Investigation and Energetic Analysis of a Novel Hydraulic Hybrid Architecture for On-Road Vehicles

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Abstract

Hydraulic hybrid transmissions in on-road vehicles have been proven to significantly reduce fuel consumption. Existing hydraulic hybrid transmissions have shown fuel savings of 30-50% [1] with higher savings predicted when using advanced architectures [2]. However while these results are promising there exists room for improvement. Consider the series hybrid architecture which is currently the most common full hydraulic hybrid configuration. This system requires over-center units which increase expense and are relatively uncommon especially for high performance bent axis units. Series hybrids may also possess a synthetic feel due to the high compliance inherent in their accumulators. Further efficiency suffers as the hydraulic units in series hybrids are often forced to operate at high pressures and low displacements. A novel hydraulic hybrid configuration is analyzed in this paper which may reduce costs, improve response, and increase efficiency under certain conditions. An automatic transmission, a series hydraulic hybrid, and the novel blended hybrid architecture were simulated for a class II truck and fuel consumption rates compared. Dynamic programming was used to optimally control all three transmissions thereby removing the effect of controller design of fuel consumption. Simulation results show a 44.8% increase in fuel efficiency for the series hybrid and a 37.0% increase with the proposed system architecture. While the proposed architectures currently lags the series hybrid in fuel economy, there exists sufficient benefits to merit further studies.

Keywords: hydraulic hybrids, blended hybrid, dynamic programming

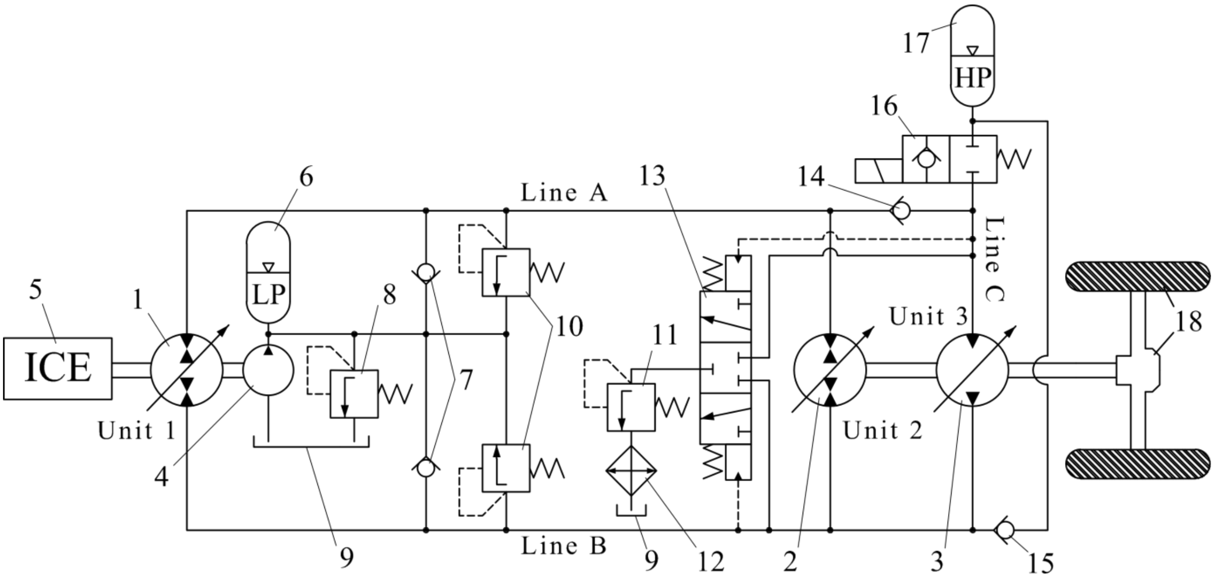
1 Introduction

In a world of rapidly expanding population and diminishing oil reserves, increasing the efficiency of on-road vehicles is now more of a priority than ever. One approach that has shown considerable potential is vehicle hybridization through a hydraulic hybrid transmission. Existing hydraulic hybrid transmissions have shown fuel savings of 30-50% [1] with higher savings predicted when using advanced architectures [2]. Further research has shown that the benefits gained from hydraulic hybridization increase as vehicle mass grows due to the increased availability of braking energy [3].

Today hydraulic hybrid transmissions can be grouped into three main categories: parallel, series, and power split. And while each of these architectures has advantages in specific applications, there exists room for improvement. Consider the series hybrid, currently the most popular architecture for full hybrids. This transmission requires over-center units which are less common and more expensive than non over-center units and especially true of the more efficient bent axis variety. Series hybrids are also often forced to operate inefficiently at high pressures and low displacements due to the accumulator's state of charge. Another drawback of series hybrids is the potential for a "synthetic" or "spongy" feel which becomes more pronounced as vehicle, and consequently accumulator size, increases. This spongy feel originates in the relationship between maximum wheel torque and current system pressure. In order to increase system pressure, more flow must enter the accumulator than exits. In series hybrids a considerable delay may be felt in achieving desired wheel torque when system pressure is below what is required resulting in a spongy feel. This inherent delay in increasing system pressure is in contrast to hydrostatic transmissions which increase pressure virtually instantaneously, a property that forms the basis for a novel system architecture presented in this paper.

In order for a new transmission to gain acceptance it must possess a similar feel and response to current systems [1]. To improve response, i.e. stiffness, to that of a mechanical transmission a new hydraulic hybrid architecture was created. This Blended Hybrid is so named for the blending of a hydrostatic transmission with a parallel hybrid. Because fuel economy is of principle concern, this paper will focus on overall vehicle efficiency. Simulation models have been created of a conventional automatic transmission, a series hybrid transmission, and the new blended hybrid and fuel consumption rates compared on a standard driving cycle.

Dynamic programming was used to optimally control all three transmissions thereby removing the effect of controller design of fuel consumption and ensuring a fair comparison between all three transmission architectures.



1	hydraulic unit 1	2	hydraulic unit 2	3	hydraulic unit 3
4	charge pump	5	engine	6	low pressure accumulator
7	low pressure check valve	8	low pressure relief valve	9	reservoir
10	high pressure relief valve	11	flushing relief valve	12	oil cooler
13	flushing valve	14	check valve	15	check valve
16	on-off check valve	17	high pressure accumulator	18	axle and wheels

Figure 1: Blended hybrid circuit

2 Blended Hybrid

A detailed description of the blended hybrid architecture was first proposed by Sprengel and Ivantysynova in [4]. A schematic of the blended hybrid is located in fig. 1. The blended hybrid is in essence a hydrostatic transmission with an additional unit attached to the transmission output shaft. A series of check valves connects this third unit to either Unit 1 thereby increasing the hydrostatic transmission's displacement, or to a high pressure accumulator which allows for secondary control of the unit. An additional check valve connects Line B to the high pressure accumulator while braking thereby permitting energy recovery without the need for over-center units.

The blended hybrid can operate in several distinct modes:

2.1 Forward driving without using the high pressure accumulator

The engine provides power to Unit 1 which supplies flow to Line A. With on-off check valve (16) closed, check valve (14) opens and connects Line C to Line A. Both Units 2 and 3 use this flow to rotate at a speed determined by a combination of their displacement. In effect both units operate as a single motor.

2.2 Forward driving using the high pressure accumulator

With on-off check valve (16) enabled, Line C is exposed to the high pressure accumulator's (17) pressure. As long as the pressure in Line C is higher than Line A, check valve (14) remains closed. Unit 3 is then able to use energy from the accumulator to supply torque to the wheels. Power can also be supplied from the engine by increasing Unit 1's displacement above zero. Flow from Unit 1 is then used to turn Unit 2. Pressure in Line A is a function of Unit 2's displacement and the resistive load on the wheels minus the torque contribution made by Unit 3. If pressure in Line A exceeds that of the high pressure accumulator, check valve (14) opens and on-off check valve (16) closes causing both Units 2 and 3 to behave as a hydrostatic transmission.

2.3 Braking while in forward motion

While braking high pressure automatically switches from Lines A and C to Line B (a feature inherent of hydrostatic transmission). Flow from Units 2 and 3 in Line B can leave through Unit 1, thereby powering parasitic loads, or through check valve (15). Pressure in Line B is a function of flow between all three hydraulic units. If flow from Units 2 and 3 exceeds that removed by Unit 1, pressure will rise. When the pressure exceeds that of the high pressure accumulator, check valve (15) opens allowing flow from Units 2 and 3 into the high pressure accumulator.

2.4 Reverse operation

To reverse Unit 1 moves over-center and supplies flow to Line B. Unit 2 then uses this flow to drive the wheels. If Unit 2 requires a pressure in Line B higher than that of the high pressure accumulator, check valve (15) opens and flow from Unit 1 is used to charge the accumulator until a suitable pressure for Unit 2 is achieved. When braking in reverse high pressure switches to Line A and can be used to power patristic loads on the engine or pass through relief valve 10.

3 Investigation Vehicle

A class II pickup truck was used as a base vehicle in which to test the three transmissions. Vehicle parameters can be found in tab. 1.

Table 1: Investigation vehicle parameters

Engine:	225 kW	Vehicle mass:	2525 kg
Axle ratio:	3.55:1	Tire rolling radius:	0.364 m
Frontal area:	3.6 m^2	Coefficient of drag:	0.416
Rolling resistance:	0.01		

Simplifications and assumptions were made in all three transmission models to either remove components that were not directly related to this paper's investigation or estimate the values of parameters which were not available. Modifications to all three models include the use of an engine map from a similarly size, although different, engine. Additionally all auxiliary loads on the engine such as the alternator and water pump were omitted.

It should be noted that the authors' purpose in this paper is not to predict fuel economy for a given transmission in the reference vehicle, but instead use the results to fairly compare all three transmission against each other. As such, simplifications which have been made hold valid because they have been applied to all three transmissions equally.

3.1 Automatic Transmission

An automatic transmission is the style of transmission typically found in class II pickup trucks and will serve as a baseline for comparing the fuel consumption rates of the two hybrid transmissions. A schematic of the automatic transmission is located in fig. 2.

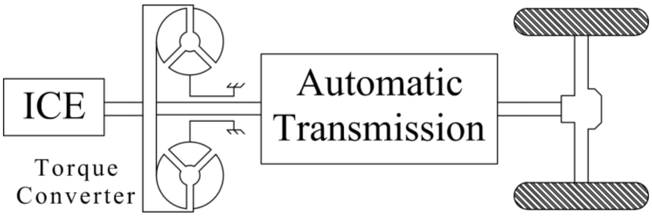


Figure 2: Automatic transmission architecture

Gear ratios for the automatic transmission are located in tab. 2.

Table 2: Automatic transmission parameters

1^{st}	4.17:1	2^{nd}	2.34:1
3^{rd}	1.52:1	4^{th}	1.14:1
5^{th}	0.86:1	6^{th}	0.69:1

Due to a lack of information the torque converter was modeled after a torque converter from a similarly sized vehicle. A lock-up clutch was not included in the torque converter model which would serve to rigidly couple the impeller and turbine. Additionally the hydraulic pump for the automatic transmission has been omitted.

3.2 Series Hybrid

A series hybrid with two units connected to the output shaft was used for the presented research study. This dual configuration is common in series hybrids and allows for a better comparison between it and the blended hybrid. A schematic of the series hybrid is located in fig. 3.

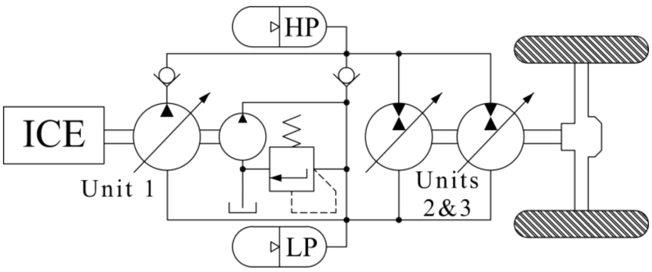


Figure 3: Series hybrid architecture

Parameters for the series hybrid are located in tab. 3.

Table 3: Series hybrid transmission parameters

Unit 1:	65 cc	Unit 2:	45 cc
Unit 3:	45 cc	Charge pump:	10 cc
Max pressure:	360 bar	Low pressure:	10 bar
HP accumulator pre-charge:	100 bar	HP accumulator volume:	30 l
HP accumulator min pressure:	130 bar	Polytropic coefficient:	1.3

Unit 1 was modeled using a 42 cc swashplate style unit while Units 2 and 3 were based on a 110 cc bent axis unit. Linear scaling laws were applied to all three units' empirically derived loss models.

3.3 Blended Hybrid

System parameters for the blended hybrid were kept as close to the series hybrid as practical for comparison purposes. While both hybrid architectures may benefit from somewhat different sizing's, by maintain identical unit sizes any differences in fuel economy can be attributed exclusively to differences in their architectures.

Parameters for the blended hybrid are located in tab. 4.

Table 4: Blended hybrid transmission parameters

Unit 1:	65 cc	Unit 2:	45 cc
Unit 3:	45 cc	Charge pump:	10 cc
Max pressure:	360 bar	Low pressure:	10 bar
HP accumulator pre-charge:	100 bar	HP accumulator volume:	30 l
HP accumulator min pressure:	130 bar	Polytropic coefficient:	1.3

4 Reference Cycle

All three transmissions were simulated using the industry standard UDDS driving schedule seen in fig. 4.

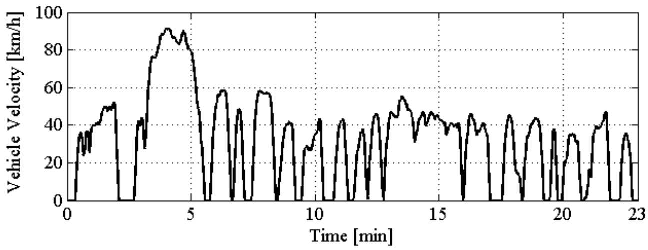


Figure 4: UDDS driving schedule

5 System Modeling

The transmissions were modeled in Matlab Simulink using governing equations.

Engine dynamics are modeled as a force balance between combustion torque, inertia, and load torque.

$$J_{\text{eng}}\dot{\omega}_{\text{eng}} = u_{\text{eng}}M_{\text{eng,max}}\left(\omega_{\text{eng}}\right) - M_1 \tag{1}$$

where J_{eng} is the engine's inertia, $\dot{\omega}_{\text{eng}}$ is the engine's acceleration, u_{eng} is the throttle, $M_{\text{eng,max}}\left(\omega_{\text{eng}}\right)$ is the engine's wide open throttle curve, and M_1 is Unit 1's torque or the torque converter's torque depending on the transmission. The engine's wide open throttle (WOT) curve and fuel consumption map are based on measured data for a 201 kW engine which has been scaled up to 225 kW. Friction is included within the engine's WOT curve and therefore not considered separately.

Vehicle dynamics are modeled as a force balance between propulsion torque provided by the driveline and the load torque generated by vehicle's inertia, rolling resistance, and aerodynamic drag.

$$m_{\text{veh}}r_{\text{roll}}^2\dot{\omega}_{\text{wheel}} = M_{\text{axle}} - \left(F_{\text{aero}} + F_{\text{roll}}\right)r_{\text{roll}} \tag{2}$$

where m_{veh} is the vehicle's mass, r_{roll} is the tire's dynamic rolling radius, $\dot{\omega}_{\text{wheel}}$ is the wheel's acceleration, M_{axle} is the torque provided to the wheels by the axle, F_{aero} is the vehicle aerodynamic drag, and F_{roll} is the tire's rolling resistance.

The torque converter is modeled using a K-Factor modeling approach [6]. Torque on the engine from the impeller is calculated using eq. (3). While torque on the transmission from the turbine is calculated using eq. (4).

$$M_{\text{eng}} = M_{\text{ratio}}\left(\frac{\omega_{\text{trans}}}{\omega_{\text{eng}}}\right)\cdot\left(\frac{\omega_{\text{eng}}}{K_{\text{fact}}\left(\frac{\omega_{\text{trans}}}{\omega_{\text{eng}}}\right)}\right)^2 \tag{3}$$

where M_{eng} is the torque on the engine, M_{ratio} is a torque converter characteristic and a function of the torque converter's turbine speed ω_{trans} and the impeller speed ω_{eng}. The torque converter's second characteristic parameter is K_{fact} and also a function of ω_{trans} and ω_{eng}.

$$M_{\text{trans}} = \left(\frac{\omega_{\text{eng}}}{K_{\text{fact}}\left(\frac{\omega_{\text{trans}}}{\omega_{\text{eng}}}\right)}\right)^2 \tag{4}$$

The automatic transmission is modeled as a set of gear ratios using eq. (5) and (6).

$$M_{\text{prop}} = M_{\text{trans}}\zeta_{\text{i}}\eta_{\text{trans}} \tag{5}$$

where M_{prop} is the torque on the propeller shaft, M_{trans} is the torque from the transmission, ζ_{i} is the current gear ratio of the transmission, and η_{trans} is the transmission efficiency.

$$\omega_{\text{prop}} = \frac{\omega_{\text{trans}}}{\zeta_{\text{i}}} \tag{6}$$

Equation (6) provides the propeller shaft speed ω_{prop}, the transmission speed ω_{trans}, and the current transmission gear ratio ζ_{i}.

Similarly the axle is modeled using eq. (7) and (8).

$$M_{\text{axle}} = M_{\text{prop}} \zeta_{\text{axle}} \eta_{axle} \tag{7}$$

where M_{axle} is the axle torque, M_{prop} is the propeller shaft torque, ζ_{axle} is the axle ratio, and η_{axle} is the axle efficiency.

$$\omega_{\text{axle}} = \frac{\omega_{\text{prop}}}{\zeta_{\text{axle}}} \tag{8}$$

where ω_{axle} is the axle speed, ω_{prop} is the propeller shaft speed, and ζ_{axle} is the axle ratio.

The hydraulic units are modeled using governing equations and empirically derived loss models. Effective unit flow is given by:

$$Q_{\text{eff}} = \frac{V\omega\beta}{2\pi} \pm Q_{\text{s}} \tag{9}$$

where Q_{eff} is the effective flow, V is the unit displacement, ω is the unit speed, β is the percentage of maximum unit displacement, and Q_{s} is the empirically derived leakage.

Effective torque is given by:

$$M_{\text{eff}} = \frac{V\Delta p\beta}{2\pi} \pm M_{\text{s}} \tag{10}$$

where M_{eff} is effective torque, V is the unit displacement, Δp is differential pressure, β is the percentage of maximum unit displacement, and M_{s} is the empirically derived torque losses.

Pressure build up in the lines is calculated using:

$$\Delta\dot{p} = \frac{1}{C_H}\left(\Delta Q\right) \tag{11}$$

where $\Delta\dot{p}$ is the change in pressure, C_H is the hydraulic capacitance, and ΔQ is the change in fluid volume within the line.

Accumulator capacitance is given by:

$$C_{\text{accu}} = \frac{V_0}{n}\left(\frac{p_0}{\Delta p^{n+1}}\right)^{\frac{1}{n}} \tag{12}$$

where C_{accu} is the accumulator's capacitance, V_0 is the initial gas volume, n is the polytropic coefficient, p_0 is the pre-charge pressure, and Δp is the change in pressure.

6 System Control

How a transmission is controlled has a major impact on its fuel economy. A poor system architecture with a good control scheme can outperform a superior system architecture with a substandard controller, thereby obscuring which system is truly better. One means of counteracting this problem is to optimally control the system essentially removing control as a factor affecting fuel economy [7]. In this paper the authors have used Dynamic Programming (DP) to optimally control all three transmission architectures.

6.1 Dynamic Programming

Dynamic programming is a powerful technique for solving dynamic optimization problems. DP's strength lies in its ability to easily handle non-linear dynamics and constrains while insuring a global optimal solution is reached. DP is based on the principle of optimality as proposed by Bellman in [5]: "An optimal policy has the property that, whatever the initial state and optimal first decision may be, the remaining decisions constitute an optimal policy with regard to the state resulting from the first decision".

Discrete time Dynamic Programming utilizes a state space modeling approach and requires both the continuous time and state spaces to be discretized. The granularity by which the system is divided has a direct impact on both solution accuracy and computational expense [8]. And while DP ensures a global optimal control strategy, it only holds true down to the level the system is discretized. Computational expense is a key drawback of DP which can be attributed to the "Curse of Dimensionality". As the number of states variables and their discretization increases, the required computation expense grows exponentially.

While DP's computational expense grows exponentially with the number of states, it grows linearly with the number of time steps to be simulated. However a caveat applies to the previous statement. DP requires that a state be capable of fully transitioning from one state to another during a single time step. If not, a state once inadmissible will remain inadmissible until it can fully project to an admissible state. A smaller DP time step reduces flow into the accumulator and consequently limits the maximum change in pressure. To satisfy the previously mentioned requirement, as the DP time step decreases the accumulator pressure discretization must proportionally increase. Balancing these requirements a one second primary DP time step was used in this paper.

Two distinct time steps and a solver were employed within the DP algorithm. First optimal controls were evaluated and held constant for the aforementioned one second primary DP time step. However one second is too long of a time step for accurate integration of rapid system dynamics. Therefore a 0.25 second simulation time step was used to increase numerical accuracy. Finally the differential equations were solved using Heun's Method (ODE2) which contains an internal time step of 0.125 seconds. The two time steps and solver were chosen to balance solution accuracy, numerical accuracy, and computational expense.

Formulating the DP problem begins by creating a cost function to optimize. As fuel economy is the principle metric by which these three transmissions are being compared, the cost function to be minimized consists primarily of the fuel consumed during the drive cycle.

$$J^*_{N\text{-}k,N} = \min_{u_{\mathrm{k,i}} \in U_{\mathrm{k}}} \left[\begin{array}{c} \underbrace{g\left(X_{N\text{-}k}, U_{N\text{-}k}\right)}_{\text{transitional cost}} + \\ \underbrace{J^*_{N\text{-}k\text{-}1,N}\left(L\left(X_{N\text{-}k\text{-}1}, U_{N\text{-}k\text{-}1}\right)\right)}_{\text{optimal cost from k-1 to final stage}} \end{array} \right] \quad (13)$$

where J^* is the optimal cost, N is the number of stages, k is a stage counter, u is a control effort, i is a control space counter, U is the control space, g is the transitional cost function, X is the state space, and L is the cost to finish.

Constraint functions are included in the cost function to ensure the cycle is met. While required wheel torque is cycle defined, the torque applied by the transmission is unlikely to exactly match. Therefore a constraint function is applied which increases cost the further apart the required and actual torque are, up to a point where the control effort is considered inadmissible.

The cost function also includes certain penalty functions which act to shape system response. One example is a penalty function imposed on changing engine speed. This function discourages the DP algorithm from applying a high throttle at one time step, thereby efficiently storing energy in the engine's inertia, and then low throttle at the next time step dropping engine speed and recovering the stored energy. Care must be taken when crafting penalty functions that shape system control. Fuel economy is highest when the DP algorithm is unconstrained and free to operate as desired. However such operation may lead to control actions which are impractical or infeasible due to effects not captured in the model. Therefore the influence of penalty functions should be minimized to prevent obscuring control strategies which are valid but unorthodox.

6.2 System Modeling for Dynamic Programming

All three transmissions are modeled in Simulink and represented in the state space form. This approach is a departure from the more commonly applied method of modeling the system in lines of code. Using Simulink offers several benefits including many engineers increased comfort with graphical programming languages (an attribute which should not be over looked). Additionally many system models for which DP is well suited are already constructed in Simulink and converting these models into line of code is a laborious and error-prone task. Finally Simulink features an advanced ODE solver which increases the fidelity of the simulation results.

Coupling Simulink to the DP driving algorithm requires using Simulink in a somewhat unconventional manner. The DP algorithm determines the current time step, states, and controls. Then the Simulink model is initialized at these states with the appropriate control efforts and simulated for a single time step.

Simplifications have been made to the Simulink models to reduce complexity and computational expense. The simplifications made are detailed below but generally involve either neglecting dynamics which are faster than the DP time step, or imposing certain deterministic properties.

Deterministic properties result from the assumption that the drive cycle is being tracked. Consequently the wheel speed is known at every instance in time. Similarly required wheel torque is calculated using the change in vehicle speed over the next time step along with other vehicle loads.

6.2.1 Automatic Transmission

The automatic transmission's state space representation is given by:

$$X \equiv \begin{bmatrix} \omega_{\mathrm{eng}} \\ \omega_{\mathrm{wheel}} \end{bmatrix} \qquad U \equiv \begin{bmatrix} u_{\mathrm{eng}} \\ i_{\mathrm{gear}} \end{bmatrix} \quad (14)$$

where ω_{eng} is the engine speed, ω_{wheel} is the wheel speed, u_{eng} is the engine throttle, and i_{gear} is the transmission gear.

The reduced state space representation used in the DP algorithm is given in eq. (15).

$$X \equiv \left[\omega_{\mathrm{eng}}\right] \qquad U \equiv \begin{bmatrix} \omega_{\mathrm{eng,des}} \\ i_{\mathrm{gear}} \end{bmatrix} \quad (15)$$

where ω_{wheel} has been removed due to being cycle defined. Engine throttle u_{eng} has also been replaced with the desired engine speed $\omega_{\mathrm{eng,des}}$. This simplification has been made to reduce computational expense by decreasing the size of the throttle control mesh. Engine speed changes based on the difference between combustion and load torques (eq. (1)). Over a time step even a minor difference in the torques can lead to a significant change in engine speed. Due to the system's stiff nature the throttle would need to be highly discretized in order to correctly match required torque thereby significantly increasing computation expense.

A different approach is used in this paper is which the desired engine speed is the control input. An equation within the simulation model then calculates the required throttle based on the initial and desired engine speeds. This method remains valid because it is based only on the control input and current states. Therefore when the optimal control path is eventually generated, the desired engine speed control will generate the same throttle input as was done during the recursive operation. This method of using feed forward control coupled with the control space is used in all three simulation models to overcome issues related to stiff systems and large time steps.

6.2.2 Series Hybrid

The low pressure system in the series hybrid is neglected (charge pump losses remain). Instead it is assumed that the low pressure line remains at 10 bar. Similarly the high pressure relief valve is ignored by saturating the high pressure at the high pressure relief valve's cracking pressure.

Pump dynamics are neglected with the pump's displacement being set by the DP algorithm. Pump dynamics are generally much faster than the DP time step so including these dynamics is unnecessary. Similarly engine throttle dynamics are much faster than the DP time step and are not included for any of the models.

The series hybrid's state space representation is given by:

$$X \equiv \begin{bmatrix} \omega_{\text{eng}} \\ \omega_{\text{wheel}} \\ p_{\text{acm}} \end{bmatrix} \qquad U \equiv \begin{bmatrix} u_{\text{eng}} \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} \tag{16}$$

where ω_{eng} is the engine speed, ω_{wheel} is the wheel speed, p_{acm} is the accumulator's pressure, u_{eng} is the engine throttle, and β_1, β_2, β_3 are the percentage of maximum unit displacement for Units 1-3 respectively.

The reduced state space representation used in the DP algorithm is given in eq. (17).

$$X \equiv \begin{bmatrix} \omega_{\text{eng}} \\ p_{acm} \end{bmatrix} \qquad U \equiv \begin{bmatrix} \omega_{\text{eng,des}} \\ \beta_1 \end{bmatrix} \tag{17}$$

where ω_{wheel} has been removed due to being cycle defined. As was done in the automatic transmission u_{eng} was replaced with desired engine speed $\omega_{\text{eng,des}}$. Required wheel torque is defined by the cycle, however the manner in which this torque is split between Units 2 and 3 is not. To reduce computational expense, especially for the blended hybrid, both units are controlled to provide 50% of the required torque. A feed forward controller within the Simulink model calculates the required Unit 2&3 displacement based on pressure, speed, and desired torque. If insignificant torque is generated the state is considered inadmissible.

6.2.3 Blended Hybrid

Modeling the blended hybrid in a form suitable for dynamic programming required a number of simplifications to be made. These simplifications were required due the stiff nature of the hydrostatic transmission and the presence of hydraulic logic elements (i.e. check and flushing valves).

First, as with the series hybrid, the low pressure system dynamics were neglected and assumed to maintain a 10 bar setting. Next only two pressure build-up equations were used: one for Line A, and one for the high pressure accumulator. Line C's pressure was set to either Line A's pressure or the HP accumulator's pressure depending on the on-off valve's position and related check valves' states. Line B's pressure was normally set to 10 bar unless both Line C was at 10 bar (due to the flushing valve) and the net flow between all three units in Line B was positive (i.e. braking). While these conditions were met Line B's pressure was set to the HP accumulator's pressure. This simplification assumes that Line B is infinitely stiff and therefore immediately transitions between 10 bar and the accumulator's pressure while braking. For an energetic comparison this is a valid assumption. Flow in and out of the two control volumes follows the same logic as the pressures.

The blended hybrid's state space representation is given by:

$$X \equiv \begin{bmatrix} \omega_{\text{eng}} \\ \omega_{\text{wheel}} \\ p_{\text{A}} \\ p_{\text{acm}} \end{bmatrix} \qquad U \equiv \begin{bmatrix} u_{\text{eng}} \\ \beta_1 \\ \beta_2 \\ \beta_3 \\ u_{\text{enab}} \end{bmatrix} \tag{18}$$

where ω_{eng} is the engine speed, ω_{wheel} is the wheel speed, p_{A} is Line A's pressure, p_{acm} is the accumulator's pressure, u_{eng} is the engine throttle, β_1, β_2, β_3 are the percentage of maximum unit displacement for Units 1-3 respectively, and u_{enab} is the on-off check valve's setting.

Hydrostatic transmissions are generally considered to be flow controlled, that is flow from Unit 1 must pass through Unit 2 and the system pressure is set by the wheel load. However numerically there is no difference between a HST and a series hybrid. The line's equivalent capacitance does not fundamentally change the system's nature. The conventional method of thinking can be explained by realizing flow control is only a deterministic control when an additional constraint that constant pressure be maintained is applied to the system. In order to increase pressure additional energy must be placed into the system. During normal operation only the engine, and not the wheels, can accomplish this. Therefore the displacement of Unit 1 should be a control variable and not a deterministic quantity.

Line A's stiffness does increase complexity when using Unit 1's displacement as a control variable. A displacement even several hundredths of a percent off of that required to maintain a constant pressure will significantly alter the system pressure over a single time step. Consequently applying a finite range of displacements (as done with the series hybrid) will never yield satisfactory results in the DP algorithm.

For the blended hybrid Unit 1's control variable indicated the desired pressure. The flow required to maintain the current Line A pressure was first calculated. Next the flow required to change Line A from its current pressure to the desired pressure was determined. This value was then added to the flow required to maintain the initial pressure. Finally a feed forward controller was used to calculate the required displacement of Unit 1.

The reduced state space representation used in the DP algorithm is given in eq. (19).

$$X \equiv \begin{bmatrix} \omega_{\text{eng}} \\ p_{\text{A}} \\ p_{\text{acm}} \end{bmatrix} \qquad U \equiv \begin{bmatrix} u_{\text{eng}} \\ p_{\text{A,des}} \\ u_{\text{enab}} \end{bmatrix} \tag{19}$$

where ω_{wheel} has been removed due to being cycle defined. As was done in the automatic transmission, u_{eng} was replaced with desired engine speed $\omega_{\text{eng,des}}$ and β_2 and β_3 were removed and treated as deterministic quantities. β_1 is also replaced with the previously discussed $p_{\text{A,des}}$ which indicates desired Line A pressure.

6.3 Dynamic Programming Algorithm

Dynamic programming is a recursive algorithm which starts at the final stage to be simulated and steps backwards through time while projecting forwards. DP decomposes the optimal control problem into a series of one stage (time step) sub-problems; each containing the model's discretized state space representation.

Beginning at the final time step a series of control efforts are applied to each state within the stage. The goal of which is to determine the optimal control effort for a given state that minimizes the one stage sub problem's cost. At the final stage the sub problem's cost is equal to the transitional cost between the final stage and the final time. Included in the transitional cost is the fuel consumed during the time step along with any other constraint or penalty functions which were applied. After the optimal control efforts and cost have been determined for every state the DP algorithm steps backwards in time to the previous time step.

In the intermediate time step a similar process is performed to determine the optimal controls with one additional cost function. Along with the transitional cost, there is now a cost to finish. As the simulation model runs it determines the amount of fuel consumed as well as the states at the end if the time step. These final states are projected onto the next stage and used to determine the cost to finish the entire cycle. It is precisely this idea of a cumulative cost function which is at the heart of DP and eliminates the need for full cycle enumeration.

DP's recursive method of stepping backswords though time repeats until the initial time step is reached. DP records the optimal control effort at every state but does not save where each state is projected. To determine the optimal path, control efforts, and fuel consumption, the simulation model is ran forward in time. Optimal controls at each state and time step are provided by look up tables generated through the recursive DP algorithm.

DP's computation expense is most clearly illustrated with the blended hybrid requiring over 35 billion dynamic simulations. Consequently any technique which decreases runtime is valuable. One approach taken was to use matrix operations whenever possible. MATLAB (MATrix LABoratory) is optimized for such operations which resulted in significant time savings compared against the commonly used method of nested loops. Simulink also benefits from matrix operations. Simulink requires significant overhead to initialize, consequently individually running each simulation would be infeasible. Instead the entire transmission model is placed within a repeating subsystem block in Simulink (proposed in [9]). An array is then created that contains all combinations of states and controls for a given time step and then subdivided into sets which contain ~500,000 simulations. Simulink is then run for each subset with all 500,000 simulations running simultaneously. Once complete, Simulink outputs an array containing the cost for each simulation run. Finally the DP algorithm records the minimum cost and respective control effort for each state, thereby generating the optimal control history. Another benefit of this approach is that each simulation subset is independent. Therefore each can be simulated concurrently on multiple processors using parallel computing which significantly reduces runtime.

Previous research using nested Matlab loops for performing DP on a series hybrid achieved a simulation rate of ~740 dynamic simulations per second [10]. Using the technique proposed in this paper simulation rates of up to 45,000 dynamic simulations per second were achieved using a single processor core.

7 Simulation Results

Dynamic programming determines the optimal control effort for every state during each time step. However it is the optimal control path and efforts which are of the most interest and presented in the results.

7.1 Automatic Transmission

An automatic transmission was modeled then simulated with dynamic programming for two functions. First simulation results provide a baseline by which to compare the two hybrid transmissions. Second modeling the transmission and comparing fuel economy results with published values serves to validate the simulation model and methodology. Understandably the results were somewhat different due to optimal control and simplifications that have been made. However the relatively close correlation with published values seen in this paper indicates a sufficiently accurate model has been constructed.

Running the automatic transmission on the 7.45 mile UDDS cycle consumed 0.4838 gallons of fuel corresponding to a fuel consumption rate of 15.4 mpg (miles per gallon). Published measurements certified by the US Environmental Protection Agency for the investigation vehicle following the UDDS cycle yielded 17 mpg. It was expected that the DP approach would yield a better fuel consumption rate than the sub optimally controlled measured value. However a portion of this discrepancy can be attributed to using a fuel consumption map from an older, and presumably less efficient, engine. Additionally the lack of a lock-up clutch, and a control law prohibiting aiding torque from being placed on the engine (common to all three transmissions), also explain the reduced fuel economy. Regardless the simulated and published values are close enough to considerer the simulation model and methodology reasonable.

Figure 5 shows a plot of the automatic transmission's vehicle speed, engine speed, throttle, and transmission gear.

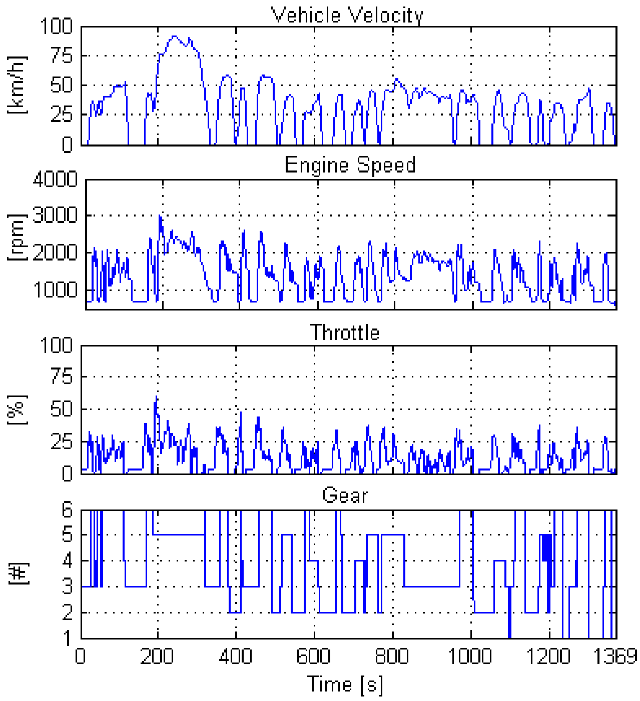


Figure 5: Automatic transmission DP results

It can be seen that the DP controller prefers to minimize engine speed by normally running in as high of a gear as possible. However the engine and wheels are linked by a mechanical ratio which restricts the engine's freedom of operation.

An engine operation map for the automatic transmission during the UDDS cycle is located in fig. 6. The white circles denote where the engine operated with their size proportional to the operation's duration. Contour lines indicate the engine's efficiency in terms of grams of fuel consumed per kWh. Like most gas engines the engine used in this paper is most efficient at low to medium speeds and medium to high throttle. However due to the finite gear ranges, and the nature of the torque converters operation, the automatic transmission's engine is often forced to operate in an inefficient manner. It is partially this poor engine management that allows the series hybrid to significantly outperform the baseline automatic transmission.

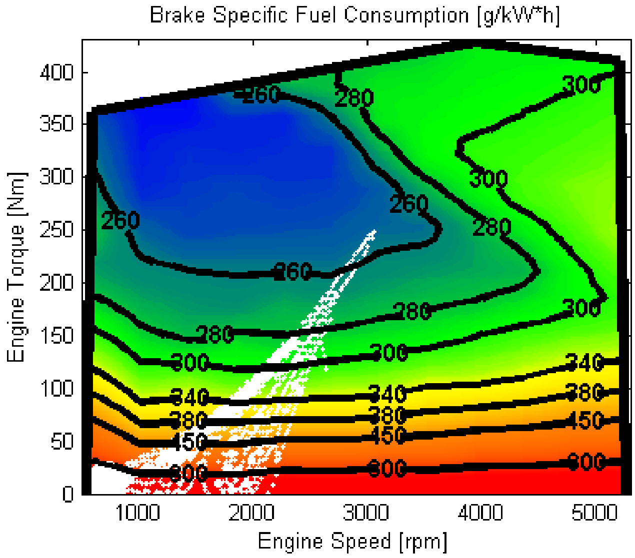


Figure 6: Automatic transmission engine operation map

7.2 Series Hybrid

Dynamic programming naturally determines the optimal control path for the series hybrid begins with a full accumulator. However because the fuel which would have been consumed to fill the accumulator is never accounted for, the resulting fuel economy would be artificially increased. Therefore both the series and blended hybrids are started with an empty accumulator and minimum engine speed.

During the UDDS cycle the series hybrid consumed 0.3341 gallons of fuel corresponding to a fuel consumption rate of 22.3 mpg. This represents an improvement of 44.8% over the baseline automatic transmission.

Figure 7 shows a plot of the series hybrid's vehicle speed, engine speed, throttle, accumulator pressure, and unit displacements. Clearly apparent are the differences in engine speed and throttle between the series hybrid and the automatic transmission. One of the series hybrids key advantages is the decoupling of the engine and wheel speeds. This permits increased control over engine management allowing the engine to operate in a more efficient manner.

It can be seen from fig. 7 that the series hybrid uses the entire range of accumulator pressures from 130 to 360 bar. The series hybrid's efficiency it directly related to the accumulator pressure with low pressures and high displacements being favored over high pressures and low displacements. However in order to recover energy while braking a certain minimum pressure is needed to provide the requisite braking torque. Dynamic programming has a priori knowledge of the future cycle which permits it run the accumulator at low pressures when possible. Before braking the DP algorithm will increase accumulator pressure to ensure full recovery of the available kinetic energy. Such a control strategy may be difficult to include in an implementable controller.

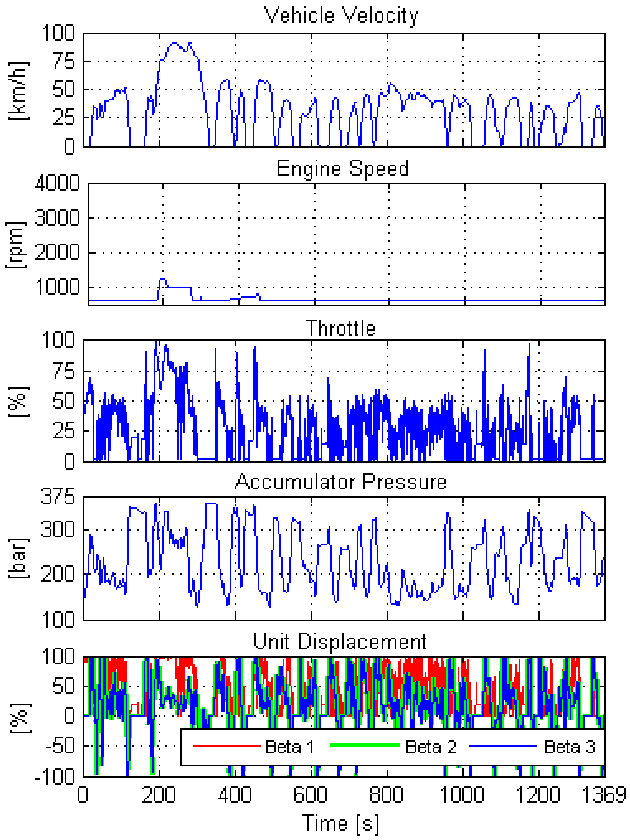


Figure 7: Series hybrid DP results

The series hybrid generally operates at the lowest allowable engine speed with a low to medium throttle (fig. 8). This is in stark contrast to the automatic transmission's operation and partially explains the series hybrids increase in fuel economy.

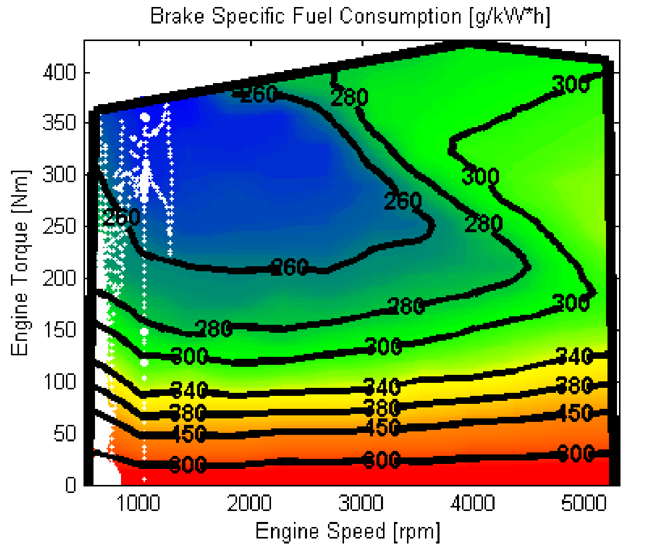


Figure 8: Series hybrid engine operation map

7.3 Blended Hybrid

Under the UDDS cycle the blended hybrid consumed 0.3526 gallons of fuel equating to a fuel consumption rate of 21.1 mpg. This represents a 37.0% improvement over the automatic transmission.

Figure 9 shows a plot of the blended hybrid operation during the UDDS cycle.

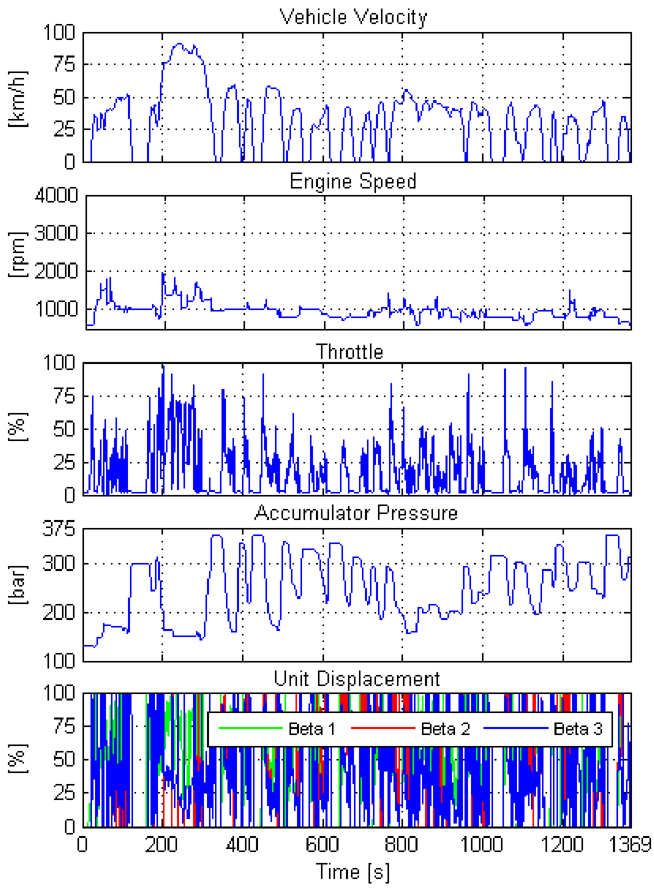


Figure 9: Blended hybrid DP results

The blended hybrid's accumulator maintains a generally higher pressure than the series hybrid. This is partially because the blended hybrid is not required to always operate at the accumulator's current pressure. Therefore there is not a need to increase efficiency by reducing pressure during extended periods of low torque requirement. The blended hybrid also maintains a higher minimum pressure to ensure all of the available braking energy is captured.

Unlike the series hybrid, the blended hybrid is not capable of increasing accumulator pressure while driving. This is due to a lack of a direct connection between Unit 1 and the accumulator. A characteristic which also prevents the transmission from storing excess energy from the engine in the accumulator. The diminished potential for engine management in the blended hybrid helps to explain the 5.4% decrease in fuel efficiency when compared to the series hybrid.

The blended hybrid's engine operation map is shown in fig. 10.

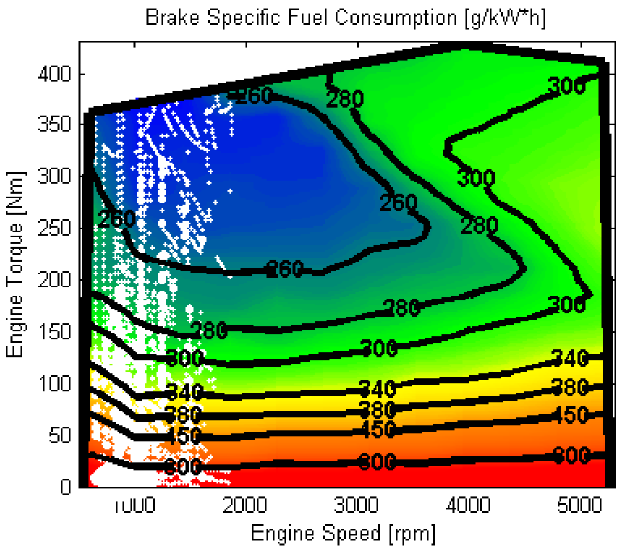


Figure 10: Blended hybrid engine operation map

A deeper understanding of the blended hybrid's operations can be gained by examining the braking and acceleration event shown in fig. 11. While braking both Units 2&3 are at Line B's (i.e. accumulator) pressure. Both units are also at the same displacement because of a control law in place which dictates both units provide 50% of the required torque. Around 728s the enabling valve opens and the vehicle begins to accelerate. Initially both Line's A & C are above the accumulator pressure and the transmission is operating in the HST mode. As the required torque decreases the pressures falls until Line's A & C are equal to the accumulator's pressure. At this point Unit 3 (Line C) connects to the accumulator and operates under secondary control while Unit 2 (Line A) remains connect to Unit 1 and operates as a stiff HST. At 746s the enabling valve closes and both Units 2 & 3 are once again operating as a HST, albeit at a pressure lower than is possible with the series hybrid.

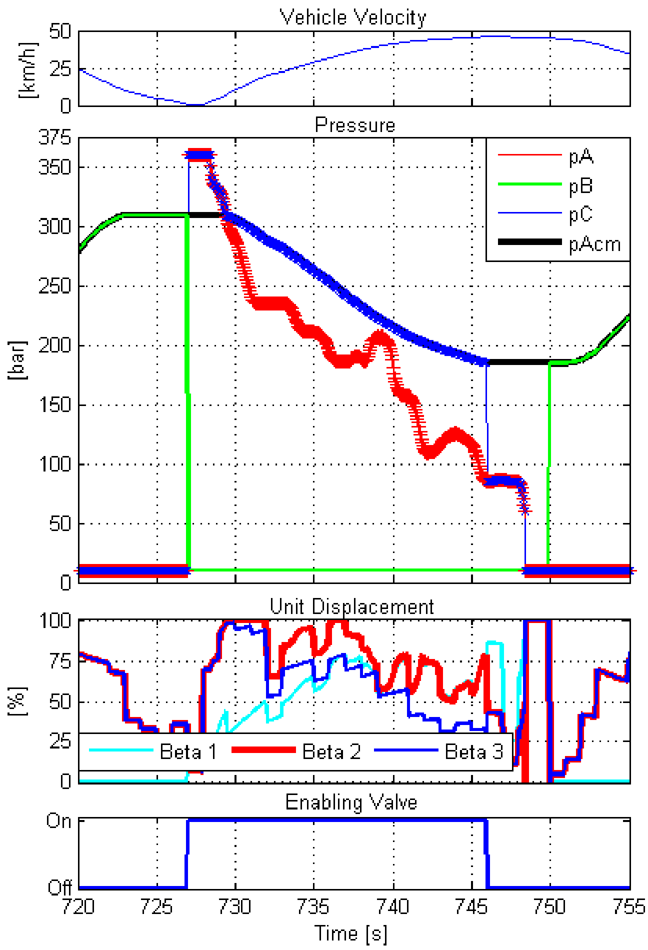


Figure 11: Detailed blended hybrid operation

Clearly apparent are the complexities of design and control which have not been fully explored through the optimal control study presented herein. There remains a question of the impact of sizing on the hybrid architectures. Both architectures will likely benefit from different unit sizes, accumulator sizes, and precharge pressures. There also remains a question of whether both Units 2 & 3 should be same size or would different unit sizes benefit the blended hybrid. In order to reduce computational expense by avoiding an additional control, the percentage of torque supplied by each unit was set to 50%. However the blended hybrid would likely benefit from being able to supply all of the required torque from the accumulator under certain circumstances. Finally there remains a question of how close an implementable controller can match each of these optimally controlled designs, a problem discussed in [11].

8 Conclusion

In this paper a novel architecture for a hydraulic hybrid transmission was presented. A class II pickup truck was used as a reference vehicle following the UDDS driving schedule. A baseline automatic transmission, a series hybrid, and the novel blended hybrid were modeled and simulated in the reference vehicle. Dynamic programming was then used to optimally control all three transmissions in order to remove control as a factor affecting fuel economy. The automatic transmission achieved a fuel economy of 15.4

mpg. The series hybrid reached 22.3 mpg which represents a 44.8% improvement over the baseline automatic transmission. Finally the blended hybrid achieved a fuel economy of 21.1 mpg which is 37.0% better than the automatic transmission.

While the blended hybrid achieved a fuel economy 5.4% lower than the series hybrid there remains a number of questions regarding sizing and implementable control of both architectures which will surly alter the efficiencies. Fuel economy, while important, is not the sole consideration. The blended hybrid architecture shows considerable potential in other areas over existing series hybrid configurations. The blended hybrid's stiff nature improves user response and feel to that of a traditional mechanical transmission. Removing the requirement for over-center units attached to the wheels lowers cost and permits the use of more bent-axis units. Finally the blended hybrid increases efficiency under certain lower power operations such as driving at a constant moderate speed. During this operation the line pressure can be reduced below the accumulator's minimum pressure, a behavior not possible with series hybrids.

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