MODELING AND SIMULATION OF MULTI-ZONE BUILDINGS FOR BETTER CONTROL

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ABSTRACT
Buildings are one of the largest energy consumers in most of the countries. Building sector in the European Union (EU) is continuously expanding and currently utilizes 40% of total energy consumption in the union. Out of that, space heating energy demand is the highest. Norway, where a harsh climate predominates, uses 48% of the total energy production for both residential and commercial buildings. Recent investigations carried out in Norway showed that there is a potential of saving 65 TWh both from residential and commercial buildings in 2020.

Nowadays there is a growing trend to use building automation system (BAS) in buildings, ranging from small rooms to multi-zone buildings with diverse architectural designs. BAS helps to make the environment more efficient for occupants with better facility management. Currently, BAS lacks a building model, and the control is based on temperature zones and lowering the temperature only 5°C when the heaters are unused. A good building model may help to optimally turn the energy on and off and reach the temperature goal of the zones. This will give a better energy performance for the buildings.

This article refers to a multi-zone mechanistic building model which can be used for simulating the thermal behavior of a residential building. It consists of modeling the ventilation, thermal mass of walls, floor, roof and furniture. The model state variables are expressed using a lumped parameter approach. The temperature and relative humidity measurements acquired from a typical residential building in Norway are used to verify the model. Model simulation is carried out in MATLAB environment, and it can be applied for controlling the energy performance of complex building designs reasonably well. Hence the current research project is important as it contributes in achieving the energy saving goals determined in 2020.

Keywords: Mechanistic building model, Multi-zone building, Residential buildings, Ventilation

NOMENCLATURE

Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>$A$</td>
<td>Surface area [$m^2$]</td>
</tr>
<tr>
<td>$\tilde{c}_p$</td>
<td>Specific heat capacity of air [$J/(kgK)$]</td>
</tr>
<tr>
<td>$E$</td>
<td>Enthalpy [$J$]</td>
</tr>
<tr>
<td>$\tilde{h}$</td>
<td>Specific enthalpy [$J/kg$]</td>
</tr>
<tr>
<td>$I$</td>
<td>Internal energy [$J$]</td>
</tr>
<tr>
<td>$M$</td>
<td>Molar mass of air [$kg/mol$]</td>
</tr>
<tr>
<td>$\dot{m}$</td>
<td>Air mass flow rate [$kg/s$]</td>
</tr>
<tr>
<td>$n$</td>
<td>No. of mols [$mol$]</td>
</tr>
<tr>
<td>$P$</td>
<td>Pressure [$Pa$]</td>
</tr>
<tr>
<td>$\dot{Q}$</td>
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<td>$R$</td>
<td>Gas constant [$J/(molK)$]</td>
</tr>
<tr>
<td>$r$</td>
<td>Radius of sphere [$m$]</td>
</tr>
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<td>Temperature [$K$]</td>
</tr>
<tr>
<td>$t$</td>
<td>Thickness [$m$]</td>
</tr>
<tr>
<td>$U$</td>
<td>Overall heat transfer coeff. [$W/(m^2K)$]</td>
</tr>
<tr>
<td>$V$</td>
<td>Volume [$m^3$]</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Thermal diffusivity [$m^2/s$]</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Furniture temperature [$K$]</td>
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<tr>
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<td>Ceiling temperature [$K$]</td>
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<tr>
<td>$\psi$</td>
<td>Floor temperature [$K$]</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Density [$kg/m^3$]</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Time [$s$]</td>
</tr>
</tbody>
</table>

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\( \theta \) Inside air temperature [K]  
\( \omega \) Wall temperature [K]  

Subscripts  
c ceiling  
f Floor  
fur Furniture  
g Ground  
i Building unit  
j Adjacent room  
w Wall  
\( \alpha \) Outside environment  

Superscripts  
h Horizontal opening  
s Surface  
v vertical opening

INTRODUCTION
The total worldwide energy demand is continuously increasing owing to the population growth, economic development and the social development. With the growth of the indicated determinants, building sector has become one of the largest energy consumers and it accounts for nearly 40% of the total global energy consumption [1]. Similarly, the building energy consumption in the EU is continuously expanding and it has also risen to 40% [2]. According to the statistics in 1999, space heating was the key contributor, which accounts for 68% of total household energy consumption in the EU [3]. Among the European nations, Scandinavian countries experience comparatively harsh climate conditions during about one third of the year. Accordingly, in a country like Norway, residential households and commercial buildings consume about 48% of the total energy production [4] mainly for space heating. Recent investigations in Norway, have showed that there is a feasibility of saving 65 TWh both from residential and commercial buildings by 2020 [4], and to achieve this goal Norwegian government authorities have imposed building technical regulations. They encourage the people to save energy using renewable sources and building energy management systems (BEMS).

BEMS are a subset of BAS and they monitor and control the energy of the buildings and building services as energy efficiently as possible while reducing the utility bill without compromising the comfort level of the occupants. These systems are a rapidly expanding field over the last two decades and they have gained the attention as a standard way of controlling the buildings with regard to the classical techniques such as thermostat control [5]. Currently, the most of the BEMS systems utilize on/off control, PID control or optimum start-stop routines as the control algorithm [5]. PID control is the most used technique in such systems [5]. However the use of classical control algorithms such as PID and on/off may not be the best to be combined with BEMS [6]. In buildings, thermal interaction between different zones and HVAC (Heating, Ventilation and Air-Conditioning) systems lead to multivariable behavior. Classical control techniques have some deficiencies in handling such systems. For example, classical controllers are easy to tune for SISO (Single Input - Single Output) systems and not easy or even impossible to tune for MIMO (Multiple Input - Multiple Output) systems. Advanced control techniques appear with a mathematical model of the building and have the potential to approach these constraints [5]. The required model in advanced control could be multivariable and consequently has a higher probability of delivering improved performance with fewer setpoint deviations and high energy savings when compared with the classical control. Therefore, it is essential to choose a good quality building model to produce a favorable outcome by BAS.

Building heating models can be categorized into three broad categories: (i) mechanistic models (white box or physical models); (ii) empirical or black box models; and (iii) grey box models ([7] - [11]). Mechanistic building models are developed based on the physical principles of mass, energy and momentum transfer. They consist of several equations with numerous coefficients to represent the building geometry and the thermal properties of the building. Large number of numerical software tools are available for solving such systems. However, there are problems associated with mechanistic models regarding the calibration of the physical parameters. Software tools like Energy Plus, TRNSYS, Modelica and Fluent provide comprehensive mechanistic models for building simulation. These models may have a very high accuracy, but they may have a high computational burden when applied to online control. Further, it may not be easy to calibrate these models with respect to the experimental data. Therefore, the selection of a mechanistic
model for a building is a balance between model complexity and the desired accuracy [7]. Application of physical principles to buildings and developing mechanistic type models are available in [12] - [19]. System identification based models, regression models [20], genetic algorithm [21], fuzzy logic models [22], neural network models [23], neuro-fuzzy models [24] and support vector machine [25] are some examples of black box models. These models are generated based on the data measured from a particular building such as inside and outside temperatures, relative humidities, wind speed, solar radiation and air flow rates. Accordingly, these models do not use any physical data of the building and hence the model coefficients do not have a physical meaning. Black box models may perform better than physics based models, but it will only work for a specific building where the data is measured. When the inputs are outside the modeled data range these models may give unrealistic and non-physical results. Grey box models [26] are a combination of both mechanistic and black box models and information about these models is partly known [11]. They are mostly used for parameter estimation and only a limited work has been done on them [11].

The present study focuses on the development of simple but comprehensive mechanistic heating model for multi zone buildings, which can also be applied in online control in BAS. The development is based on the single zone building model presented in [19]. There are a number of research articles that explain the modelling of multi-zone buildings using physical principles. [12] presents the development of a multi zone building model for MATLAB/SIMULINK environment implemented into the SIMBAD Building and HVAC Toolbox. Wall thermal mass is considered in the model and it is assumed to have constant thermo physical properties for each layer of the multilayered walls. A window model and solar radiation model are also included in [12]. In [15], a reduced order state space thermodynamic model is developed. Each zone is assumed to be well mixed and inter zone air mixing, air infiltration and solar radiation also modeled. A variety of multi zone building thermal modelling techniques can be found in [27], [17], [16] and [28]. However, the indicated multi zone building models lack either one or several features: (i) zonal mass balance; (ii) thermal mass of walls, floor and roof; (iii) thermal storage capability of building furniture; (iv) solar irradiation; and (v) occupancy. Hence, it is important to develop a reliable multi zone mechanistic building model that depicts all these effects.

The rest of the paper is organized to present a detailed overview of the multi zone model development, simulation to validate the proposed approach using real experimental data and finally some concluding remarks.

MODELLING APPROACH

In this section, a mechanistic dynamic heating model for a multi zone building unit is developed. The modeled building unit is presented in Figure 1. It is connected to four adjacent rooms, outside environment and ground. Heat is transferred from the main unit to the surroundings via walls, roof and floor. The mechanical ventilation system controls the air flow rate into and out of the building unit. There is a staircase to access the room above the building unit via the horizontal opening in the ceiling. Air is exchanged in between the adjacent rooms owing to the interactions caused by vertical and horizontal openings. A heater is installed inside the building unit to supply the energy for heating. Further, the other electrical appliances discharge their waste energy which can also increase the inside temperature. The furniture inside the unit may behave as a heat sink or heat source depending on the temperature difference between the furniture and the surroundings.

Figure 1: Configuration of the building unit
Application of mass balance for ventilated spaces is vital as ventilation plays a key role in convective mode of heat transfer. The mass balance equation for the indicated multi zone building unit can be expressed as in the equation 1.

\[
\frac{d\rho_i}{d\tau} = \frac{1}{V_i} \left[ m_{\alpha,i} \dot{h}_{\alpha,i} - m_{i,\alpha} \dot{h}_i + \sum m_{\beta,j,i} \dot{h}_{\beta,j,i} - \sum m_{\gamma,j,i} \dot{h}_{\gamma,j,i} - \sum m_{\delta,j,i} \dot{h}_{\delta,j,i} \right] \tag{1}
\]

Energy balance for the building unit is derived using the standard energy balance equation, the relation \( E = I + PV \), the relation \( dE = d(m\ddot{c}_p \theta) \) and the ideal gas law \( PV = nR\theta \).

\[
\frac{d\theta_i}{d\tau} = \left[ \frac{m_{\alpha,i} \dot{h}_{\alpha,i} - m_{i,\alpha} \dot{h}_i + \sum m_{\beta,j,i} \dot{h}_{\beta,j,i} - \sum m_{\gamma,j,i} \dot{h}_{\gamma,j,i} - \sum m_{\delta,j,i} \dot{h}_{\delta,j,i}}{\rho_i V_i (\ddot{c}_p_i - R/M_i)} \right] - \theta_i \frac{d\rho_i}{\rho_i d\tau} \tag{2}
\]

Modelling the heat transfer via the building envelope is essential in thermal modelling as its thermal mass has a significant contribution to the temperature fluctuations inside the building. Walls, ceilings, roof and floor usually consist of several layers of dissimilar materials such as wooden panels and insulation materials. In this study, all the layers are recognized as one element of constant thermal properties for simplicity of the model. Transient heat equation \( \left( \frac{\partial T}{\partial \tau} - \alpha \nabla^2 T - \frac{\dot{q}}{\rho c_p} = 0 \right) \) is discretized using the finite difference method to obtain the respective energy balance equations for the walls, floor and ceiling of the building unit based on the assumption of one-dimensional heat transfer. The deduced ordinary differential equations for heat transfer through walls, floor and ceiling are given by equations 3, 4 and 5 respectively.

\[
\frac{d\omega}{d\tau} = \alpha_w \left[ \alpha_{\omega}^f \frac{2\omega - \omega^f}{(\omega^f/2)^2} \right] + \frac{\dot{q}_w}{\rho w \ddot{c}_p w} \tag{3}
\]

\[
\frac{d\psi}{d\tau} = \alpha_f \left[ \psi_{\beta}^f - 2\psi - \psi_{\alpha}^f \right] + \frac{\dot{q}_f}{\rho f \ddot{c}_p f} \tag{4}
\]

\[
\frac{d\phi}{d\tau} = \alpha_c \left[ \phi_{\gamma}^c - 2\phi - \phi_{\alpha}^c \right] + \frac{\dot{q}_c}{\rho c \ddot{c}_p c} \tag{5}
\]

Equations 1 to 6 presents the ordinary differential equations describing the model for the multi zone building unit. The rest of this section shows the algebraic equations required to obtain the complete model.

It is necessary to evaluate the air mass flow rates via vertical and horizontal openings of the building unit to the neighboring zones. Figure 2 presents the air flow pattern through a vertical and a horizontal opening.

![Air flow through (a) Vertical and (b) Horizontal openings][29]

Many authors ([29], [30], [31] and [32]) have developed equations for air flow across a vertical opening considering a constant air density for each zone. Interested readers can refer to the above mentioned references to admit a relation for the mass flow rates \( (m_{\beta,j,i}, m_{\gamma,j,i}) \) addressed in the equation 1. The mass flow rate through horizontal openings could be either one way or two way depending on the pressure difference between the zones [29]. Consequently, to determine the direction of the flow, it is necessary to understand the pressures of each zone. Equation 7 is suggested by [33] to determine the air mass flow rate along a staircase. \( A_o \) and \( H_o \) are the area and thickness of the opening while \( C_d \) is the coefficient of discharge.

\[
\dot{m}^h = \rho A_o C_d \left[ \frac{\Delta \theta g H_o}{\theta} \right]^{0.5} \tag{7}
\]
The term $\dot{Q}_i$ in the equation 2 represents the net heat flow to the building unit, and it can be approximated using the equation 8. Heat losses through walls, floor, roof, windows and doors can be estimated using the equation 9 for each component.

$$\dot{Q}_i = \left[ \dot{Q}_{\text{heater}} + \dot{Q}_{\text{solar}} + \dot{Q}_{\text{appliances}} - \dot{Q}_{w} - \dot{Q}_{f} - \dot{Q}_{c} - \dot{Q}_{\text{window}} - \dot{Q}_{\text{door}} - \dot{Q}_{\text{fur}} \right]$$  \hspace{1cm} (8)

$$\dot{Q} = UA\Delta T$$  \hspace{1cm} (9)

THE TEST BUILDING
This section yields a description of the test building which is located in Norway. It is a three storeyed residential building located near Langesund and built in 1987.

![Sketches of the test building](image)

Figure 3: Sketches with the inner dimensions of the test building (a) second floor (attic) (b) first floor (main floor) (c) basement. All the dimensions are in cm.

The building inner dimensions, window and door dimensions are given in Figure 3. The three storeys are accessible via two inner staircases. The main floor and the attic are equipped with a mechanical ventilation system while the basement is not provided with mechanical ventilation. Total average air inflow rate into the building is 230 $m^3/h$. There is a heat exchanger installed across the ventilation system to heat the incoming air using the outgoing air. This heat recovery system has an efficiency of 90%. The exterior walls of the attic have a thickness of 30 cm (average) and the roof thickness is 30 cm. It has a volume of 93 $m^3$. Both attic and main floor are constructed using wood and mineral wool insulation. The furniture volume inside the second floor is estimated to be 3 $m^3$. There are no heaters fixed in this storey, but four personal computers are running all the time which supply around 700W.

The main floor has the same roof thickness similar to attic with a volume of 196 $m^3$. It’s wall thickness is 15 cm. It is filled up with 25 $m^3$ of furniture and 3200 W power is supplied for heating purposes. The electrical heaters are controlled by a simple BAS with a set temperature of 20$^0C$ when the building is occupied. In the simulations, the heater is precisely controlled by an On/Off controller having an operating band of $\pm 0.25^0C$, to maintain the temperature at 20$^0C$. In addition to the electrical heater, wood firing is used to heat the building only during the colder periods, which is not modelled in this article.

The thicker walls of the basement and it’s ground floor are built using concrete and the rest is wood. Outer wooden walls of the basement have a thickness of 20 cm and concrete walls have a thickness of 40 cm. The thickness of the ceiling is 30 cm, and the wall height is 235 cm. The total volume of the basement is 185.5 $m^3$ and the furniture accounts for 40 $m^3$. There are four heaters installed in the basement. Out of that, two are wall heaters (2x750 W), controlled by the same BAS in the first storey. The others are full time running floor heaters. The floor heaters have switches to turn them OFF or ON(1), ON(2) and ON(3). ON position 1 (160+390 W) is the lowest power usage and ON position 3 is the highest power usage. All the floor heaters are running at position 1 for most of the time. However, in reality, these heaters are manually brought to position 2 depending on the outside temperature.

The experiment of the test building is carried out for 79 days/1897 hours in the period 24 October 2013 - 10 January 2014. The locations where the temperatures and relative humidities are measured, are...
symbolized in the sketch (Figure 3). No measurements were collected from the attic of the building during the test period. To eliminate the outliers and the noise present in the data, they are smoothed using 30th order Savitzky-Golay filter. For the considered period, solar irradiation measurements are not available. Hence, it is roughly estimated using the instant outside temperatures.

A simplified form of the equation 7 has been used to determine the value of the convective air mass flow rates inside the building in between each storey. To define the air movement direction, it is essential to recognize the pressure and density fluctuations of each storey. However, pressure measurements are not logged in this experiment. Therefore, the pressure increment in each zone is calculated by analyzing the volume of air flowing into each storey through the ventilation system.

### RESULTS AND DISCUSSION

In this section, the performance of the developed multi-floor building model is analyzed for the selected test building after its implementation in MATLAB.

The thermal properties like thermal conductivity, specific heat capacity and density of the building materials are obtained from the literature, and they are used to calculate the thermal diffusivity of the building components. The overall heat transfer coefficients are determined using the experimental data. Parameter identification from test data has revealed that calibrating the parameters presented in the model to normal operating data from a building may lead to grossly inaccurate estimates. The predicted overall heat transfer coefficients, which can admit a favorable solution to the proposed criteria, and the computed thermal diffusivities are given in Table 1. It should be noted that only the thermal parameters observed in the model equations which are acknowledged to be significant are tabulated.

The predicted temperatures of the three storeys of the building are presented in the Figure 4. According to the figure, it can be noticed that the inside temperatures have a close relationship with the outside temperature fluctuations.

In the second storey, only the predicted temperature is shown because of the unavailability of sensor measurements. The temperature is wavering while maintaining 20°C, which is acceptable according to the residents’ feedback.

The first storey of the building maintains 20°C throughout the 79 days, which can be observed by the measured temperature profile. The predictions also produce a consistent 20°C for more than 90% of the time with the benefit of an ON/OFF temperature controller. Even though the measured temperature is restricted to 20°C, the predicted temperature is considerably lower close to day 30 and day 45. The low outside temperature predominating over the period is the reason for this scenario. However, in actuality the inside temperature is preserved at the set temperature by wood firing, which is not reflected in the simulation.

The deviations of the basement temperature profiles are proportionately higher compared to the first storey. The maximum divergence between the predicted temperature and the measured temperature approaches 4.5°C at day 47. The discrepancies could be owing to the action of floor heaters at ON position 2 during the cold periods.

### CONCLUSION

Mechanistic building heating models have speeded up the design, construction and operational activities of buildings and succeeded in establishing the new technologies in building operation. Hence, the identification of a proper model of the heat dynamics of a building based on frequent readings will be very useful in defining the energy performance of the building, forecasting the energy consumption and controlling the indoor environment.

Table 1: Model parameters of the test building

<table>
<thead>
<tr>
<th>Parameter</th>
<th>2nd Storey</th>
<th>1st Storey</th>
<th>Basement</th>
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<tr>
<td>$\alpha_{\text{w,wood}}$</td>
<td>$9.5 \times 10^{-7}$</td>
<td>$8.3 \times 10^{-7}$</td>
<td>$8.3 \times 10^{-7}$</td>
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<tr>
<td>$\alpha_{\text{w,concrete}}$</td>
<td>-</td>
<td>-</td>
<td>$4.9 \times 10^{-7}$</td>
</tr>
<tr>
<td>$\alpha_{f}$</td>
<td>-</td>
<td>-</td>
<td>$4.9 \times 10^{-7}$</td>
</tr>
<tr>
<td>$\alpha_{c}$</td>
<td>$1 \times 10^{-6}$</td>
<td>$1 \times 10^{-6}$</td>
<td>$1 \times 10^{-6}$</td>
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<tr>
<td>$\alpha_{\text{fur}}$</td>
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<td>$1.7 \times 10^{-7}$</td>
<td>$1.7 \times 10^{-7}$</td>
</tr>
<tr>
<td>$U_{\text{w,wood}}$</td>
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<td>$U_{\text{w,concrete}}$</td>
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<td>$U_{\text{doors}}$</td>
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<td>$U_{\text{windows}}$</td>
<td>1.2</td>
<td>1.2</td>
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</table>
Figure 4: Inside and outside temperature variations of the test building. No temperature measurement data is available for the attic. The main floor and the basement temperature predictions closely follow the measured temperature profiles.
In this paper, a lumped parameter model illustrating the long term heat dynamics of a residential building based on first principles is presented. The model also takes the thermal mass of the building envelope and vertical air mixing into consideration. Hence, it has been shown that the used methodology can provide rather detailed knowledge of the heat dynamics of the building. Moreover, the proposed criteria is simple, computationally attractive and requires limited input information. The developed model accomplishes the application independability and, therefore, the application of this methodology to a broad class of building types is straightforward.

The deficiencies met in the model validation are: requirement for more temperature sensors at representative positions in each floor; solar irradiation measurements; and pressure measurements. These deficiencies can be eliminated, and accurate thermal simulation can be achieved if sufficient and precise input data of the building is available.

Integration of the developed model with a BAS may help to optimize the usage of energy consumption. Further, it will help to achieve the temperature goal of each zone with less energy compared to using a time schedule to control the temperature.

REFERENCES


