the process. It is highly recommended that the model of the process should be used for tuning of the PID controllers. In contrast, it was relatively easier to tune the model based controllers.

When the setpoints on the pitch angles (yaw angles) were changed, all the three controllers could achieve the new setpoints while at the same time keeping hold or without losing control of the yaw angles (pitch angles), despite the presence of a strong cross coupling effect between the inputs and the outputs.

As has also been described before, in real life the pitch angle and the yaw angles are not changed in steps but are instead ramped. The model predictive controller showed a good response to the ramped setpoints. One benefit of ramping the helicopter angles is that the control inputs (voltages to the motors) are not suddenly changed by a large amount but instead they are changed gradually. This in practice may/will generate smoother response.

5.2 Disturbance rejection performance

Disturbances were applied manually by the user. Big disturbances (more than $\pm 50^0$ deviations) were applied to both the pitch and the yaw angles. All three controllers showed relatively equal and satisfactory performance in compensating the disturbances.

5.3 Comments on state estimation

The presence of dead band and the mismatch between the mathematical model and the real process makes it interesting for state estimation. A standard Kalman filter was applied to estimate the unmeasured states and the disturbances for the model based controllers (LQR+I) and MPC. In Figures 3, 6 and 10 (for the LQR+I controller), it can be seen that there is a small offset between the estimated angles (θ : *est* and ψ : *est*) and the angles measured by the sensors (θ : *real* and ψ : *real*). However, in Figures 4, 7 and 11 (for the MPC controller), it can be seen that there is no offset between the estimated angles (θ : *est* and ψ : *est*) and the angles measured by the sensors (θ : *real* and ψ : *real*).

With the MPC, the system states are augmented by a disturbance model (see Equation 15) and the disturbances are estimated using a Kalman filter. The estimated disturbances accounts for the dead band and the model mismatch and hence compensates for any offsets, thus producing zero offset between the estimated and the measured values. On the other hand, with the LQR+I controller, the system states are not augmented with any kind of disturbance model. The presence of dead band and model mismatch are hence not compensated, thus producing small offset between the estimated and the measured values. This clearly indicates the fact that due to model mismatch, such offset can be expected as an output from a Kalman filter algorithm. The important thing is to judge whether such offsets are important for the control/estimation purpose at hand. If they are not important, and the closed loop response is stable and correct, such offsets may simply be discarded.

6 Conclusion

This paper has shown experimental results obtained from applying different control structures to a real process. The control structures used in this paper are standard algorithms. They are relatively easier to understand and to implement. To solve the QP optimal control problems, open source solver which supports code generation is chosen. This allows us to implement the model based control structures to a real process using Simulink. The built-in QP solvers in MATLAB/Simulink does not support code generation and hence cannot be used for real time control of processes with fast dynamics such as the helicopter unit.

For this particular process at USN, the classical PID controllers show relatively as good performance as the advanced model based control structures. However, the control inputs generated by PIDs are noisy. Chattered input signals applied to the motor cause vibrations and can induce mechanical damage to the unit with time. In addition, model based control such as the MPC has the added advantage of including process constraints directly into the optimization problem. With the PID and LQR+I, constraints on the inputs were implemented as ad-hoc if-else conditions. Finally, the paper also justifies that such experimental units can be built from the scratch and can be used for pedagogic purpose with both the classical and the advanced control algorithms.

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