

Short-term production planning for district heating networks with JModelica.org

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Abstract

The short term thermal production planning problem is solved in two steps by integrating physical plant models into the standard approach. The first step aims at solving the discrete variables from the unit commitment sub-problem (UCP) using standard mixed integer linear models and optimization techniques. The second step focuses on the economic dispatch sub-problem (EDP) described by high-fidelity, continuous time, physics-based Modelica models together with nonlinear optimization techniques from the JModelica.org platform. The output of the second step includes optimized power flows but also highly relevant variables such as supply temperature, supply flow rate, turbine by-pass valve in the cogeneration plant. The optimization is formulated as a maximization of the benefit from heat and electricity sell over a finite time-horizon.

The proposed method is validated in several test cases using experimental data from a plant in Nyköping. The optimizations demonstrate the feasibility and the high economic potential of the proposed approach when comparing with measurement data and the standard optimization techniques. The optimized planning schedules result in a balance between produced and consumed heat, priority to low-cost boilers and maximization plant revenue. Compared to measurement data, the optimizations result in a significantly lower supply temperature, a more extensive usage of the external cooler for higher efficiency and higher electricity production, fewer starts of units as well as an appropriate use of the accumulator tank.

The high-level description of optimization problems using JModelica.org provides useful means to speci-

fy flexible optimization problems including constraints on arbitrary process variables such as heat load of the production units, supply temperature and flow rate, pressures.

Keywords: production planning; nonlinear optimization; district heating; physical modeling; unit commitment

1 Introduction

1.1 Background

1.1.1 Production planning

Production planning in district heating systems aims at finding a cost optimal scheduling of the heat and power production plants, which satisfies both the network load demand and operational constraints. Scheduling refers to the status of the production unit (on-off, a discrete variable), and the produced power (a continuous variable). The resulting optimization problem that involves both discrete and continuous variables is referred to as mixed integer non-linear problem (MINLP), for which no robust algorithm is available. It is therefore necessary to make reasonable assumptions on both the modeling and the computation approaches to get a tractable optimization problem. The problem can be described as being composed of two sub-problems:

- The Unit Commitment Problem (UCP), in which decisions are taken on whether a plant should be running or not. The main difficulty lies in the combinatorial nature of the problem.
- The Economic Dispatch Problem (EDP) in which the load decisions for all active plants are taken. The main difficulty lies in the nonlinearity of the plant.

A good survey of the available approaches for short-term production planning is given in [15].

As an input to the production planning problem, a predicted heat load over the entire optimization horizon should be provided. This is often generated by a load prediction model that typically includes a description of the district heating network and the effect of outdoor temperature. Here it is assumed that a perfect load prediction is available over the entire optimization horizon.

1.1.2 Common approach

The standard formulation of the thermal production planning problems relies on a simplified representation of the model equations. The plant models are typically linearized and the resulting problem becomes a Mixed Integer Linear Program (MILP). The continuous decision variables of the optimization problem are the energy flows whereas the influence of the supply temperature and mass flow are usually not modeled. This represents a limitation since e.g., supply temperature affects many critical parameters such as the amount of energy that can be stored in the network or the accumulator, the heat loss in the network and the electric efficiency of the co-generation plants. To maintain a low model complexity, it is also common to describe all processes by static relationships except for the storage dynamics (heat and fuels) and eventually the transport delays in the distribution network. The linearization process, which is a necessary and critical part of the MILP approach, is consequently a trade-off between model accuracy and tractable model complexity.

1.2 Proposed approach

The proposed approach is based on the natural separation of the discrete problem (UCP) from the continuous one (EDP).

- UCP. The entire optimization problem is formulated using simple piecewise linear models and solved using a MILP solver. The main result of this stage is the status of every plant (on/off) over the optimization horizon.
- EDP. The desired load is dispatched between the running production plants to meet all plant operational and safety constraints. The status (on/off) of the plants and the start values of continuous control signals are given by the solution of the UCP.

The aim of the second step is to optimize the non-linear plant model based on physical laws without

any major simplification. The plants are described by mass and energy balances, in terms of enthalpy, mass flow rate and pressure. Dynamics can be included without restrictions to match the real dynamic behavior of the plants. The output of this second step includes optimized power flows, but also highly relevant variables that affect the production economy such as supply temperature, supply flow rate or turbine by-pass valve in the co-generation plant. This model complexity yields however a non-linear dynamic optimization problem and requires another type of solver than MILP solvers, see [1] for an overview of the available strategies. One reliable and efficient method to solve dynamic optimization problems that is based on non-linear programming solvers is the so-called collocation method. Control signals to be optimized and model equations are parameterized by a smaller number of variables, reducing considerably the complexity of the non-linear optimization problem. The original continuous-time optimization problem is transformed into a (discrete-time) Non-linear Programming (NLP) problem that can be efficiently solved using commercial or open-source solvers. The authors have applied the collocation method for dynamic optimization of a Carbon Capture plant, see [2]. Other successful applications of this optimization technique have been reported in the literature, see [3] for a list of applications where IPOPT (Interior Point Optimizer), an open-source NLP solver, was used. To the authors' knowledge, limited work on applying large-scale NLP methods for solving the economic dispatch problem has been performed.

2 Optimization tools and languages

Two environments were used to define and solve the production planning problem:

- Dymola [9] was the chosen platform to derive, calibrate and simulate the physical Modelica [10] models defining the economic dispatch sub-problem
- Python was the chosen platform to solve both optimization problems and to do the post-processing

The unit commitment problem (UCP) was formulated in Python using the package PuLP [4] that is a light weight package that allows modelers to easily express mathematical programming problems, including mixed integer linear programs. It uses a high level modeling language and has been built to interface with commercial and open-source solvers. In the present work the solver CBC has been used, see [5].

3.2.2 Co-generation plant P3

The produced heat Q_{P3} and electric power P_{el} are influenced by the boiler load, the position of the turbine by-pass valve as well as the flow and temperature of the district water. It is by far too complex to describe this relationship by simple (piecewise) linear functions. By using a physics based Modelica plant model and varying the four main variables, a feasible region in the Q_{P3} - P_{el} -plane has been created to describe the behavior of the cogeneration plant. The resulting feasible region was described by four inequalities of the form

$$a_i P_{el} + b_i Q_{P3} + c_i \geq 0$$

where the points a_i , b_i and c_i are scalar coefficients.

The UCP optimization should optimize Q_{P3} and P_{el} with the constraint that they should be in the feasible region described by the inequalities.

3.3 Models for EDP

The models used to formulate the economic dispatch problem are implemented in Modelica.

3.3.1 Medium model

Two different types of media models are used in the model package:

1. Simple water media – a water media with constant specific heat capacity and density, used to describe the water in the district heating network.
2. Advanced water media – a water media with polynomial functions approximating IF97 reference functions, used to describe liquid and vapor water in the cogeneration plant, see [17] for more details.

3.3.2 Cogeneration plant

The co-generation (heat and electricity production) unit P3, shown in Figure 2, is described by the following models

- One high pressure turbine and two low pressure turbines, all with bleed streams
- Two condensers
- By-pass valve for by-passing steam directly to the condensers
- A lumped bleed streams to describe the flow to the de-aerator and the high-pressure pre-heater
- Control volumes

The characteristics that are important to capture with respect to the optimization is the influence of the turbine by-pass valve, the district heat network flow and temperature and the boiler load on the produced heat and electricity. It is not necessary to describe e.g., the furnace, instead, focus has been directed towards the vapor cycle. The main modeling simplifications on the vapor cycle is that it is not closed, which results in the following assumptions:

- The vapor characteristics (pressure and enthalpy) at the boiler outlet are constant and the boiler load linearly affects the boiler mass flow rate.
- The feed water heaters that are downstream of the condensers are not modeled. All the bleed streams that normally go to the non-modeled pre-heaters are represented by a single stream connected to a constant pressure source.
- The condensate leaving the condenser is assumed to be at saturation pressure.

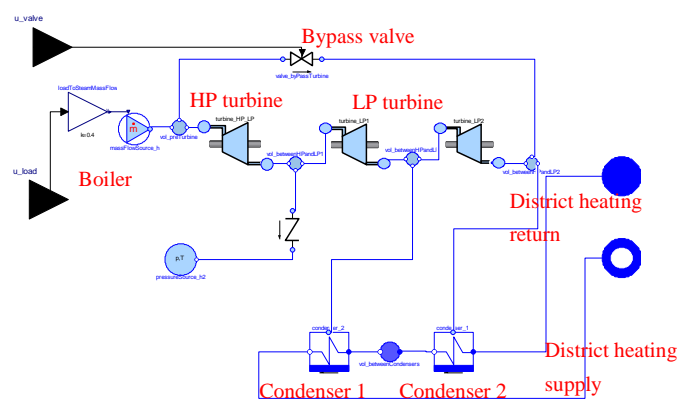
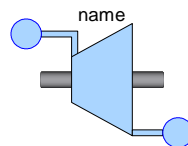


Figure 2. Dymola model of the cogeneration plant P3 with connections to the district water.

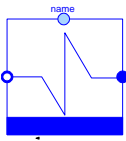
3.3.3 Fluid component models

3.3.3.1 Turbine

Physics-based model defined by an isentropic efficiency to calculate the outlet enthalpy and turbine work. The mechanical power generated from the steam is calculated using a mechanical efficiency and the pressure drop is related to the flow rate using Stodola's law. The electric power is calculated from the mechanical power using an efficiency parameter (generator losses).

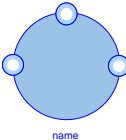


3.3.3.2 Condenser



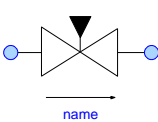
The heat flow rate transferred to the district water is driven by the temperature difference between the incoming water and the saturation temperature in the condenser. This heat flow rate is further used to compute the condensation rate that drives the bleeding flow from the turbine.

3.3.3.3 Control Volume



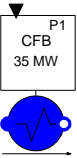
The control volume is a straightforward implementation of dynamic mass and energy balances expressed using pressure and enthalpy as states. Temperature is computed using pressure and enthalpy. The model requires partial derivatives of density with respect to enthalpy and pressure.

3.3.3.4 Valve and pressure loss



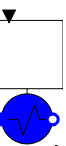
The mass flow through the valve is computed using the pressure difference, the valve opening and data from a nominal point. A standard quadratic equation relates mass flow and pressure drop.

3.3.3.5 Circulating Fluidized Beds, P1 and P2



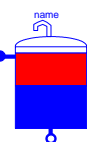
The model is not physics-based and heat transferred to the water is calculated by first-order filtering of the load and using parameters for efficiency and maximum heat transfer.

3.3.3.6 Electric Boiler and Oil Driven Boilers



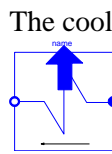
The model is not physics-based and heat transferred to the water is calculated by the load and using parameters for efficiency and maximum heat transfer.

3.3.3.7 Accumulator



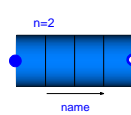
The accumulator is modeled using a finite-volume approximation that neglects buoyancy effects, i.e. no mixing is assumed when the accumulator is not charging or discharging. The accumulator is charged and discharged from the top and bottom. Return water enters from the bottom. Heat loss has been neglected.

3.3.3.8 External cooler



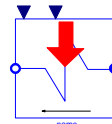
The cooler is modeled as a lumped epsilon-NTU heat exchanger where heat transfer is driven by the difference between the inlet temperatures. It is assumed that the minimal heat capacity flow is always on the district side flow.

3.3.3.9 Transport Pipe



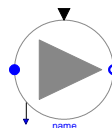
It is a finite volume implementation of a pipe with control volumes in series. The nominal number of discretization segments is 4 and the pipe diameter and length are parameters.

3.3.3.10 Flue Gas Condenser



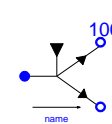
The model is not physics-based and is an ideal heat source which produces constant heat as long as P3 is running.

3.3.3.11 Pump



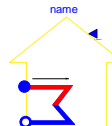
The pumps are ideally modeled, delivering a specific mass flow rate depending linearly on the control input, i.e., pump speed.

3.3.3.12 Splitter/Valve



The splitter/valve is ideally modeled and splits the incoming flow into two flows which sizes depend on the control signal to the splitter.

3.3.3.13 Customer



The customer is modelled with a fixed outlet temperature and the heat absorbed is calculated by the temperature difference between in- and outlet and the flow that is merely propagated through from in- to outlet.

4 Problem formulation

4.1 Degrees of Freedom

The discrete time optimization (UCP) includes the power heat flows, the electricity production rate as well as the status (on/off) of every unit as optimiza-

tion variables. The status of each unit is fixed in the continuous optimization.

The continuous time optimization problem contains 11 degrees of freedom, namely:

- $U_i(t)$ fuel load of production unit i , $1 \leq i \leq 6$
- $U_{BP}(t)$, turbine by-pass valve position
- $U_{CP}(t)$, Speed of circulation pump
- $U_{DP}(t)$, Speed of distribution pump
- $U_{split}(t)$, ratio between flows going to customer and by-passing customer respectively
- $U_{cooler}(t)$, ratio between flows going to external cooler (Beriden) and by-passing it

In the optimizations however, the decision variables used by the optimization routine will be the derivatives of the above inputs. Thus, an equation on the form

$$U_j(t) = \int_t \dot{U}_j(t) dt$$

is introduced for each input. This extension makes it easy to set minimum and maximum constraints on the input signal derivatives in the optimizations. In Modelica code, for a general input u with minimum and maximum values u_{min} and u_{max} and minimum and maximum derivative values \dot{u}_{min} and \dot{u}_{max} , an example is the following:

```
model inputExample
  parameter Real u_min = -1;
  parameter Real u_max = 1;
  parameter Real u_der_min = -1;
  parameter Real u_der_max = 1;
  input Real u_der(min = u_der_min,
max = u_der_max);
  Real u(min = u_min, max = u_max);
equation
  der(u) = u_der;
end inputExample.
```

Another possibility is to utilize the constraint-section in the optimization model.

4.2 Cost function

The goal of the optimization, both in the UCP and EDP, is to produce enough heat to follow the customer heat load over time and at the same time do it as economically beneficial as possible. The EDP problem considers fuel costs $U_i(t)p_i$ and incomes from selling heat $P_{el}(t)p_{el}$ and electricity $Q(t)p_Q$ to describe the plant economy. Note that operational costs for e.g., pumps are not considered. The cost for

starting a production unit $start_i[t]s_i$ is only included in the objective function of the UCP sub-problem. Hence, the revenue at a time instant can thus be formulated as

$$R(t) = P_{el}(t)p_{el} + Q(t)p_Q - \sum_i U_i(t)p_i - start_i(t)s_i,$$

A minor cost on the input derivatives must be used for regularity reasons. This cost at a certain time instant is formulated as

$$\dot{W}(t) = \sum_{j=All\ model\ inputs} q_{\dot{U}_j} \dot{U}_j(t).$$

where $q_{\dot{U}_j}$ is the weight for derivative \dot{U}_j .

The cost function to be minimized, considering the cumulative revenue over the optimization horizon, can thus be formulated as

$$J = \int_{t_0}^{t_f} (\dot{W}(t) - R(t)) dt$$

where the optimization interval is $[t_0, t_f]$ and 24 h long. This is as long as the optimization horizon in the UCP and also as the prediction time series provided by the heat load prediction model. Without the extra term of the input derivatives, this cost function is the continuous time counter part of the cost function in the UCP except for the start-up costs.

4.3 Constraints

4.3.1 Common constraints for EDP/UCP

For fulfillment of the heat demand from the customers, the following constraint is used in both EDP and UCP:

$$-Q_{dev.max} \leq Q(t) - Q_d(t) \leq Q_{dev.max}$$

where Q is the heat delivered to the customers, Q_d the desired heat load from the prediction model. The maximum deviation $Q_{dev.max}$ is set to 1 MW. The supply temperature is not a direct function of the outdoor temperature, which is common practice. Instead, the supply temperature depends on the supply flow and heat demand, where the latter is a prediction depending on predicted outdoor temperature. A lower bound on the supply temperature has also been introduced and this could be replaced by a lower bound on the temperature at the customer substation if the district heating network is modelled.

An accumulator energy end-point constraint is introduced in both EDP and UCP to avoid that the accumulator gets empty at the end of the optimization interval:

$$E_{acc}(t_f) \geq E_{acc}(t_0)$$

The input derivatives, i.e., the decision variables for the optimizer, are given minimum and maximum values. A maximum change of $\pm 2\%/min.$ has been set for all input derivatives relative to the maximum value of the input to the model. These limits can be set both from a physical perspective but also from a numerical perspective such that highly changing control signals are avoided. In the optimization results presented in this report, the derivative constraints are far from being active.

4.3.2 UCP specific constraints

The start and stop of large solid fuel boilers is time-consuming and it is therefore critical to model the delay between the start/stop decision and the time the boiler is running at minimum/zero load. All units that are starting or stopping are required to follow a pre-defined trajectory $Q_{i,start}[t]$ or $Q_{i,stop}[t]$, respectively. The constraints are formulated as

$$\begin{aligned} Q_i[t] &= Q_{i,start}[t], \\ t &\in [t_{start}, t_{start} + t_{startdelay}] \\ Q_i[t] &= Q_{i,stop}[t], \\ t &\in [t_{stop}, t_{stop} + t_{stopdelay}] \end{aligned}$$

where $t_{startdelay}$ and $t_{stopdelay}$ are the lengths of start and stop sequences in time. The details on how the timing variables $start_i[t]$, $stop_i[t]$, $starting_i[t]$ and $stopping_i[t]$ relate to the starting and stopping sequence equations above can be found in [11].

The accumulator storage capacity and heat flow are constrained with minimum and maximum values as

$$\begin{aligned} E_{acc,min} &\leq E_{acc}[t] \leq E_{acc,max} \\ Q_{acc,min} &\leq Q_{acc}[t] \leq Q_{acc,max} \end{aligned}$$

4.3.3 EDP specific constraint

Normally, the supply temperature is chosen as a function of the outdoor temperature. In the continuous optimization, the supply temperature is optimized and allowed to vary between 74.5°C and 110°C . Also the flow to the customers, essentially the distribution pump flow, is given minimum and maximum values. These are 0 kg/s and 550 kg/s .

5 Optimization example

Different test cases based on measurement data from Idbäcken plant were considered to evaluate the production planning strategy. All optimizations, both UCP and EDP, are performed using an optimization horizon of 24 h. The UCP sampling time is 0.5 h while the number of elements in the collocation scheme is 72, i.e., the length of each element is 20 minutes. One test case with two load peaks is presented in the paper.

5.1 Initialization

All dynamic models were initialized using experimental data of the real plant. The variables of the UCP model that require initialization are the status of each unit, the heat and electricity production of the co-generation plant as well as the accumulator energy. The physical model of the plant used to formulate the EDP sub-problem contains much more states to be initialized. The start values were either directly taken from the measurement data or computed using an FMU simulation of the plant.

5.2 Computational statistics

In all optimization cases, the UCP optimization formulation results in a MILP of approximately 4000 decision variables and 7000 constraints. The sampling time considered is 30 minutes and the optimization horizon is 24 h. With the CBC MILP solver, see [5], the solution time is less than 25 s for all optimization cases.

The resulting NLP, after discretization of the EDP optimization problem using 72 elements each of a length 20 minutes, is solved using IPOPT v3.10.0 running with the linear solver MA27, see [12] and [13]. The NLP contains approximately 130 000 variables for each of the considered optimization cases, which can be considered a small to medium sized problem for IPOPT. Much larger problems have been solved using the JModelica.org framework and IPOPT, see [14] and references therein. The solution time is approximately 2-5 minutes and depends on e.g., initial guesses number of decision variables and number of active constraints.

5.3 Specific test case

5.3.1 Experimental data

In this test case, the co-generation plant P3 is the only running unit at the beginning of the considered

time interval. The heat load profile showed in the figure below displays two peaks that require the start of additional units. To meet that increasing customer demand, the heat production in the cogeneration plant was first prioritized by fully opening the by-pass valve before both P1 and P2 were started, see dotted curves in Figures 4 and 5.

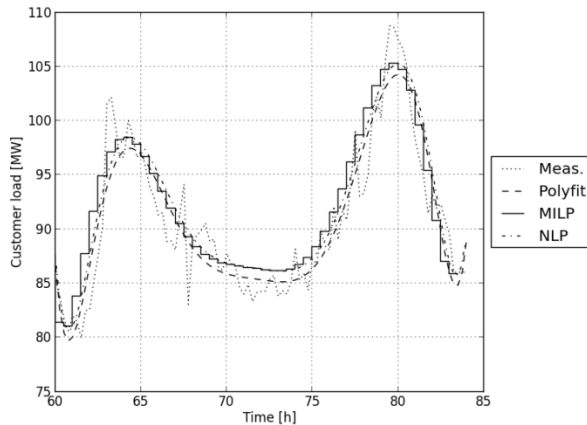


Figure 3. Profile of the customer heat load: measurement, polynomial approximation, MILP and NLP solutions.

5.3.2 Optimization results

The solution of the unit commitment problem is characterized by the start of P2 at the first sample to meet the increasing customer demand. The start of P1 does not seem to be necessary. As the first peak of the customer heat demand comes before P2 is fully available, the accumulator is used and its energy level is significantly decreased during the first half of the optimization interval. P2 stays at its minimum capacity during the first 12h, a stop and re-start would be a more expensive alternative. During the second half of the optimization, P2 is used to load the accumulator and provide heat to the network. The optimization leads to a constant and maximum heat electricity production as it is economically beneficial. The turbine by-pass valve is therefore kept closed during the entire optimization interval. The variation in the customer load is met by changing the load in P2 and by using the accumulator. Figure 6 shows one typical advantage of applying nonlinear optimization for the economic dispatch: the supply temperature and flow are optimized to maximize the benefits. Note that none of these variables are in the UCP formulation as the UCP only contains heat and energy variables. For minimal fuel consumption, the supply temperature is kept low, about 5 degrees lower than in measurement data, the flow rate to the

network is instead increased to meet the heat power demand. Note that the distribution pump operates close to its maximal capacity except when the accumulator needs to be charged. Accumulator loading requires indeed a difference in the flows delivered by the circulation and the distribution pumps. At $t=70\text{h}$, the circulation pump operates at its maximum and the distribution pump is therefore forced to be decreased for accumulator loading.

5.4 Conclusion from other test cases

The production planning strategy has been successfully tested and compared with experimental data in 6 different cases, see [18] for more details. The following conclusions can be drawn:

- It is fully possible to integrate nonlinear optimization techniques from JModelica.org into the standard production planning approach for more accurate and more informative production plans.
- One of the main advantage of combining physical modeling and nonlinear optimization techniques is the optimized supply temperature and mass flow rate as well as a more accurate description of the accumulator.
- In the considered network, the optimization showed a high potential income related to the use of the external cooler that reduces the return temperature for a higher electric efficiency and a higher electricity production. This only occurs at low customer loads, i.e., when P3 is not required to run on full load. If customer load is high, then P3 should already be running at full load and the electricity production maximized.
- Handling of constraints on physical variables such as temperature, pump capacity and also constraints related to the way the accumulator is connected, improves substantially the quality of the optimization results.
- Supply temperature can be decreased by approximately 4°C compared to experimental data. With a perfect load prediction, the savings compared to measurement data is about 8%, which is related to the extensive use of the external cooler.

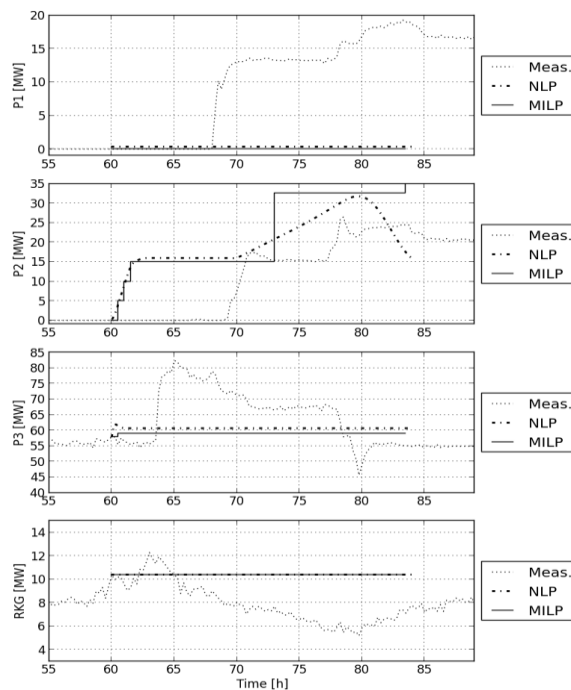


Figure 4. Heat production of the main units: measurement (dotted), MILP (solid) and NLP (dashed).

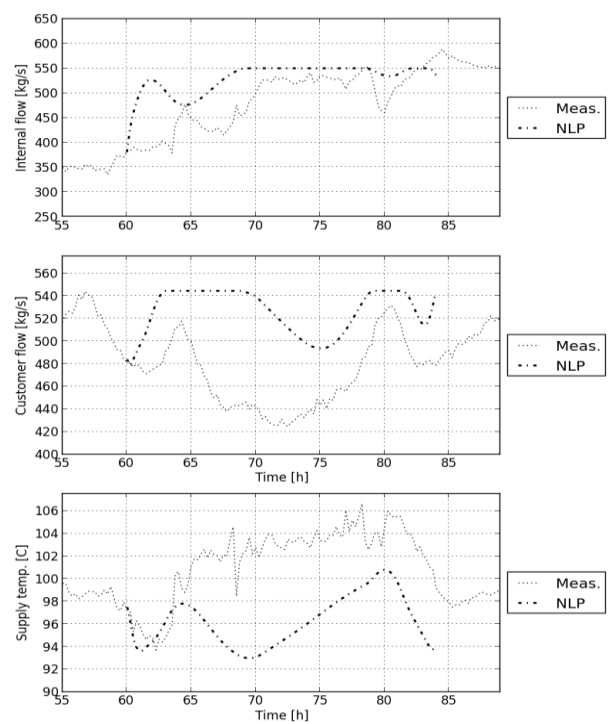


Figure 6. Circulation flow, flow to network and supply temperature. Measurement (dotted) and NLP (dashed).

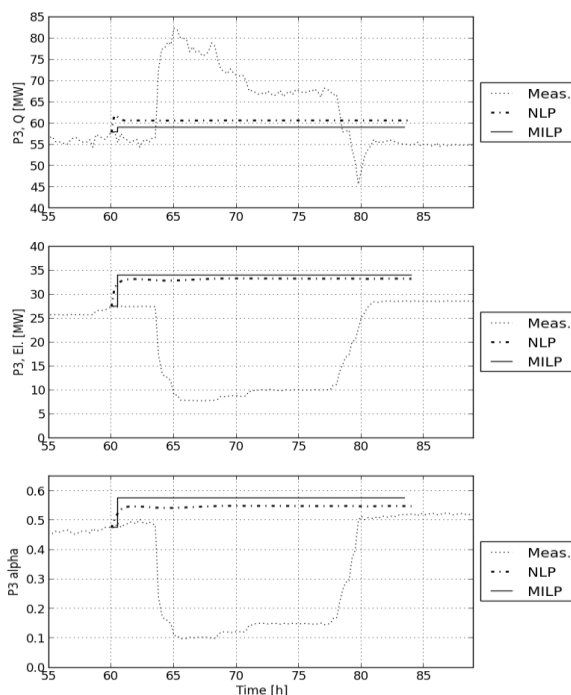


Figure 5. Heat, electricity production rates and alpha-value of the cogeneration plant P3.

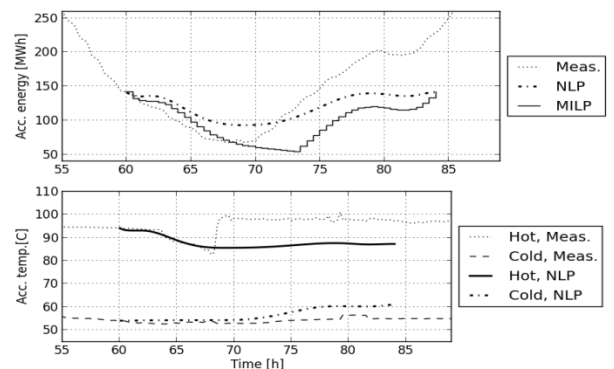


Figure 7. Accumulator energy and temperatures.

6 Conclusions

A method that substantially improves the standard approach for short-term production planning has been proposed in the paper. It is based on the use of physical plant models and nonlinear optimization techniques from JModelica.org to solve the economic dispatch sub-problem. The strategy has been validated using experimental data from a plant in Nyköping.

ping Sweden. Compared to measurement data, the method results in a significantly lower supply temperature, a more extensive usage of the external cooler for higher efficiency and higher electricity production. The flexible optimization platform JModelica.org makes it possible to optimize the plant economy and introduce constraints on critical variables such as temperature, pressure or flow.

7 Acknowledgement

Värmeforsk, "The Swedish Thermal Engineering Research Institute" and Energimyndigheten "The Swedish Energy Agency" are gratefully acknowledged for their financial support (project P12-203).

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