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A Comparison of Volumetric Illumination Methods by Considering their Underlying Mathematical Models

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Abstract

In this paper, we study and analyze seven state-of-the-art volumetric illumination methods, in order to determine their differences with respect to the underlying theoretical mathematical models and numerical problems potentially arising during implementation. The chosen models are half angle slicing, directional occlusion shading, multidirectional occlusion shading, shadow volume propagation, spherical harmonic lighting, dynamic ambient occlusion and progressive photon mapping. We put these models into a unified mathematical framework, which allows them to be compared among each other as well as to the global volume rendering equation. We will discuss the mathematical differences of the compared models and describe the numerical implications of the simplifications made by each method.

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1. Introduction

Volumetric data sets are acquired in many scientific disciplines, and direct volume rendering (DVR) is frequently used to transform them into images. In recent years, several interactive volumetric illumination methods have been proposed in order to improve the perceptual qualities of DVR images. All of these methods are based on an optical model which maps data values in a volumetric data set to optical properties, such as color and opacity. Thus, these models describe how light particles inside the volume are generated, reflected or scattered, and simulation of multiple scattering, volumetric shadows and color bleeding are some of the effects that can be created by these advanced illumination methods.

In this paper, we analyze the underlying mathematical models of seven state-of-the-art volumetric illumination methods. We put all these models into a unified mathematical framework and compare them with respect to their mathematical simplifications, which are usually made in order to decrease rendering times. This enables us to compare the features of volumetric illumination methods by comparing their mathematical components to a reference model that includes all illumination features.

The illumination models that have been selected to be analyzed in this paper are half angle slicing [KPHE02], directional occlusion shading [SHB*08], multidirectional occlusion shading [SPBV10],shadow volume propagation [RDRS10], spherical harmonic lighting [KJL*12], dynamic ambient occlusion [RMSD*08] and progressive photon mapping [HOJ08]. These methods are selected based on their novelty, the number of citations as well as actual spread in real-world applications [LR11].

The paper is structured as follows. In the next section we will review work related to our approach. In Section 3 we will go through the volume rendering integral and introduce the unified mathematical framework that will be used to describe the mathematical models of the selected DVR techniques. In Section 4, we discuss the details on how to describe the selected illumination models within the unified mathematical framework. Section 5 contains a discussion of the different equations (models), the difference between them and what impact the mathematical simplifications have on the possible illumination effects. Finally, Section 6 concludes by summarizing the results.

2. Related Work

In recent years several interactive volumetric illumination models have been proposed [JSYR13]. In this work we have chosen to study seven models that are representatives from each group of direct volume rendering techniques. We refer to [JSYR13, KFC*10, GJJD09] for a comprehensive overview of the methods and here mainly focus on related work in other areas.

A perceptual comparison of the images generated by many of the methods used in this work has been analyzed by Lindemann et al. [LR11]. Other perceptual studies include the work of Wanger et al. [WFG92], which investigated spatial relations in images and concluded that, for instance, shadows and perspective were important cues. Hubona et al. [HWSB99] studied the perception of relative position and size of objects in 3D space and found that, for instance, the use of shadows increase the accuracy of object positioning, but not the speed.

Some of the work not included in this comparison are for instance Vicinity Shading proposed by Stewart et.al. [Ste03], which simulates illumination of isosurfaces by taking into account neighboring voxels. Also in the area of computing occlusion are Penner and Mitchell [PM08] and Hernel et al. [HLY07]. The formed proposed a technique to compute the visibility around a voxel based on histograms and the latter presented a technique that computes local visibility of each voxel by integrating the opacity of each voxel in its surrounding sphere. None of the techniques presented here utilizes splatting, which for instance Zhang and Crawfis [ZC02] exploit to create shadows in volume rendering.

3. Volume Rendering Equation

To be able to describe each of the analyzed model within a unified mathematical framework, we first need to clarify the notation for the volume rendering equation considering global illumination. The following mathematical derivation of the volume rendering equation builds upon the work of Jensen and Jarosz et al. [JC98, Jar08], as it is a widely accepted notation for describing optical models. Light that travels through a participating medium is affected by emission, in-scattering, absorption and out-scattering. By taking these four terms into account, the change in radiance in direction $\vec{\omega}_o$ at the point x can be described by:

$$(\vec{\omega}_o.\bigtriangledown)L(x,\vec{\omega}_o) = -\sigma_a(x)L(x,\vec{\omega}_o) - \sigma_s(x)L(x,\vec{\omega}_o) + \sigma_a(x)L_e(x,\vec{\omega}_o) + \sigma_s(x)L_i(x,\vec{\omega}_o). \quad (1)$$

Here $\vec{\omega}_o$ is the viewing direction, $\sigma_a(x)$ and $\sigma_s(x)$ are the absorption and scattering coefficients, $L_e(x,\vec{\omega}_o)$ is the emitted radiance and $L_i(x,\vec{\omega}_o)$ is the in-scattered radiance. Integrating both sides of the Equation 1 along a straight path gives us the following integral:

$$\begin{aligned} L(x,\vec{\omega}_o) = {} & L(x_0,\vec{\omega}_o)\tau(x_0,x) \\ & + \int_{x_0}^{x} \tau(x',x)\sigma_a(x')L_e(x',\vec{\omega}_o)\,\mathrm{d}x' \\ & + \int_{x_0}^{x} \tau(x',x)\sigma_s(x')L_i(x',\vec{\omega}_o)\mathrm{d}x'. \end{aligned} \quad (2)$$

$$L_i(x',\vec{\omega}_o) = \int_{\Omega} f(x',\vec{\omega}_i,\vec{\omega}_o)L(x',\vec{\omega}_i)\mathrm{d}\omega_i,$$

where $L(x,\vec{\omega}_o)$ is the radiance scattered from point x inside the volume in direction $\vec{\omega}_o$ (the viewing direction), x_0 is the point behind the medium (the end point of the viewing direction), and $\vec{\omega}_i$ is the incoming direction (towards the point). $\tau(x,x')$ is the transmittance, which gives us the fraction of incident light that can travel unobstructed between two points x and x' in the medium along the straight line, and can be computed by the following equation:

$$\tau(x,x') = e^{-\int_{x'}^{x}\kappa(t)\mathrm{d}t}, \quad (3)$$

where $\kappa(x) = \sigma_a(x) + \sigma_s(x)$ is the extinction coefficient. The term $\int_{x'}^{x}\kappa(t)\mathrm{d}t$ is called optical thickness or optical depth.

Equation 2, which is the integral form of the radiative transfer equation, is called the volume rendering equation. The first term of the equation is the reduced surface radiance, the second term is the accumulated emitted radiance and the last term represents the accumulated in-scattered radiance inside the medium. We use the notation derived in this section to integrate the selected volumetric illumination models into a unified mathematical framework.

4. Unified Mathematical Framework

In this section the derivation of mathematical models for each of the seven chosen direct volume rendering techniques will be discussed in detail.

4.1. Half Angle Slicing

Half angle slicing is a slice-based approach introduced by Kniss et al. [KPHE02]. This model captures the appearance of translucency by approximating multiple scattering. Scattering effects can be fully captured using Equation 2 when taking the incoming light from all directions into account. However, this is generally expensive and the Half Angle Slicing method therefore makes an approximation by assuming that the light only scatters in forward direction. More specifically, the light is assumed to propagate within a cone in the direction of the light source only, instead of considering the incoming light from all directions.

The model can be expressed using the following equation:

$$L(x,\vec{\omega}_o) = L(x_0,\vec{\omega}_o)\tau(x_0,x) + \int_{x_0}^{x} \tau(x',x)\sigma_s(x')L_i(x',\vec{\omega}_o)\mathrm{d}x'. \quad (4)$$
$$L_i(x',\vec{\omega}_o) = \int_{\Omega} f(x',\vec{\omega}_i,\vec{\omega}_o)L(x',\vec{\omega}_i)\mathrm{d}\vec{\omega}_i.$$

We can imediately see that the emission term is removed when comparing it to Equation 2. The change that enforces forward scattering is less obvious and comes from limiting the evaluation of $L(x',\vec{\omega}_i)$ to only consider incoming light directions within a cone with an apex angle θ in the direction of the light source, which in turn is enabled by assuming a cone phase function:

$$f(x',\vec{\omega}_i,\vec{\omega}_o) = \begin{cases} \frac{1}{2\pi\cdot(1-\cos(\theta))} & \text{if } \vec{\omega}_i\cdot\vec{\omega}_l < cos(\theta) \\ 0 & \text{otherwise.} \end{cases}, \quad (5)$$

where w_l is the light direction.

4.2. Directional Occlusion Shading

Directional occlusion shading is a method that is able to create soft shadow effects of ambient occlusion. Similar to Half Angle Slicing, a specialized phase function is used to derive an occlusion factor. This phase function is known as a backward-peaked cone phase function of user specified aperture angle.

This model can be described by the following equation:

$$L(x,\vec{\omega}_o) = L(x_0,\vec{\omega}_o)\tau(x,x_0) + \int_{x_0}^{x} \tau(x,x')\sigma_s(x')L_i(x',\vec{\omega}_o)\mathrm{d}x'. \quad (6)$$
$$L_i(x',\vec{\omega}_o) = L_b \int_{\Omega} \tau(x',x_0')f(x',\vec{\omega}_i,\vec{\omega}_o)\mathrm{d}\vec{\omega}_i.$$

When comparing the above equation to Equation 2 we can see that the emission term is dropped and a term $L_b = 1$ is introduced, which represents the constant background intensity. Ω is also limited to the angles covered by the backward-peaked cone phase function, expressed using the following equation:

$$f(x',\vec{\omega}_i,\vec{\omega}_o) = \begin{cases} \frac{1}{2\pi\cdot(1-\cos(\theta))} & \text{if } \vec{\omega}_i\cdot\vec{\omega}_o < cos(\theta) \\ 0 & \text{otherwise.} \end{cases}, \quad (7)$$

4.3. Multidirectional Occlusion Shading

This method extends the directional occlusion shading technique allowing the light source to be placed anywhere within the hemisphere defined by the view vector. Even though it in practice and implementation wise is different from directional occlusion shading they are actually theoretically equivalent in our formulation. Thus, Equation 6 is also used to describe the light propagation together with the same cone shaped phase function in Equation 5.

4.4. Shadow Volume Propagation

Shadow volume propagation is an illumination model that supports scattering and shadowing effects. However, in order to have hard shadow borders that results in improvement of depth perception, the shadow computation is decoupled from scattering [RDRS10]. In addition, similar to Kniss et al. [KPHE02] the in scattered radiance is calculated by blurring the incoming light within a given cone centered about the incoming light direction instead of considering the scattering of light coming from all directions on the unit sphere.

This model can be specified using the following equation:

$$L(x,\vec{\omega}_o) = L(x_0,\vec{\omega}_o)\tau(x,x_0) + \int_{x_0}^{x} \tau(x',x)L_e(x',\vec{\omega}_o)\mathrm{d}x' + \int_{x_0}^{x} \tau(x',x)L_i(x',\vec{\omega}_o)\mathrm{d}x'.$$
$$L_i(x',\vec{\omega}_o) = \lambda(L(x',\vec{\omega}_l)) \int_{\Omega} f(x',\vec{\omega}_i,\vec{\omega}_o)\gamma(L(x',\vec{\omega}_i))\mathrm{d}\vec{\omega}_i, \quad (8)$$

where $\gamma(L(x',\vec{\omega}_i))$ and $\lambda(L(x',\vec{\omega}_l))$ are the functions that return the chromaticity and luminance, respectively.

As mentioned earlier, in order to generate hard shadow borders, the blurring does not apply to luminance part of the incoming light direction. In order to blur chromaticity, they use a strongly forward-peaked phase function:

$$f(x',\vec{\omega}_i,\vec{\omega}_o) = \begin{cases} C.(\vec{\omega}_i.\vec{\omega}_o)^{\beta} & \text{if } \vec{\omega}_i.\vec{\omega}_o < \cos(\theta); \\ 0 & \text{otherwise.} \end{cases}$$

The phase function is a Phong lobe whose extent is controlled by the exponent β and restricted to the cone angle θ which is used to control the amount of scattering. The constant C is chosen with respect to β.

4.5. Spherical Harmonic Lighting

Using several light sources and moving them can be challenging with respect to performance in DVR. Spherical harmonic lighting techniques allow different types of dynamic light sources to be used without increasing the computational burden much. Real time performance is achieved by decoupling visibility information and the light information.

This illumination model can be expressed using the following equation:

$$\begin{aligned}
L(x,\vec{\omega}_o) &= L(x_0,\vec{\omega}_o)\tau(x_0,x) \\
&+ \int_{x_0}^{x} \tau(x',x)L_e(x',\vec{\omega}_o)\mathrm{d}x' \\
&+ \int_{x_0}^{x} \tau(x',x)L_i(x',\vec{\omega}_o)\mathrm{d}x'. \\
L_i(x',\vec{\omega}_o) &= \int_{\Omega} f(x',\vec{\omega}_i,\vec{\omega}_o)L_a(x_0,\vec{\omega}_i)V_R(x',-\vec{\omega}_i)\mathrm{d}\vec{\omega}_i. \\
V_R(x',\vec{\omega}_i) &= \tau(x',x'+R\vec{\omega}_i).
\end{aligned} \tag{9}$$

Here, R is the distance from x' in direction of $\vec{\omega}_i$, and a spherical harmonic basis function is used to store visibility information, $V_R(x',\vec{\omega}_i)$, and the radiance distribution $L_a(x_0,\vec{\omega}_i)$. $V_R(x',\vec{\omega}_i)$ estimates the local visibility and shows how much of the incoming light reaches the point x' in direction $\vec{\omega}_i$. Efficient rotation in the spherical harmonic basis is enabled by assuming that an isotropic phase function:

$$f(x',\vec{\omega}_i,\vec{\omega}_o) = \frac{1}{4\pi}.$$

Spherical harmonics (SH) are orthonormal basis defined over the unit sphere S^2, where the 2D domain can be seen as the set of all possible directions. They allow both the visibility and lighting to be projected into spherical harmonic basis functions. In this case the original functions can be approximated as:

$$\begin{aligned}
L_a(x_0,\vec{\omega}_i) &= \sum_{l=0}^{n}\sum_{m=-l}^{l} L_{l,m}Y_l^m(\vec{\omega}_i) \\
V_R(x',\vec{\omega}_i) &= \sum_{l=0}^{n}\sum_{m=-l}^{l} V_{l,m}Y_l^m(\vec{\omega}_i),
\end{aligned} \tag{10}$$

where Y_l^m are the spherical harmonics basis functions by the degree l and order m with $l \geq 0$ and $-l \leq m \leq -l$. The coefficients are calculated using the following equation:

$$\begin{aligned}
L_{l,m} &= \int_S LY_l^m(\vec{\omega})\mathrm{d}\vec{\omega} \\
V_{l,m} &= \int_S VY_l^m(\vec{\omega})\mathrm{d}\vec{\omega}.
\end{aligned} \tag{11}$$

In order to find a function approximation of a finite number of coefficients the outer sum in 10 is truncated. For interactive purposes no more than $n_l \leq 3$ is being used, resulting in 16 coefficients. Orthonormality of the SH basis function provides us with efficient integral evaluation of in-scattered radiance (L_i) in equation 9. We can compute the integral using truncated SH expansions $L_a(x_0,\vec{\omega}_i) \approx L(x_0,\vec{\omega}_i)$ and $V_R(x_0,\vec{\omega}_i) \approx V(x_0,\vec{\omega}_i)$ as:

$$L_i(x',\vec{\omega}_o) = \frac{1}{4\pi}\sum_{l=0}^{n}\sum_{m=-l}^{l} L_{l,m}V_{l,m} \tag{12}$$

The implementation of the method uses a pre-processing stage, where local visibility is projected into the spherical harmonics space and then global visibility spherical harmonics expansions are computed by integrating the stored local visibility. The lighting environment is projected into spherical harmonic space as well. Equation 12 is evaluated at each step along the ray during ray casting in order to compute the radiance reaching the eye.

4.6. Dynamic Ambient Occlusion

Although the naming of this technique implies support for ambient occlusion only, it also provides an approximation of color bleeding.

The mathematical model can be described as follows:

$$\begin{aligned}
L(x,\vec{\omega}_o) &= L(x_0,\vec{\omega}_o)\tau(x_0,x) \\
&+ \int_{x_0}^{x} \tau(x',x)L_i(x',\vec{\omega}_o)\mathrm{d}x' \\
L_i(x',\vec{\omega}_o) &= \int_{\Omega}\int_{x'}^{(x'+R\vec{\omega}_i)} |x'-x''|\sigma_a(x'')\mathrm{d}x''\mathrm{d}\vec{\omega}_i.
\end{aligned} \tag{13}$$

In order to find the in-scattered radiance L_i for a the position x', the scattering of light coming from all directions on the sphere with radius R is considered. L_i is approximated numerically by integrating the occlusion over all the voxels lying in a distance R from x', weighted based on their distance and their absorption coefficient.

4.7. Progressive Photon Mapping

Progressive photon mapping (PPM) is a volume rendering technique proposed by Hachisuka et al. [HOJ08] and later reformulated by Knaus et al. [KZ11]. PPM naturally inherits the properties of standard photon mapping by Jensen [Jen96] and is thus capable of rendering complex illumination in participating media and in practice can be used to create unbiased renderings. Thus, Equation 2 can be solved without any simplifications, although the emission term is usually not solved using photon mapping.

The progressive photon mapping formulation by Knaus et al. [KZ11] is based on a probabilistic analysis of the error stemming from using a kernel radius to represent the photon extent. The variance (noise) and expected value (bias) of this error are studied. The main idea of PPM is to reduce both variance and expected error continuously by averaging the results generated using independent photon maps.

The average error of the photon map radiance estimate over N samples can be expressed as:

$$\bar{\varepsilon}_N = \frac{1}{N}\sum_{i=1}^{N}\varepsilon_i, \tag{14}$$

where ε_i represents the error of radiance estimation in the pass i.

Convergence can be achieved by making sure that both the noise and bias goes to zero simultaneously as the number of passes goes to infinity ($N \to \infty$):

$$\begin{aligned} Var[\bar{\varepsilon}_N] &\to 0 \\ E[\bar{\varepsilon}_N] &\to 0, \end{aligned} \tag{15}$$

where

$$\begin{aligned} Var[\bar{\varepsilon}_N] &= \frac{1}{N^2}\sum_{i=1}^{N} Var[\varepsilon_i] \\ E[\bar{\varepsilon}_N] &= \frac{1}{N}\sum_{i=1}^{N} E[\varepsilon_i]. \end{aligned} \tag{16}$$

The main idea of PPM is to increase the variance in each step such that in average it still goes to zero. This will decrease the radius used for radiance estimates, which leads to a reduction of the expected error. Thus, there will be a trade-off between noise and bias in the radiance estimation.

5. Discussion

Comparing the half angle slicing mathematical model to Equation 2, the volume rendering integral, it is clear that the half angle slicing model assumes that particles inside the volume do not emit light. In addition, it only considers the incoming light within a cone in the direction of the light source when computating the in-scattered radiance. The assumption of forward scattering and the use of a specialized cone shaped phase function allow it to be implemented using a sweeping pass in the light direction.

Our derivation of the directional occlusion shading mathematical model shows that it is a specialization of the half angle slicing method and therefore inherits the same properties, i.e. the medium does not emit light and scatters light only in directions within a cone. The main difference is that the light direction must be aligned and point in the opposite view direction. Also, it only takes into account first order scattering and the in-scattered radiance is the attenuated constant background radiance scaled by the cone-shaped phase function. This model is capable of producing soft shadow effects of ambient occlusion.

As mentioned earlier, multidirectional occlusion shading is an extension of the directional occlusion shading model but in this model the light can be placed anywhere within the hemisphere defined by the view vector. A different numerical evaluation of the cone-shaped phase function make this possible, we refer to [SPBV10] for details.

The shadow volume propagation model is also similar to the mathematical model of half angle slicing. Both models solve the in-scattered radiance numerically by blurring the incoming light within a cone centered on the incoming light direction instead of considering the scattering of light coming from all directions on the unit sphere. However, shadow volume propagation decouples the chromaticity and luminance when computing the in-scattered radiance. Blurring applied to the chromaticity part, while the luminance is used directly without blurring. This produces hard shadow borders, which is shown to improve the depth perception [RDRS10].

The mathematical model of the spherical harmonic lighting [KJL*12] differs in that it assumes an isotropic phase function and assumes single scattering. The isotropic phase function allows the discretization of the mathematical model to use rotation properties of the spherical harmonic basis function in an efficient manner. Furthermore, by using the spherical harmonic basis functions to represent visibility and radiance separately, the light sources can be changed withouth recomputing the visibility.

Dynamic ambient occlusion approach is restricted to capture light interactions between adjacent voxels only since, in order to compute in-scattered radiance at a given voxel x, it only considers the voxels that lying in a certain distance from x. This model is the only model among other studied advanced illumination techniqus in this paper that the whole dataset does not affect the lighting in it.

6. Conclusion

We chose seven direct volume rendering approaches to study in this paper from several advanced illumination models that have been proposed recently. Three of them are slice-based techniques and the other ones can be combined with ray-casting based methods. The selected models are half angle slicing, directional occlusion shading, multidirectional occlusion shading, shadow volume propagation, spherical harmonic lighting, dynamic ambient occlusion and progressive photon mapping.

The mathematical model for each of the chosen illumination models has been derived. One can easily notice the differences of these models as they are written using a consistent mathematical formulation. Also, we can figure out the conditions and assumptions of each of these models by comparing their mathematical models with the most general volume rendering integral.

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