Railway Infrastructure Capacity Utilization Description through Data Integration in Blocking Time Theory

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Abstract
We propose a method to describe capacity utilization for railway infrastructure that applies blocking time theory to managing train runs. Different from traditional capacity evaluation, infrastructure capacity utilization description shows detailed information on infrastructure utilization hidden in timetabling data instead of sheer number of trains that can be operated, or capacity consumed. Using a function system defined upon necessary operational inputs for timetabling in blocking time theory, we can obtain the feasibility condition for operating consecutive trains. Thus, the method to identify critical block section can be deduced from the feasibility condition. Structural indication determines the capacity utilization of consecutive train paths, which can be further integrated into a bi-directional graph to model infrastructure capacity utilization description followed by infrastructure time allocation. Consumed capacity of railway infrastructure by operating train runs can be formulated. Besides, a general procedure is proposed to analyse the sensitivity of consumed capacity to operational inputs. An experimental case study is conducted to demonstrate the application of this method in analysing the impact of speed and recovery time.

Keywords
Blocking time theory, Capacity analysis, Infrastructure capacity utilization description, Timetabling data

1 Introduction

Railway capacity analysis or calculation is almost an ancient problem in the field of railway operations. Yet it has not been eliminated from a rather critical role in infrastructure utilization management and rolling stock utilization. And the increasing emphasis on energy efficiency, CO_2 emission reduction, and environmental protection can be better met by new generation railway services, especially the so-called high-speed railway. With the fourth railway package issued, a more open and competitive railway market reinforced by relevant administrative and technical artifices can be expected. The increasing demands on railway services also require highly efficient managerial techniques of railway infrastructure and rolling stocks for proper strategizing as a potential reply.

Along with the introduction of German railway reform, blocking time theory was employed in service planning by the German railway infrastructure manager DB Netz AG, when developing its computer-aided timetabling system in the late 1990s. Blocking time theory was developed by Happel (Happel, 1959) and is now widely used for timetabling in Europe. It allows the description of train runs on different railway networks with different signalling and control systems. It also enhances railway operations with competitive edges over train diagramming in many ways, one of which is that blocking time theory visualizes the infrastructure occupation by a specific train. And this is especially significant in terms of conflict detection and resolution in a competitive business environment under the duality of infrastructure-operation. Blocking time theory will continue to dominate European railway operations in a foreseeable future, which further drives managerial innovations.

The overall structure of this paper is organized as follows. Section 2 reviews related literatures. Section 3 introduces the operational inputs required to perform the analysis. Section 4 presents the method to critical block section identification and parametrically indicating the structure of feasible train paths, providing the theoretical foundation for the description of infrastructure capacity utilization. Section 5 integrates the results achieved in previous steps into infrastructure capacity utilization description and gives the method to calculate fragmented infrastructure capacity, which constitutes the allocation of infrastructure time along with structural indicators. An experimental case study is reported in section 6, and the impact of parameters, such as speed and recovery time, on consumed is discussed. And section 7 concludes this work briefly.

2 Literature Review

Numerous methods have been developed to analyse and calculate railway capacity. Many scholars have classified these methods from different perspectives. Pachl (2018) classified the capacity methodologies as two major classes: analytic and simulation. Another well-received overview concerning railway capacity issues was provided by Abril et al. (2008), which further divided the relevant methods into three levels: analytical, optimization, and simulation methods. Sameni et al. (2011) categorized capacity evaluation methods to be timetable-based and non-timetable-based. Among all classifications, the classification presented by Abril et al. is most widely noted (Abril et al., 2008), based on which this research summarizes existing methodologies on capacity researches.

2.1 Analytical Methods

Analytical methods are designed to model the railway environment by means of algebraic expressions or mathematical formulae (Abril et al., 2008). They usually obtain theoretical capacities and determine practical capacities either as a percentage of the theoretical capacity or by including regularity margins (Yaghini et al., 2014). The UIC method proposed by the International Union of Railways (UIC) is an important one within this category, which is based on visually compressing timetable (UIC, 2004). This method measures the consumed capacity of sections for a given infrastructure based on pre-determined timetable (Landex et al., 2006; Jensen et al., 2017), though it is also argued that the method can also be applied when the infrastructure is not divided into sections (Landex, 2008). Many researches have been produced in terms of analysing the method (Landex, 2009), including propositions to improve it following different ideologies, such as (Lindner

& Pachl, 2010; Lindner, 2011), which eventually resulted in an improved update (UIC, 2013). Other important analytical methods include the subtraction factor method (Yan, 1997; Zhao, 2001), the minimum interval method (Zhang, 2015; Jamili, 2018), and parametric method (Lai and Barkan, 2009; Lai and Barkan, 2011).

In general, analytical methods are useful for the calculation of railway capacity at a planning level, as well as for the identification of bottlenecks in the infrastructure. However, different methods may provide very different results when studying the same line since they are very sensitive to the parameters used and variations in the composition of trains (Riejos et al., 2016).

2.2 Optimization Methods

Optimization methods are designed to provide more strategic methods for solving the railway capacity problem other than purely analytical formulae (Abril et al., 2008). The ideological basis of optimization methods is usually timetable saturation through mathematical programming. Optimization techniques, such as tabu search (Higgins, 1998), branch-and-bound (Higgins et al., 1996), Lagrangian relaxation (Caprara et al., 2002) and heuristic algorithms (Carey and Lockwood, 1995) are designed to solve railway capability analysis problems.

The railway capacity optimization methods can be roughly divided into deterministic optimization methods and stochastic optimization methods. For deterministic optimization method, an initial timetable is required. Recent contributions that belong to deterministic optimization method include Yaghini et al. (2014), Harrod (2009), Petering et al. (2015), and Burdett (2015). But for stochastic optimization method, it does not need an initial timetable, instead it requires the probability distribution of relevant time variables and dwell times (de Kort et al., 2003). Recent papers of this type include Burdett and Kozan (2005). Kroon et al. (2008), and Medeossi et al. (2011).

Optimization methods may be useful for problems of uncomplicated nature, but it could be very difficult to solve a model with very complex capacity and traffic constraints.

2.3 Simulation Methods

Simulation methods are usually provided a model as close to reality as possible, to validate a given timetable (Abril et al., 2008). These methods attempt to replicate the actual operation of trains within a line or a railway network (Riejos et al., 2016). Excellent surveys of the railway capacity simulation methods have been done by Pouryousef et al. (2015). There are two basic simulation models: microscopic and macroscopic model. And some works are based on the integration of both models (Kettner et al., 2003). While most simulation models fall into these two categories, mesoscopic models can be created by simplifying microscopic model or enriching details in macroscopic model with proper skills (Gille et al., 2008; Marinov and Viegas, 2011; Jensen et al., 2017).

Realization of simulation models requires specific tools. Current mainstream railway capability simulation software includes SIMONE, RailSys, and OpenTrack. More information about railway simulation tools can be found in Barber et al. (2007).

Simulation is most effective method to analyze capacity for infrastructure of limited size (Lai et al., 2014), and they become computationally intensive when applied in network level. In addition, these models are sensitive to data because of their dependency of complex operational data as inputs, such as geometrical configuration, velocity of trains, and movement rules.

In conclusion, existing methods for capacity analysis basically focus on infrastructure capacity determination or evaluation in terms of time consumed or numbers of trains that can be operated based on ready timetables or other operational parameters. However, other useful information in timetabling data remains unrevealed. We feel that capacity researches can be approached from another angle where the deterministic relationship between timetabling data and infrastructure capacity utilization can be clarified and utilized. For instance, formulations for timetable optimization programs are generally based on timetabling, which lacks the insight from this connectivity that can potentially simplify computations. Therefore, this paper proposes a description of capacity utilization for railway infrastructure that applies blocking time theory.

3 Operational Inputs

This paper considers one direction of double-track railway infrastructure whose operation is based on blocking time theory.

An arbitrary train i operating on the infrastructure is always defined in section $[O_i, D_i]$, where O_i and D_i denote the first and last block section on the operation route of train i. The operation route of train i does not necessarily overlap the infrastructure which we analyze.

In order to clarify the relationship between consecutive train paths, define two train paths as a train pair if they meet following conditions:

(i) they directly follow each other over certain section of railway infrastructure;

(ii) the lower blocking time of the leading train can be scheduled at the same time as the upper blocking time of the following train in certain block section, without causing conflicts to any other train.

Condition (i) demands that train i is followed by $i+1$ during a certain section on the infrastructure. Interpretation of condition (ii) involves feasibility issues and can be referred to section 4.2 and 4.3. Let train i and $i+1$ form a train pair, denoted as $(i, i+1)$, on their common operation route $[o_{i,i+1}, d_{i,i+1}]$. Noticeably, a train pair is sequence-relevant. There is always an arbitrary block section $j \in [o_{i,i+1}, d_{i,i+1}]$ when we talk about train pair $(i, i+1)$ unless specified otherwise.

And the information required for analyzing the utilization of infrastructure from timetabling data can be called operational inputs, as in the following explanations.

(i) Time for signal setup A_i^j denotes the time needed to set up the signal to operate in block section j for train i.

(ii) Time for signal confirmation B_i^j denotes the time needed for the train driver to confirm the signal to approach in block section j for train i.

(iii) Approach time C_i^j denotes the time needed for train i to end block section j.

(iv) Running in a block section r_i^j denotes the time needed for train i to cover the whole length of block section j. It is usually the sum of pure calculated running time and recovery margin which makes up certain percentage of the total running time.

(v) Time for clearance D_i^j denotes the time needed for train i to clear block section j.

(vi) Time for release E_i^j denotes the time needed for railway operation system to release the signal of block section j after the traverse by train i.

(vii) Scheduled stop d_i^j denotes the duration of a scheduled stop of train i at station in block section j.

(viii) Operation sequence $(1,\cdots,i,\cdots,m)$ denotes the sequence of train departing from certain block sections of the infrastructure.

(ix) Overtaking arrangement $(i,i+1)\xrightarrow{j}(i+1,i)$ denotes a change of operation sequence from $(i,i+1)$ to $(i+1,i)$ at station in block section j. It is noteworthy that $(i,i+1)$ and $(i+1,i)$ should be treated as two train pairs on different sections.

All mathematical notations used in this paper are listed in the Appendix.

4 Capacity Analysis of Consecutive Train Runs

Before analysing infrastructure capacity utilization that is determined by its timetable, the method to study its occupation that is determined by the structure of consecutive train runs is introduced in this section.

4.1 Function System

The time spent from the departure of a train at a certain node to another node on its route of operation, can be calculated and used to describe the temporal proceeding of that train. It is called process time, different from departure and arrival time in a ready timetable, with which timetable structure can be restated.

(1) Single train path

Define the process time of train i when entering block section $j\in[O_i,D_i]$ from block section O_i as the entry process time of train i in block section j, denoted as $p_i^{O_i,j}$, and it can be given by

$$p_i^{O_i,j}=\sum_{k=O_i}^{j-1}(r_i^k+d_i^k), \tag{1}$$

where k is the universal serial number. Process times are but intermediate to model train runs in blocking time theory, so that capacity analysis can be performed. Since planning timetables in blocking time theory relies on blocking time, the upper blocking time of train i in block section $j\in[O_i,D_i]$ from block section O_i can be given by

$$b_{i,up}^{O_i,j}=p_i^{O_i,j}-A_i^j-B_i^j, \tag{2}$$

where $j=O_i$, or $d_i^{j-1}\neq 0$.

And

$$b_{i,up}^{O_i,j}=p_i^{O_i,j}-A_i^j-B_i^j-C_i^{j+h}, \tag{3}$$

where $j\neq O_i$, or $d_i^{j-1}=0$. And the lower blocking time of train i in block section $j\in[O_i,D_i]$ from block section O_i can be given by

$$b_{i,low}^{O_i,j}=p_i^{O_i,j}+D_i^j+E_i^j+r_i^j+d_i^j. \tag{4}$$

(2) Train pair

Let the blocking time difference of train pair $(i,i+1)$ on block section $j\in[o_{i,i+1},d_{i,i+1}]$

be $t_{i,i+1}^{o_{i,i+1},j}$, given by

$$t_{i,i+1}^{o,j} = b_{i+1,up}^{o,j} - b_{i,low}^{o,j}, \tag{5}$$

where the subscript $i, i+1$ of notation $o_{i,i+1}$ is intentionally left out given there is no confusion, just in case equations get too long and unreadable. Replacing the right-hand sided blocking times of equation (5) with equation (2-4) yields

$$t_{i,i+1}^{o,j} = p_{i+1}^{o,j} - p_i^{o,j} - (A_{i+1}^{j+1} + B_{i+1}^{j+1} + C_{i+1}^{j+1}) - (D_i^{j+1} + E_i^{j+1}) - (r_i^j + d_i^j). \tag{6}$$

4.2 Feasibility

A complex train path structure can always be decomposed into several train pairs, each of which be conflict-free, when analyzing the infrastructure occupation of train paths. Suppose that train pair $(i, i+1)$ exists in section $[o_{i,i+1}, d_{i,i+1}]$ and its blocking time difference at block section j can be given by

$$t_{i,i+1}^{o,j} = p_{i+1}^{o,j} - p_i^{o,j} - (A_{i+1}^{j+1} + B_{i+1}^{j+1} + C_{i+1}^{j+1}) - (D_i^{j+1} + E_i^{j+1}) - (r_i^j + d_i^j). \tag{7}$$

The departure time when train i entries section $j \in [o_{i,i+1}, d_{i,i+1}]$ can be denoted as $y_{i,j}$. Departure time $y_{i,1}$ denotes the departure time of train i from its origin block section. It is obvious that

$$y_{i,j} = y_{i,j-1} + r_i^j + d_i^j. \tag{8}$$

And departure times can be calculated using process times using following equations

$$y_{i,j} = y_{i,o} + p_i^{o,j}, \tag{9}$$

$$p_i^{o,o} = 0. \tag{10}$$

Using equation (11), the first two items on the right-hand side of equation (9) can be written as

$$p_{i+1}^{o,j} = y_{i+1,j} - y_{i+1,o}, \tag{11}$$

$$p_i^{o,j} = y_{i,j} - y_{i,o}. \tag{12}$$

Substituting equation (13) and (14) into equation (10) yields

$$t_{i,i+1}^{o,j} = (y_{i+1,j} - y_{i+1,o}) - (y_{i,j} - y_{i,o}) - (A_{i+1}^j + B_{i+1}^j + C_{i+1}^j) - (D_i^j + E_i^j) - (r_i^j + d_i^j). \tag{13}$$

Denote the mark of the lower blocking time of train i in block section j on the time axis as $m_{i,j}^{low}$, and it can be expressed by

$$m_{i,j}^{low} = y_{i,j} + (D_i^j + E_i^j). \tag{14}$$

Denote the mark of the upper blocking time of train i in block section j on the time axis as $m_{i,j}^{up}$, and it can be expressed by

$$m_{i+1,j}^{up} = y_{i+1,j} - (A_{i+1}^j + B_{i+1}^j + C_{i+1}^j). \tag{15}$$

Substituting equation (14) and (15) into equation (13) yields

$$m_{i+1,j}^{up} - m_{i,j}^{low} = t_{i,i+1}^{o,j} + y_{i+1,o} - y_{i,o} - r_i^j - d_i^j. \tag{16}$$

Train pair $(i, i+1)$ is feasible if and only if the $m_{i+1,j}^{up} - m_{i,j}^{low}$ is nonnegative for $\forall j \in [o_{i,i+1}, d_{i,i+1}]$, or the right-hand side of equation (16) being nonnegative.

Thus, the nonnegativity of $m_{i+1,j}^{up} - m_{i,j}^{low}$ can be called the feasibility condition of train pair $(i,i+1)$ in block section $j \in [o_{i,i+1}, d_{i,i+1}]$

4.3 Critical Block Section

There is at least one block section on the common operation route of a train pair, where their blocking time squares elapse earlier than those in other block sections when pushing their train paths closer together. It supports the train path structure of a train pair and determines the occupation of infrastructure by them. This section presents the method to its identification using operational data.

Non-Overtaking Operation

Let train i and $i+1$ depart into section $o_{i,i+1}$ at the same time, meaning $y_{i+1,o} = y_{i,o}$. Train i and $i+1$ are obviously conflicted in section $[o_{i,i+1}, d_{i,i+1}]$. Thus, a value must be added to the right-hand side of equation (16), which is denoted as $I_{i,i+1}^{g_{i,i+1}}$, and equation (16) transforms into

$$m_{i+1,j}^{up} - m_{i,j}^{low} = t_{i,i+1}^{o,j} - r_i^j - d_i^j + I_{i,i+1}^j. \tag{17}$$

Substituting $m_{i+1,j}^{up} = m_{i,j}^{low}$ into equation (17) yields

$$I_{i,i+1}^j = d_i^j + r_i^j - t_{i,i+1}^{o,j}. \tag{18}$$

Equation (18) gives the minimum value needed to make train pair $(i,i+1)$ feasible in section $j \in [o_{i,i+1}, d_{i,i+1}]$, and anything more than that might be considered as buffer time.

Adding a positive value to the right-hand side of equation (17) signifies letting the train path $i+1$ translate away from train path i by that value. Notice that during the transition of train pair that follows a fixed operation sequence on the time axis, their blocking time differences on block sections within a fixed section increase or decrease proportionately during the process. This shows the structural stability of a train pair given their parameters constant, meaning $\{I_{i,i+1}^j\}$ is certain. In order to make train path i and $i+1$ conflict-free, a value large enough should be added. There could be more than one case that can make them so, and we define the section that is traversed the latest among the sections with the same largest $I_{i,i+1}^j$ to be the critical block section of train pair $(i,i+1)$, which can be mathematically expressed as

$$g_{i,i+1} = max\{j \mid max[I_{i,i+1}^j]\} \quad j \in [o_{i,i+1}, d_{i,i+1}]. \tag{19}$$

Notice that it is unnecessary to distinguish between homogeneous and heterogeneous train operations for a train pair, since the method presented can treat them in general.

Complex Overtaking Operation

There are often complex overtakes which involve more than two train paths, when scheduling timetables for railway network of limited scope. There should be quite some instances that are of this kind when considering railway network covering a considerably large area. The method to obtain a feasible schedule is to examine all trains according to the operation sequence.

Consider the scenario that train i acts as the leading train in the train pair formed with train $\mu \in \{\mu\}$ in section $[o_{i,\mu}, d_{i,\mu}]$. Thus, the critical block section of train pair (i,μ) can be expressed as $g_{i,\mu} \in [o_{i,\mu}, d_{i,\mu}]$. And their largest value to be added can be given by $I^{g}_{i,\mu}$.

Assume that trains within set $\{\mu\}$ are scheduled conflict-free and involving overtakes. All train pair involving train i and $\mu \in \{\mu\}$ with train i being the leading train, if and only if a critical block section g_i exists for the complex structure and satisfies the following

$$g_i = max\{k \mid max[I^{g_{i,\mu}}_{i,\mu}]\}. \tag{20}$$

4.4 Structural Indication

The structural stability of a train pair can be exploited to describe the capacity utilization of a train pair. For this purpose, define the structural indicator of train pair $(i, i+1)$ on block section $j \in [o_{i,i+1}, d_{i,i+1}]$ to be the difference between the added value of train pair $(i, i+1)$ on its critical block section $g_{i,i+1} \in [o_{i,i+1}, d_{i,i+1}]$ and block section $j \in [o_{i,i+1}, d_{i,i+1}]$, and its mathematical expression can be written as

$$s^{j}_{i,i+1} = I^{g}_{i,i+1} - t^{j}_{i,i+1}. \tag{21}$$

Structural indicator $s^{j}_{i,i+1}$ can be used to denote the minimum infrastructure time interval to operation two consecutive train paths that form a train pair on their common section $[o_{i,i+1}, d_{i,i+1}]$.

5 Infrastructure Capacity Analysis

We consider describing infrastructure capacity utilization based on analytical results from previous steps. And a general method to analyse the impact of timetabling data is summarized based on the formulation of consumed capacity.

5.1 Infrastructure Capacity Utilization Description

As suggested by equation (17), adding $I^{g}_{i,i+1}$ to its left-hand side is same as to move train path $(i, i+1)$ away so that they can be feasible, thus producing a compressed train pair. Repeat the process so that all the train pairs are feasible. And the utilization of infrastructure capacity by train pairs can all be indicated using methods presented in section 4.

The blocking time graph originally calculated by blocking time theory can be improved by integrating structural indicators into an infrastructure capacity utilization description, abbreviated as ICUD. As can be seen in Fig. 1, denote the two edges of a time square that are parallel to the time axis of the timetable as time edges, and the two edges of time square that are parallel to the distance axis as distance edges. The distance edge of any time square does not concern capacity analysis and therefore is deemed 0.

Denote the blocking time square representing the occupation of infrastructure by train path i in block section j as $U(i, j)$. The weight of the time edge of $U(i, j)$ can be given by

$$L(i, j)^{U} = \sum_{j} (r_i^{j} + d_i^{j}) + A_i^{j} + B_i^{j} + C_i^{j} + D_i^{j} + E_i^{j}. \tag{22}$$

Define the time square representing the occupation of infrastructure by train pair

$(i,i+1)$ in block section j as structural time square, which is also suggested by its structural indicator $s_{i,i+1}^{j}$. The weight of the time edge of $V(i,i+1,j)$ can be given by

$$L(i,i+1,j)^{V}=s_{i,i+1}^{j}. \tag{23}$$

With the weight given by equation (22) and (23), relevant information on ICUD is sufficiently provided. And an obvious and useful property of graph ICUD is its strong connectedness. It is easily noticeable that ICUD is uniquely defined by operational inputs (or timetabling data). Notice that the method introduced in this paper should be performed on a relatively integral infrastructure, which is illustrated in section 6. See (Lindner, 2011) for more details.

5.2 Infrastructure Time Allocation

As in Fig. 1, there are several time squares that are neither blocking time square nor structural time square. And notice that train $i-1$ and $i+1$ do not constitute a train pair in block section $j+1$, neither can train $i+1$ and $i+2$ in block section $j-1$ or j.

They are either the product of imperfect timetabling in terms of capacity utilization, or the result of acceptable marketing strategies. And those infrastructure time squares can sometimes be used to operate other trains, and sometimes not. They can be intuitively regarded as infrastructure time fragments. This happens when the operation routes of two or more trains partially overlap or overtakes occur.

Define the time square formed by train path i and $i+1$ in block section j, where train i and $i+1$ do not form a train pair in block section j, as fragment time square, and denote as $W(i,i+1,j)$. This is the reason why compression cannot be conducted partially on certain section of infrastructure, namely the nature of infrastructure time utilization in

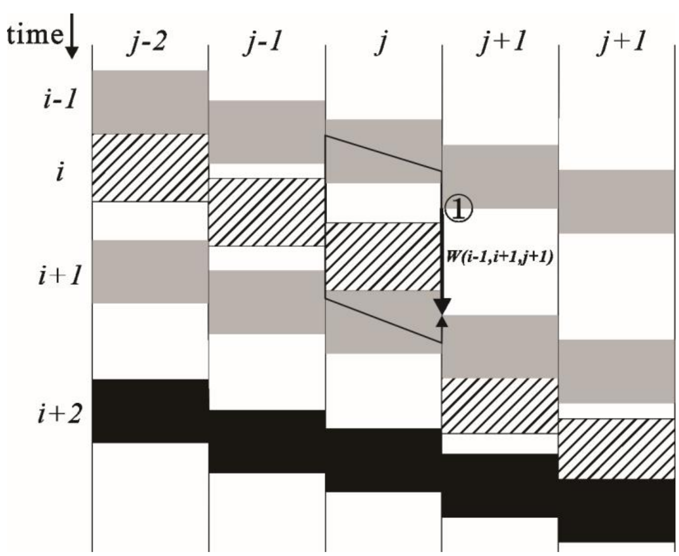


Figure 1: Infrastructure capacity utilization description

railway operations is not all identical. Mixing fragmented infrastructure time with structural indicator only produces meaningless results.

The lower blocking time of train $i-1$ in block section $j+1$ corresponds to time $y_{i-1,j+1}+r_{i-1}^{j+1}+d_{i-1}^{j+1}+D_{i-1}^{j+1}+E_{i-1}^{j+1}$, and the upper blocking time of train $i+1$ in block section $j+1$ corresponds to time $y_{i+1,j+1}-A_{i+1}^{j+1}-B_{i+1}^{j+1}-C_{i+1}^{j+1}$. The weight of the time edge on fragment time square $W(i-1,i+1,j+1)$ can be expressed as

$$L(i-1,i+1,j)^{W}=(y_{i+1,j+1}-A_{i+1}^{j+1}-B_{i+1}^{j+1}-C_{i+1}^{j+1})-(y_{i-1,j+1}+r_{i-1}^{j+1}+d_{i-1}^{j+1}+D_{i-1}^{j+1}+E_{i-1}^{j+1}). \quad (24)$$

A path made of relevant elements, which are calculable using the function system and the given information in ICUD, can be found in ICUD that linking the upper- and lower-time edges of the fragment time square. Path ① calculates the weight of the time edge of $W(i-1,i+1,j+1)$.

Incorporating information on fragment time squares produces an improved ICUD that can better visualize the allocation of infrastructure time.

5.3 Consumed Capacity

Consumed capacity, or capacity consumption, is used to express the total consumption of infrastructure capacity due to certain purpose of calculation. The consumed capacity of an infrastructure during a time period can be expressed as time needed to correspondingly go through the first occupation of the infrastructure till the last occupation of the infrastructure concerned on ICUD.

Like the calculation of infrastructure time fragment, a path can be found linking the upper time edge of time square denoting the first occupation of the infrastructure and the lower time edge of the time square denoting the last occupation of the infrastructure. And all elements can be calculated based on applying function system on given information. As in Fig. 2, a bolder polyline linking the upper edge of Ub and the lower edge of Ue presents the consumed capacity determined by timetabling data, where Ub and Ue denote the first

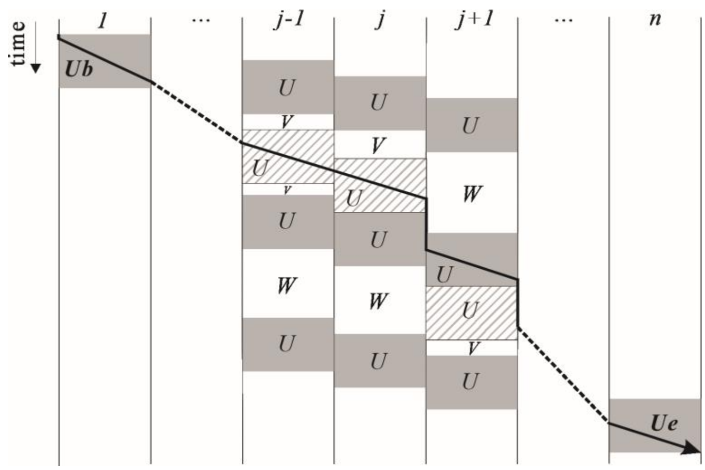


Figure 2: Calculation of consumed capacity

and last occupation of the infrastructure respectively.

As a matter of fact, more than one path of the like can be found. Among them, one path uniquely made up of only blocking time squares and critical block sections of all trains operating on the infrastructure, which we denote as the critical path and denote as P_c. Denote the distance of P_c as $L(P_c)$, and it can also be calculated in a vector-based way.

5.4 Sensitivity Analysis

In order to address the constantly required changes in operational inputs in real operations, impact of operational inputs on consumed capacity should be considered. For that purpose, the connection between capacity utilization and operational inputs must be shown.

Suppose that the operational inputs of train i are changed, which mainly includes (iv) and (vii) as in section 3. Other terms are rarely subject to changes in the short run, which can be dealt with in the same vein. A general procedure is proposed as follows:

a) Renew the sets of feasibility additives, typically set $\{I_{i-1,i}^{j}\}$ and $\{I_{i,i+1}^{j}\}$;

b) Renew the critical block sections of relevant train pairs, typically train pair $(i-1,i)$ and $(i,i+1)$;

c) Renew the blocking time squares of train i, structural time squares of relevant train pairs, typically $(i-1,i)$ and $(i,i+1)$, in ICUD;

d) Renew infrastructure time allocation in ICUD;

e) Renew P_c', and calculate $L(P_c')$, where P_c' denotes the renewed critical path.

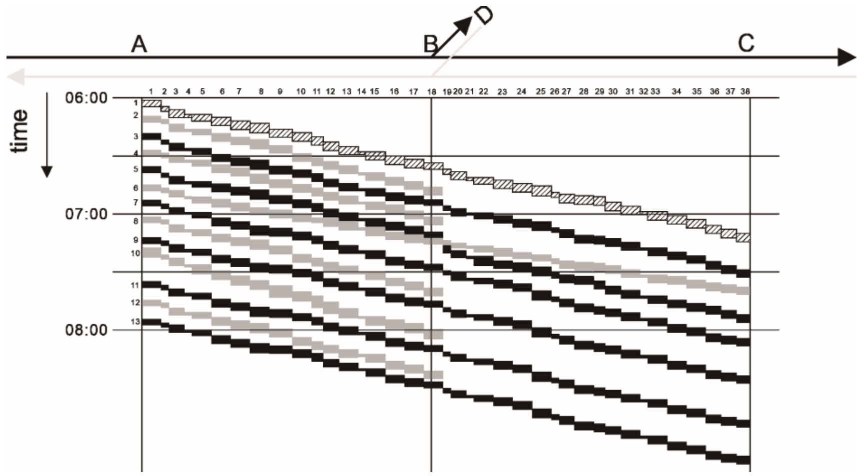
In real application of this method, step d) can be skipped when only $L(P_c')$ is required, since the renewed P_c' share certain section of the original path P_c.

6 Case Study

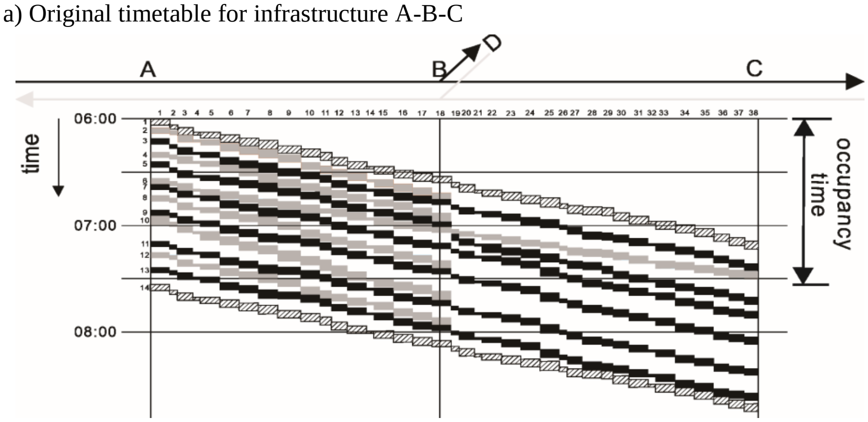
In order to demonstrate the application of proposed method in analyzing railway capacity, including the calculation of consumed capacity and its relationship with relevant parameters, an experimental case in analyzing one direction of a double-track railway infrastructure's capacity is considered in this section.

6.1 Calculation of Consumed Capacity

We consider analysing the capacity utilization of railway infrastructure from A-B-C from 06:00 to 08:00, as shown in Fig. 3 a). As an example, the timetabling follows the basic structure of blocking time theory. Station A, B, and C are terminals, and in between there are intermediate stations that operate passenger transport. As presented in the figure, there is an extra double-track railway line linking station D that is also a terminal, which in real operation causes fragmented use of railway infrastructure. On infrastructure A-B-C, there operate 13 trains of 4 types from 06:00 to 08:00 according to the definition of UIC code



a) Original timetable for infrastructure A-B-C



b) Graphic output of ICUD (compressed timetable)

Figure 3: Capacity analysis of infrastructure A-B-C

406 (UIC, 2013). Regional services 1, 3, 5, 7 ,9 11, 13 operate in section A-B-C. Regional services 2, 4, 8, 12 operate in section A-B-D. Intercity service 5 operates in section A-B-C on the infrastructure. And freight train 10 operates in section A-B on the infrastructure.

The compressed timetable on infrastructure A-B-C is presented in Fig. 3 b). Train 14 acts as the repeated train path of train 1. The first occupation of infrastructure A-B-C is by train 1 in block section 1, and the end of occupation is denoted by the upper blocking time of train 14 in block section 1. The occupancy time in section A-B-C is 93.4 min, which accounts for 77.8% of the chosen period.

Graphic representation of ICUD is a saturated timetable, which is the same as the compressed timetable generated by UIC compression. The difference of ICUD to the compressed timetable is that time edge' weight of all the time squares formed by train 1 to 13 and section 1 to 38 are calculated (which is impossible to show in the picture), presenting the capacity utilization pattern determined by timetabling data.

Using ICUD to formulate the consumed capacity according to the definition of UIC compression, the path found to calculate the consumed capacity when operating timetable shown in Fig. 3 a) can be either one that links the beginning of blocking time square (1,1) and the beginning of blocking time square (14,1). As reported in table 1, the column contribution expresses that respective train's blocking time square path contributes to the overall consumed capacity positively or negatively. The calculation result based on ICUD is $L(P_c) = 93.4\,min$, the same as that from UIC compression. As a matter of fact, the consumed capacity can be viewed as the sum of time components in vectors that denote complete infrastructure occupation. Thus, the calculation process of UIC compression can be regarded as a simplified calculation process using ICUD.

Table 1: Blocking time squares defining critical blocking time path P_c

Train	Blocking Time Squares		Contribution
	From	To	
1	U(1,1)	U(1,8)	+
2	U(2,8)	U(2,18)	+
3	U(3,18)	U(3,7)	-
4	U(4,7)	U(4,8)	+
5	U(5,8)	U(5,18)	+
6	U(6,18)	U(6,5)	-
7	U(7,5)	U(7,8)	+
8	U(8,8)	U(8,18)	+
9	U(9,18)	U(9,1)	-
10	U(10,1)	U(10,18)	+
11	U(11,18)	U(11,8)	-
12	U(12,8)	U(12,10)	+
13	U(13,10)	U(13,29)	-
14	U(14,29)	U(14,1)	-

Data source of UIC compression is ready timetable, or timetable information such as departure and arrival times at each block sections, while ICUD is based on processing operational inputs using function system. UIC compression generates compressed timetable to determine the infrastructure occupancy so that the utilization rate of the whole infrastructure can be analysed. In the meantime, ICUD comprehensively presents the utilization of railway infrastructure through the distribution of structural time squares and fragmented time squares, which can be used for various purposes.

6.2 Speed and Consumed Capacity

Consider increasing the running time of train 4 within all block sections by 3%, which influences train pair $(3,4)$ and $(4,5)$. Apply the procedure for sensitivity analysis as follows:

a) Renew set $\{I_{3,4}^j\}$ and $\{I_{4,5}^j\}$, where $j \in \{1,\ldots,18\}$;

b) Renew the critical block sections of train pair $(3,4)$ and $(4,5)$, and they are respectively $g_{3,4} = 6$ and $g_{4,5} = 18$;

c) Renew the blocking time squares of train 4, structural time squares of relevant train pair $(3,4)$ and $(4,5)$, in ICUD;

d) Renew infrastructure time allocation in ICUD;

e) After increasing running time, train 4 contributes more to the total consumed capacity. And the distance of renewed critical path is around 94.3 min, or 78.5% in terms occupancy rate.

The previous analytical process shows that ICUD can present the impact of timetabling parameters on infrastructure utilization as well as on consumed capacity. The advantage of ICUD lies in the unnecessity to repeat the whole analytical process to generate a complete new ICUD. Instead, it is done in a rather limited scope which only involves trains whose timetabling data is changed.

6.3 Recovery Time and Consumed Capacity

Recovery time is added to train running time within a block section. Using equation (18), the feasibility constant is influenced by adding recovery margin. Thus, recovery margin influences the distribution of critical block sections. Since it changes the process times of trains, which is immediately related to the contribution in consumed capacity from that train. Therefore, the real influence of recovery time must be determined through the analytical procedure described in section 5.4. In order to show the impact of recovery margin, we present a comparison of consumed capacity with and without recovery time.

Suppose that evenly-spread regular recovery time addition in every train path is 5%. Now we consider the scenario without added recovery time. The critical path P_c'' without adding recovery time is as shown in table 2.

Table 2: Blocking time squares defining critical blocking time path P_c''

Train	Blocking Time Squares		Contribution
	From	To	
1	U(1,1)	U(1,8)	+
2	U(2,8)	U(2,18)	+
3	U(3,18)	U(3,7)	-
4	U(4,7)	U(4,18)	+
5	U(5,18)	U(5,18)	+
6	U(6,18)	U(6,5)	-
7	U(7,5)	U(7,8)	+
8	U(8,8)	U(8,18)	+
9	U(9,18)	U(9,1)	-
10	U(10,1)	U(10,18)	+
11	U(11,18)	U(11,8)	-
12	U(12,8)	U(12,18)	+
13	U(13,18)	U(13,29)	-
14	U(14,29)	U(14,1)	-

Based on equation (18), running time in block section influences the calculation of feasibility constants. In this case, the adding running times only changes the critical block section of train pair (12,13). The consumed capacity without recovery time is 88.8 min, which accounts for the 74.0% within the total 2 hours. In comparison with the consumed capacity with recovery time, a 5% recovery time addition with evenly spread pattern to the timetabling data partake 4.6 min of the total consumed capacity in real timetable, which takes up 4.92% of the total consumed capacity, slightly less than 5%. Therefore, it is easy to conclude that there is no linear correlation between the claimed percentage of recovery time addition and its real influence due to the existence of blocking time elements other than running time in a block section. And it is foreseeable that the percentage representing the real influence will be smaller as more trains are included in the analysis.

7 Conclusion

In this paper, we propose an analytical tool to present capacity utilization of railway infrastructure whose operation is based on blocking time theory. The basic assumption concerning the structure of timetable is that operational inputs, mainly comprised of timetabling data, remain constant during scheduling, thereupon the sensitivity of consumed capacity to operational inputs can be considered. The critical block section of train pair is determined through comparing its feasibility additives in all block sections and can then be used for describing the capacity utilization of a train pair. A simple overtake can be viewed to be composed of several train pair during analysis, while a complex overtake can be analysed by examining the structure of each train pair composing the complex overtake. Infrastructure capacity utilization can be formulated as a graph of distributed blocking time squares and structural time squares, which can be improved by an infrastructure time allocation process that determines fragmented infrastructure usage. Based on ICUD, the overall consumed capacity can be computed, along with the general procedure to analyse the impact of parameter variations on the utilization of infrastructure capacity.

An experimental case study was reported to support the method, in which the differences of this method to UIC compression were demonstrated. Based on the results, ICUD can be used for calculating consumed capacity. And it proved to be a better tool that presents infrastructure capacity utilization when it comes to utilization analysis of railway infrastructure whose operation is dependent on a conflict-free timetable. And parametric connection between operational parameters and consumed capacity was also tested. In the paper, speed and recovery time correspond to running time a in block section. It was demonstrated in both cases that the proposed formulation of ICUD is capable of presenting influence of important parameters owing to the connection between operational inputs (mainly timetabling data) and the utilization of infrastructure.

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Appendix

Mathematical notations used in this paper are listed as follows:

Notation	Description
i	An arbitrary train operating on the infrastructure
O_i	The first block section on the operation route of train i
D_i	The last block section on the operation route of train i
$[O_i, D_i]$	The operation route of train i
$(i, i+1)$	Train pair comprised of train i and $i+1$
$o_{i,i+1}$	The first block section on the common operation route of train pair $(i, i+1)$
$d_{i,i+1}$	The last block section on the common operation route of train pair $(i, i+1)$
$[o_{i,i+1}, d_{i,i+1}]$	The common operation route of train pair $(i, i+1)$
j	An arbitrary block section on the operation route of train i or train pair $(i, i+1)$
A_i^j	The time needed to set up the signal to operate in block section j for train i
B_i^j	Time needed for the train driver to confirm the signal to approach block section j for train i
C_i^j	The time needed for train i to end block section j
r_i^j	The time needed for train i to cover the whole block section j
D_i^j	The time needed for train i to clear block section j
E_i^j	The time needed to release the signal of block section j after the traverse by train i
d_i^j	The duration of a scheduled stop of train i at station in block section j
$(1, \cdots, i, \cdots, m)$	The sequence of train departing from certain block section of the infrastructure
$(i, i+1) \xrightarrow{j} (i+1, i)$	A change of operation sequence from $(i, i+1)$ to $(i+1, i)$ at station in block section j
$p_i^{O_i, j}$	The entry process time of train i from block section O_i to j
$b_{i,up}^{O_i, j}$	The upper blocking time of train i in block section j from the origin section O_i
$b_{i,low}^{O_i, j}$	The lower blocking time of train i in block section j from the origin section O_i
$t_{i,i+1}^{o_{i,i+1}, j}$	The blocking time difference of train pair $(i, i+1)$ in block section j

Notation	Description
$y_{i,j}$	The departure time of train i from block section j
$m_{i,j}^{low}$	The mark of the lower blocking time of train i in block section j on the time axis
$I_{i,i+1}^{j}$	The feasibility constant of train pair $(i,i+1)$ in block section j
$g_{i,i+1}$	The critical block section of train pair $(i,i+1)$
μ	An arbitrary train that forms a train pair with train i in a complex overtake
$s_{i,i+1}^{j}$	The structural indicator of train pair $(i,i+1)$ in block section j
$U(i,j)$	The blocking time square formed by train i in block section j
$L(i,j)^{U}$	The weight of blocking time square's time edge
$V(i,i+1,j)$	The structural time square formed by train pair $(i,i+1)$ in block section j
$L(i,i+1,j)^{V}$	The weight of structural time square's time edge
$W(i,i+1,j)$	The fragment time square form by train i and $i+1$ in block section j
$L(i,i+1,j)^{W}$	The weight of fragment time square's time edge
P_c	The critical path
$L(P_c)$	The distance of critical path P_c

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