

AN ALGORITHM FOR OPTIMAL CONTROL OF AN ELECTRIC MULTIPLE UNIT

Nima Ghaviha^{*}, Fredrik Wallin and Erik Dahlquist
Mälardalen University, Sweden
School of Business, Society and Engineering

Markus Bohlin
SICS Swedish ICT AB

ABSTRACT

This paper offers a solution for the optimal EMU train (Electric Multiple Unit) operation with the aim of minimizing the energy consumption. EMU is an electric train with traction motors in more than one carriage. The algorithm is based on dynamic programming and the Hamilton-Jacobi-Bellman equation. To model the train, real data has been used, which was provided by experts from Bombardier Transportation Västerås. To evaluate the model, some experiments have been done on the energy saving in exchange for the increase in the trip time. Moreover a simple accuracy factor is introduced to evaluate the accuracy of the model. The final goal is to use this approach as a base for a driver advisory system, therefore it is important to have the amount of calculations as minimum as possible. The paper also includes the studies done on the calculation time. The solution can be used for driverless trains as well as normal trains. It should be mentioned that this paper is a part of a research which is still in progress and the final model will also be used by Bombardier Transportation Västerås as an evaluation tool for the propulsions systems and trains.

Keywords: Dynamic programming, Hamilton-Jacobi-Bellman equation, DAS, EMU

INTRODUCTION

The rail system is both a big energy consumer and at the same time the most energy efficient means of transportation. The subject of optimization of energy efficient train systems can be divided into two categories: optimization of train units themselves, and optimization of train unit operation, i.e., driving style. The goal in this paper is to work on energy efficient driving style of the train. The main problem is to plan the train movement during a trip with the aim of minimizing the energy

consumption. In other words we are looking for an optimal speed profile of a train to minimize the energy consumption for traction force subject to different constraints like travel time, speed limits etc.

Over more than 50 years different approaches have been used to solve this problem. Maximum principle, non-linear programming, dynamic programming and genetic algorithms are among the most widely used approaches. The solution based on Maximum Principle consists of four stages of maximum acceleration, constant speed, coasting

^{*} Corresponding author: Phone: +46(0)21 101368,
Email: nima.ghaviha@mdh.se

and maximum deceleration [1]. The problem of optimal train control can also be considered as a nonlinear optimization problem. Sequential quadratic programming is among the nonlinear programming algorithms used to solve this problem [2]. In the dynamic programming approach all the variables are discretized and the best solutions are selected among all possible combinations [1]. Genetic algorithms have also been used to solve this problem [3].

In order to use the solutions on real trains, they are implemented in a form of driver advisory systems (DAS). Currently there are many solutions and DAS's in the market. In [4] a complete study is done on different driver advisory systems and in [5] different solutions are assessed based on different criteria. One of the main obstacles for implementation of different algorithms in a form of DAS is the limitation on the amount of calculation that can be done on a train and during a trip.

In this paper an approach is introduced for the problem of energy efficient driving of a train with the aim of having minimum amount of calculations during the trip. The model should be extendable to handle other constraints such as limitations on the power we can get from the line, maximum acceleration and retardation, regeneration rate etc. The approach is based on a simplified version of the problem which was previously solved in [6].

OBJECTIVE

The objective is to minimize the energy consumption of an EMU (Electrical Multiple Unit) during a specific trip. The distance and trip time are fixed. The aim is to find the optimal speed profile for the train with the minimum energy consumption. The train is equipped with regenerative brakes which can be used in higher speeds. It is not possible to use regenerative brakes at lower speeds, therefore the only available alternative is the mechanical brake. Later on the results shall be used for a driver advisory system to help the driver reach the destination on time and at the same time minimize the energy consumption.

MODEL

The mass of the train is assumed to be concentrated on the gravity center of the train; therefore we treat the train as a single mass point. During the trip there are several forces affecting the train:

- Tractive effort (F_t)
- Running resistance (F_{rr})
- Gradient resistance (F_g)
- Curve resistance (F_c)
- Aerodynamic resistance (F_{ae})

In the current model curve resistance and aerodynamic resistance are assumed to be zero. The summation of the rest of the forces will be the acceleration force.

$$\sum F = m * a \quad (1)$$

$$m * a = F_t - F_{rr} - F_g - F_c - F_{ae} , \quad (2)$$

where m is the mass of the train and a is the average acceleration.

Running Resistance

The formula used for the running resistance is as follows:

$$F_{rr} = A + B * v + C * v^2 \quad (3)$$

A , B and C are the running resistance coefficients that are dependent on the weight of the train and v is the velocity.

Gradient Resistance

The gradient resistance formula used in this model is as follows:

$$F_g = m * g * \frac{s}{1000} , \quad (4)$$

where m is the train mass, g is the standard gravity and s is the gradient.

Energy Consumption

The total amount of energy used during a trip is equal to

$$E = \int F_t * x \quad , \quad (5)$$

where E is the total energy consumption, F_t is the tractive effort and x is the distance traveled.

APPROACH

The approach used in this paper is based on dynamic programming and the Hamilton-Jacobi-Bellman equation and Bellman's backward approach [7]. Using this approach it is possible to find the best solution for all the states of the train during the trip beforehand which makes the amount of calculation needed on the train as minimum as possible. There is a maximum trip time which corresponds to the latest arrival time of the train to its destination. The idea is to have a general solution ready at the start of the trip and find the best solution at each state during the trip. The state of the train is defined with three parameters: time passed, distance traveled and current velocity. Trip time, trip distance and velocity are discretized. Therefore we can represent any state of the train using an array with dimensions of T , X and V , where T is the number of time steps, X is the number of track sections represented and V is the number of admissible train velocities. For each state three outputs are needed and each will be stored in different matrices: the minimum energy needed to reach the final destination (cost-to-go) which is stored in matrix E , the optimum tractive effort which is stored in matrix F_t and the optimum velocity in the next time step which is stored in matrix V_{op} . In the current model it is assumed that at the final time (T), the train will stop at the destination (X). Therefore the final state is $(T, X, 0)$ which has a cost-to-go of zero. For the states (T, X, v) where v has any value other than zero a high penalty is assigned. There are also penalties set for the states that represent any position other than final one (X) in the last time step (T).

Cost-to-go calculations

In the following sections the below symbols are used:

$E(i, j, k)$: Minimum energy needed for the train to reach the final state from the state of (i, j, k)

dt : duration of one time step [s]

dx : length of one distance step [m]

dv : one velocity step [km/h]

$(T, X, 0)$: final state which is the last time step, last distance step and zero velocity

Suppose that we are in a certain state (we call it start point with the parameters of t_s, x_s and v_s) and want to calculate the total energy needed to reach the final state. Suppose that all the states in the next time step (t_s+dt, x, v) are already filled with the minimum energy needed to reach the final state.

$$E(t_s, x_s, v_s) = \text{Min} \left(de(s, f) + E(t_s + dt, x_f, v_f) \right) \quad \forall x_f, v_f \quad , \quad (6)$$

where $de(s, f)$ represents the transition cost which is the energy needed for getting the train from the state (t_s, x_s, v_s) to the state (t_s+dt, x_f, v_f) .

By backward iteration starting in the final state where $t = T$ and the optimum cost is known, we calculate the optimum cost at time $t-1$ and so forth.

Transition cost

Suppose that the starting state is (t_s, x_s, v_s) and the next state is (t_f, x_f, v_f) with t_f being equal to t_s+dt .

Having v_f and dt we can calculate the x_f and later on de .

$$t_f = t_s + dt \quad (7)$$

$$v_{avg} = \frac{(v_s + v_f)}{2} \quad (8)$$

$$x_f = x_s + v_{avg} * dt \quad (9)$$

$$\Delta x = x_f - x_s \quad (10)$$

$$F_a = \frac{m*(v_f - v_s)}{dt} \quad (11)$$

$$F_{rr} = A + B * v_{avg} + C * v_{avg}^2 \quad (3)$$

As it is mentioned in above equations average velocity in one time step is used for running resistance calculations. For gradient resistance however, the gradient force for all the distance steps in one time step will be checked and the total gradient force will be calculated:

$$F_g = \sum_{x_s}^{x_f} F_{gi} \quad (13)$$

$$F_t = F_a + F_{rr} + F_g \quad (14)$$

$$de = F_t * \Delta x \quad (15)$$

Undesired states

For each state of (t_s, x_s, v_s) there will be $\frac{v}{dv}$ number of states to check in the next time step. Most of them are not feasible and should not be checked.

1. As stated before x_f is calculated based on v_f . States in which the x_f will exceed X won't be checked.
2. The train's traction motor has a certain limit on the available tractive effort based on the velocity (see figure 1). Based on the maximum tractive effort available, the gradient force, the running resistance and the mass of train, the maximum and minimum acceleration rate can be calculated. A limit on acceleration rate will set a limit on the number of different velocities the train can reach from a certain velocity in one time step.
3. There are certain speed limits during the trip. In one time step the train might encounter different speed limits in different distance steps which will reduce the

number of feasible velocities in the next time step.

4. Jerk rate is defined as rate of change of acceleration which can also be represented as rate of change of acceleration force. Having a maximum jerk rate will set another limit on the tractive effort the traction motor can provide and therefore limits the number of states the train can reach in one time step.

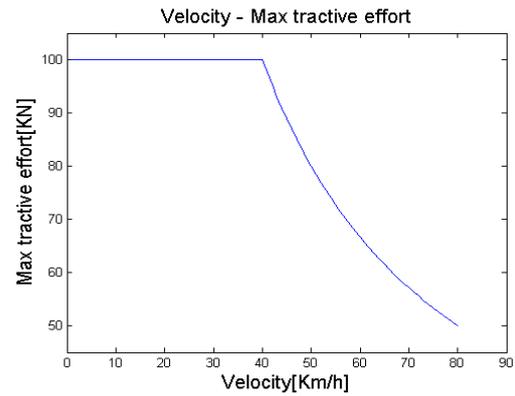


Figure 1 - maximum tractive effort available in different velocities.

The above states will not be checked by the model and the optimum state in the next time step with the minimum $de + E(t_s + dt, x_f, v_f)$ for each (t_s, x_s, v_s) will be chosen among the rest of the states.

Final matrices

Matrices filled at the end of calculations are as follows:

- Next optimum velocity (V_{op})
- Minimum total energy needed to reach the final state (E)
- Optimum tractive effort at each state (F_t)

Finding the Optimum Speed Profile

Filling in the matrices of the general solution takes a long time. However it is needed to be done only once before starting the trip. During the trip the current state of the train can be set as the input and the optimum tractive effort and velocity in the next time step and the minimum total energy needed to reach the final state will be found from the general

solution matrices. The time needed to look up the general solution and find the optimum tractive effort is short enough for the model to be used as an advisory system for the driver. This is discussed further in the next section.

RESULTS AND DISCUSSION

MATLAB is used as the programming tool in this study. Figures 2 to 4 show the speed profile for 3 states (0,0,0), (20, 500, 0) and (20, 500, 60) in the same trip. The trip time is 3.16 minutes and the distance between two stations is 3.02 kilometers. The dotted line is the speed limit in *km/h* and the dashed line represents the gradient.

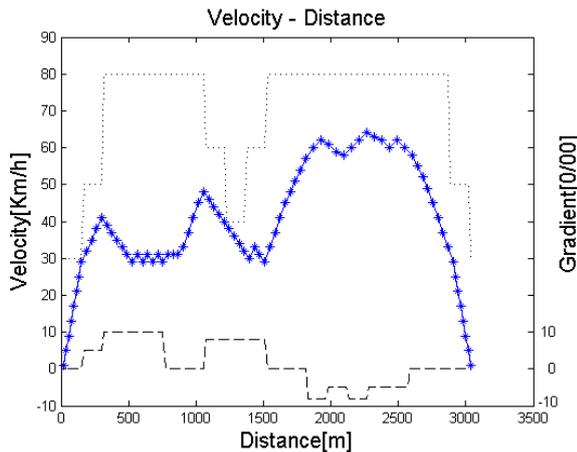


Figure 2-the optimum speed profile for the state (0,0,0) with the speed limits (dotted line) and gradient (dashed line).

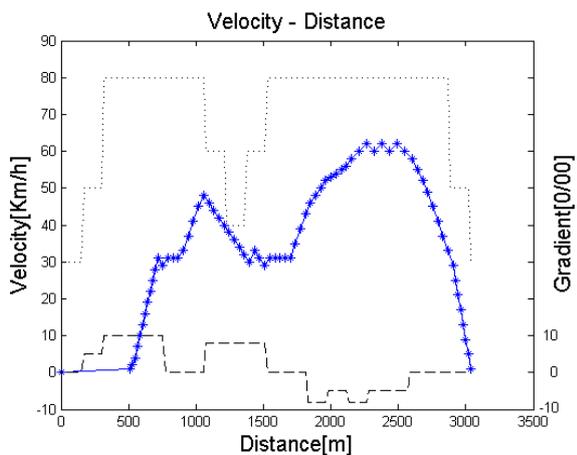


Figure 3- the optimum speed profile for the state (20,500,0) with the speed limits (dotted line) and gradient (dashed line).

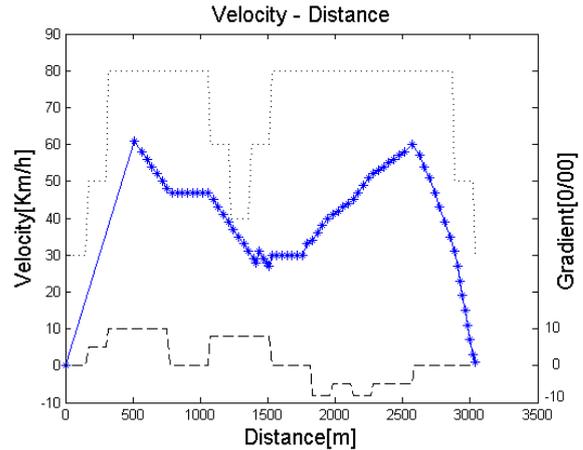


Figure 4- the optimum speed profile for the state (20,500,60) with the speed limits (dotted line) and gradient (dashed line).

Discretization

In this approach time, distance and velocity are discretized. It is obvious that with smaller intervals or steps, there will be smoother curves for speed profiles and more accurate solutions. However with higher number of intervals the calculation time will increase. Moreover the trip time and the trip distance will also affect the calculation time. To study the runtime two trips and four types of discretization have been studied. (See table 1)

	Time	Distance
Trip1	2 min	1.51 Km
Trip2	4 min	3.02 Km

Table 1 – time and distance for each trip

Table 2 shows the number of intervals in each accuracy.

	Number of time steps	Number of distance steps
Accuracy 1	10	20
Accuracy 2	20	40
Accuracy 3	40	80
Accuracy 4	80	160

Table 2 – number of time and distance steps for each accuracy.

Table 3 includes the calculation times for each combination of trip and accuracy in seconds. The calculations are done in MATLAB using a normal PC.

	Accuracy 1	Accuracy 2	Accuracy 3	Accuracy 4
Trip 1	9.9376	21.4295	43.4619	75.6426
Trip 2	15.7453	37.5917	82.8783	174.9483

Table 3 – time required to calculate the general solution in seconds for each trip and accuracy.

As we can see, with increasing the time and the distance and also increasing the number of intervals, the calculation time for the general solution increases. It can be expected that for longer trips the calculation time would increase more. There are still works in progress to decrease the calculation time. The calculation of a general solution shall be done only once and that's before starting the trip. During the trip it is only needed to input the current state of the train (time, distance and velocity) and look up the general solution to find the optimum solution for that specific state. The time needed to look up the general solution and find the optimum tractive effort and the optimum speed profile for trip two and accuracy four is around 0.03 seconds. It is to be mentioned that one time step for trip two and accuracy four is equal to 3 seconds. So it is theoretically possible to take actions when using the system during the trip. The challenges for implementation of the system is discussed under implementation subheading.

Error

To study the effects of the number of intervals for discretization, an accuracy factor is introduced. As mentioned in previous sections to calculate the energy consumption two states are needed. The first state is (t_s, x_s, v_s) and the second state is $(t_s + dt, x_f, v_f)$. The second state is chosen according to all possible velocities in the next time step. x_f is estimated based on v_f and dt ; it is calculated according to equations of motion and then rounded down to the closest distance step. The estimation is in rounding the real distance step to the integer one.

$$E_i = (x_{fR} - x_{fI})^2 \quad \text{error in time step } i \quad (16)$$

$$E_{average} = \text{Average}(E_i) \quad \text{average error}, \quad (17)$$

where x_{fR} is the real distance step and x_{fI} is the integer distance step. $E_{average}$ shows the average of deviations of the estimated distance step from the real distance steps. In order to compare four accuracies with each other, $E_{average}$ is then multiplied by the length of one distance step. The results are shown in table 4.

	Accuracy 1	Accuracy 2	Accuracy 3	Accuracy 4
Trip2	64.0364	35.2121	16.3624	8.9636

Table 4- $E_{average}$ multiplied by the length of one distance step for each accuracy for trip two.

As we can see the more distance and time are discretized the less error we will have.

Energy Calculations

In this model the train is assumed to be a single mass point and the energy calculations are based on the equations of motion. In other words the energy needed for the transition from one state to another one is only the energy needed to move a single mass from one point to another. However in a real train there are other sources of energy consumption like auxiliary systems. Moreover each component in the train has a certain efficiency which will increase the total energy consumption. Including the efficiency in the equations would be quite simple if the efficiencies were constant, but the efficiency of different components will vary in different velocities and different tractive efforts. There are currently not enough data and information on the changes of efficiencies according to velocity and tractive effort. There is work in progress to have more accurate functions ready for this purpose.

Regenerative Brake

The train modeled is equipped with a regenerative brake which works in higher velocities, for low speeds mechanical brake is used (below 7 km/h). Using regenerative brake it is possible to theoretically regenerate all the acceleration force. But in reality there are some losses when braking. Some losses are the same kind of losses as the ones discussed in the previous topic (i.e. losses when transferring energy from the wheels back to the line).

Another kind of loss happens after transferring the energy back to the line. Energy regenerated from braking can be used in different ways. It can be either used by some other train that is accelerating at the same exact moment, saved somewhere for the future use (e.g. in capacitors in certain points in line) or just going to waste. Considering all these three conditions in the model makes it more complex. At the moment we assume that 80 percent of the energy is regenerated from braking. This percentage is a rough estimation used by experts in Bombardier Transportation Västerås. Having the constant coefficient of 0.8 for the regenerated energy, makes it always favorable to brake rather than coast (F_r equals to zero). Therefore the solution will rarely include any coasting.

Validation

Currently there are no real data available to be used for validating the model. There is going to be a test run of a certain train later on during autumn 2014. It is planned that the data from that test run be used for validating the model. However from the test runs of the model the decrease in energy consumption in exchange for increase in time is obvious. The table 6 shows the change in energy consumption in different travel time.

Change in the trip time	+2 sec (1.0%)	+4 sec (2.1%)	+7 sec (3.8%)	+10 sec (5.4%)	+12 sec (6.5%)	+17 sec (9.2%)	+22 sec (12.0%)
Energy saving %	9.86 %	16.35 %	33.93 %	40.37 %	47.95 %	60.29 %	60.17 %

Table 4 – Energy saving in exchange for increase in time for trip two and accuracy four.

As it can be seen in the table above in exchange for a small increase in the trip time, the energy consumption will decrease.

Implementation

There are several methods to implement a DAS system [5]. One is to have the optimum speed profile ready and have the driver follow it. The other one is to have a system on the train to give instruction to the driver during the trip. The solution

represented in this paper can be used in both ways. However there are some points to be mentioned.

In the current model instructions are given in every time step. Given the fact that the time steps are chosen to be rather short, it is not practically possible to use the current model on a real train. Moreover the instruction given by the model for each state is the optimum tractive effort needed at the moment, which is continuous whereas in the real train, the driver has a possibility to apply a specific amount of tractive effort. In other words the control is discrete.

There are still works to be done on the model to make it ready for a practical use.

CONCLUSION

The problems addressed in this paper was to plan train movement to minimize the energy consumption. The model shall be a base for a Driver Advisory System (DAS) which is currently under development. Since its going to be used later as a DAS, the main challenge was to have the minimum amount of calculations during the trip. The main idea in the model was to have a general solution ready before starting the trip and find the best solution during the trip. There are still new features to be added to the model and it needs to be developed further for it to be used as a DAS. However, no matter the complexities of the new features, the amount of calculations on the train would still be the same or slightly more, because they will all be done before the trip and during the trip the calculations will be limited to just looking up the general solution.

ACKNOWLEDGMENT

This research is funded by VINNOVA, grant number 2012-01277, and Bombardier Transportations Västerås. Authors would like to thank Christer Holmberg and Kevin Babr from Bombardier Transportations for their help with the information and data on the train and its propulsion system during the research.

REFERENCES

- [1] Franke R, Terwiesch P, Meyer M, *an algorithm for the optimal control of the driving of trains*, proceedings of the 39th IEEE conference on Decision and Control, 2000
- [2] Miyatake M, Matsuda K. *Energy Saving Speed and Charge/discharge Control of a Railway Vehicle with On-board Energy Storage by Means of an Optimization Model*, IEEJ Transactions on Electrical and Electronic Engineering 2008 4:771 – 778
- [3] Chang C.S, Sim S.S, *Optimizing Train Movement through Coast Control Using Genetic Algorithms*. IEE Proceedings – Electrical Power Applications 1997, 144(1):65 – 73
- [4] DeltaRail Group, *Driver Advisory Information for Energy Management and Regulation*. In: Stage1 Report, RSSSB Research Program 2009, T724
- [5] Panou K, et al. *Railway Driver Advice Systems: Evaluation of Methods, Tools and Systems*. Journal of Rail Transport Planning & Management 2014, <http://dx.doi.org/10.1016/j.jrtpm.1013.10.005>
- [6] Gkortzas P. *Study on Optimal Train Movement for Minimum Energy Consumption*, Master Thesis Work, Mälardalen University 2013
- [7] Bertsekas D.P. *Dynamic Programming and Optimal Control, third edition*. Athena Scientific, Belmont, Massachusetts. 2005, ISBN 1-886529-26-4