An Optimization Model for Rescheduling Trains to Serve Unpredicted Large Passenger Flow

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Abstract
As the separation of vertically-integrated organizations in railway transportation, not only the competitive but also the collaboration between different operating companies and different modes should be considered emphatically in the rapidly changing multimodal transportation market. This paper tries to solve the Train Timetable Problem for serving Unpredicted Large Passenger Flow causing by the stop of air traffic in collaborating with air transportation companies. We address the Unpredicted Large Passenger Flow as a perturbation in normal train dispatching and solve this problem through an optimization approach. Two strategies of reassigning remaining seats and inserting new trains are adopted to establish integer programming model in dispatching to evacuate unpredicted passengers. The proposed model is solved by a standard CPLEX solver and test through a study case. The effectiveness of the proposed model is demonstrated in the study case and both two strategies take part in serving ULPF.

Keywords
Train rescheduling, Collaboration, Unpredicted large passenger flow, Inserting new trains

1 Introduction

Railway, with its capacity of transporting large passenger flow, plays an important role in the rapidly changing multimodal transportation market. However, the competitiveness of railway is receded sharply over the years. How to maintain and further improve the competitiveness are of great importance for railway companies and their operators. As the separation of vertically-integrated organizations of railway, train operating companies always concentrate on the competitiveness with others and other transportation modes. Nevertheless, not only the competitiveness but also the collaboration (i.e., complementation and connection) between different companies and transportation modes should be considered emphatically. While unavoidable perturbations (e.g. bad weather) disrupt airport causing stop of air transportation, a large amount of passengers are remained causing unpredicted transporting demand, which is called Unpredicted Large Passenger Flow (simply for ULPF) in this paper. The problem encountered by dispatchers of railway is how to rescheduled train timetables in collaborating with air transportation, for the purpose of win-win situation.

Traditionally, a sequential process consisting of line planning, train timetabling, rolling stock and crew scheduling is used for planning train operations. The outcome of each stage is used as an input of the following stage (Desaulniers and Hickman, 2007). While a planned timetable is put into operation, unavoidable stochastic perturbations (e.g., bad weather, large passenger flow, capacity breakdowns) may influence the scheduled train running and dwelling times causing delays, thus the timetables need to be rescheduled to recover common(Luan et al., 2017). Always, passenger demand is an input of a line plan rather than other stages including rescheduling.

In this paper, we focus on generating an optimal dispatching solution for serving ULPF. We solve the Train Timetables Problem for serving Unpredicted Large Passenger Flow (TTP-ULPF) through an optimization approach to explicitly consider the characteristics of passengers from the stop of air transportation. We considered ULPF as a stochastic perturbation in the normal rescheduling, and two strategies of organizing remained seats and inserting new trains, are adopted to serve ULPF. The proposed integer programming model for formulating the TTP-ULPF problem is solving by a standard CPLEX solver.

The remainder of this paper is organized as follows. Section 2 provides a detailed literature review on relevant studies. In Section 3, the ULPF is described visually. In section 4, a mathematical model is proposed to reschedule timetables for serving ULPF with the statement of rail network and model assumption, followed by a case study in Section 5, which quantify the trade-off between the delay cost of existing passengers and the revenue of increasing new passengers. Finally, conclusions and further research are given in Section 6.

2 Literature Review

2.1 Train rescheduling

The train rescheduling problem has been studied in the past few decades. Carey and Lockwood (1995) presented a mixed integer programming model and solution algorithms for the train timetabling problem on a double-track rail line. Carey (1994a) further developed an extended model to consider more general and more complex rail networks with possible choices of lines and station platforms. A companion paper by Carey (1994b) proposed an extension from one-way to two-way rail lines. Caprara et al. (2002) proposed a graph-theoretic formulation for the periodic-timetabling problem using a directed multi-graph by incompatible arcs and forbid the simultaneous selection of such arcs through a novel concept of clique constraints. This formulation is used to derive an integer linear programming model that is relaxed in a Lagrangian way, which embedded within a heuristic algorithm that makes extensive use of the dual information associated with the Lagrangian multipliers. Depending on the basic problem of TTP, Caprara et al. (2006) proposed a mathematical model incorporating several additional constraints (e.g., Manual block signalling for managing, station capacities, prescribed timetable for a subset of the trains and Maintenance operations). Meng and Zhou (2014) develop an Integer Programming model for the problem of train dispatching on an N-track network by means of simultaneously rerouting and rescheduling trains. A vector of cumulative flow variables was introduced by them to reformulate the track occupancy so that they can decompose the original complex rerouting and rescheduling problem efficiently into a sequence of single train optimization sub-problems. The decompose mechanism provide us a method to deal with large-scale optimal problems of train dispatching.

On the other hand, inserting new trains into existing timetables is a critical manner in rescheduling. Cacchiani et al. (2010) describe a problem for inserting new freight trains, which send requests for infrastructure usage, to existing passenger trains timetables. An Integer Linear Programming (ILP) model with the objective of total deviation between the actual timetable and the ideal one of all the freight trains is proposed, and solved by Lagrangian heuristic solution. It is a large-scale dispatching problem, since timetables should be rescheduled associating with new trains added. However, inserting new trains into existing timetables was used by Cacchiani et al. (2010) in the offline scheduling, while the capacities of network have not been used completely. The main goal of the study is to schedule the timetables of inserting train more close to the ideal ones, with the existing trains fixed. If we used this method to serve ULPF, it is an online scheduling, as all train timetables are on duty, and no train was fixed or has priority than others.

2.2 Railway transportation in multimodal market

Recently, the issue of competition between different operating companies received much attention in multimodal transportation market. Directive 91/440/EC (Commission of the European Communities, 1991) introduced separation of concerns between IM and TOCs. The IM holds a monopoly in the supply of access to its network and has the duty of providing fair and non-discriminatory access to the available infrastructure capacity. The TOCs are companies that compete to offer services to customers. Luan et al. (2017) focus on competition between different train operating companies. A Mixed Integer Linear Programming (MILP) model is proposed by Luan et al. (2017) to describe the trade-off between equity and delays in non-discriminatory train dispatching in multimodal transportation market. However, not only competition between different operating companies exists in the multimodal transportation market, but also the collaboration. Researchers pay more attention on competition, but less on collaboration, which reflect abilities (include stabilities and reliabilities) for the enhancement of competitiveness, as the research by Luan et al. (2017).

2.3 Paper contributions

There are three major contributions in this paper as followed:

(1) This paper focus on the train rescheduling problem with consideration of collaboration with air transportation, which is not found in previous studies to our best knowledge. It makes a step forward to perfect rescheduling trains in multimodal transportation market. It provides a model for cooperation between different transportation modes.

(2) This paper develops an ILP model considering jointly the balance of delays of existing passengers and revenue of unpredicted new passengers in the emergency situation, which are studied separately in previous researches. Thus, the trade-off between the above two represents one important contribution of this paper.

(3) In addition, the rescheduling planning generated by the model proposed in this paper, can give a supplement for existing frame of research.

3 Problem Description

Before formulating the TTP-ULPF problem, we first explain the terms used in describing the ULPF in the following formulations.

In this paper, we address the optimization problem of rescheduling trains to serve ULPF, which comes into being with the characteristics of 1) nonstop between original and destination metropolises 2) having willing to pay high cost for short travel time. Therefore, high-speed railway is first and foremost considered in this paper to serve ULPF.

As the transportation mode (e.g. travel time, stop manners, etc.) of railway has significantly different from air traffic, not all the passengers from disrupted air transportation have willing to transfer to railway. In order to contact the willing of ULPF and dispatching manners, a concept of time interval is introduced to depict the relationship. The time interval in this paper is the gap between expected arrival time of ULPF at its destination and the actual arrival time. The relationship of time interval and passengers' willing to transfer can be observed through investigation, and is regard as a linear function for assumption in this paper. Table 1 list the relationship between the volume of passenger willing to transfer and time interval at its destination (maximum volume: 100). We assigned that all the passengers have willing to transfer while the train to serve them departure from origin at the time that passengers generated and do not stop at any intermediate stations. And the volume reduced with the addition of time interval linearly by 5% per minute as shown in Table 1.

Table 1: Relationship between passengers volume and time interval

Time interval (min)	**Passengers volume**
0	100
1	95
2	90
3	85
4	80
5	75

Two strategies can used to serve ULPF transferred from air transportation: 1) organizing the seats remained in the planned trains; 2) inserting a new train. Obviously, inserting a new train is not a feasible manner to serve ULPF in congested timetables. But while the ULPF generated, the time is too close for existing trains to have enough remained seats for serving ULPF. Therefore, both of the two strategies should be used to realize the goal in this paper. It is hard to insert a new train in an existing timetable, since the timetables of some lines are too dense that there is no interspace between any of two trains to insert without changing their prescribed arrival/departure time.

The solution of inserting a new train in the congested timetables is to use the recovery time in the running and dwelling time of a train and the buffer time between two trains in the existing timetables. Fig.1 depicts a simple timetable with 3 stations and 2 segments. Three trains operate from station A to station C in the existing timetables in the Fig.1(a). It is easy to see that, both the running and dwelling time of existing trains and the headway between any of two consecutive trains are reduced to the limited value (e.g. 5min, 1min and 3min) to obtain time gap to insert a new train as illustrate in Fig.1(b). This strategy explores the trade-off between the revenue of inserting a new train and delay cost of existing trains at a part of the intermediate stations.

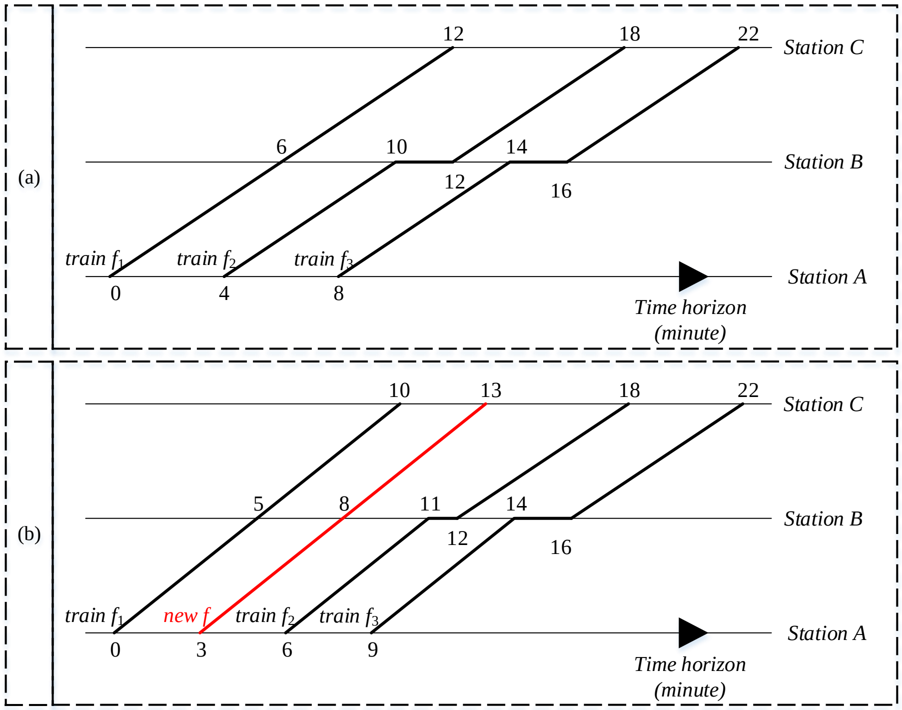

Figure 1: A sample of timetables

4 Mathematical Formulation

4.1 Description of railway network

In this paper, we focus on a simple railway network with only one line that consist of a sequence of station and double track segments between two consecutive station. Fig.2 and Fig.3 illustrates two networks for instance at microscopic level and modelling level considered in this paper respectively. In Fig.2, the railway network is consist of double track, signal and platform. The segment between two stations is divided into several block sections for the purpose of train safety. The station is also regard as two or more block sections according to the numbers of siding tracks.

The network in Fig.2 can be further simplified as shown in Fig.3, which the railway network is described as $G = (N, E)$ with a set of nodes N and a set of cells E. In order to explain the space-time network, two concepts should be introduced in this paper, i.e. node and cell. A cell represents a block section, and a node represents a beginning/ending point of block section. A station is regard as a node for simplicity, since the routing in the station make no difference to the objective and the capacity of station is assumed as sufficient in this paper. Therefore, two set of nodes are defined in our problem: a station node represents a station in physical network where trains can stop for loading/unloading and crossing which is shown as big dot in Fig.3; a segment node represents the point between two adjacent block sections where trains cannot stop which is shown as little dot in Fig.3. A cell is a vector directed from a starting node i to an ending node j, as well as

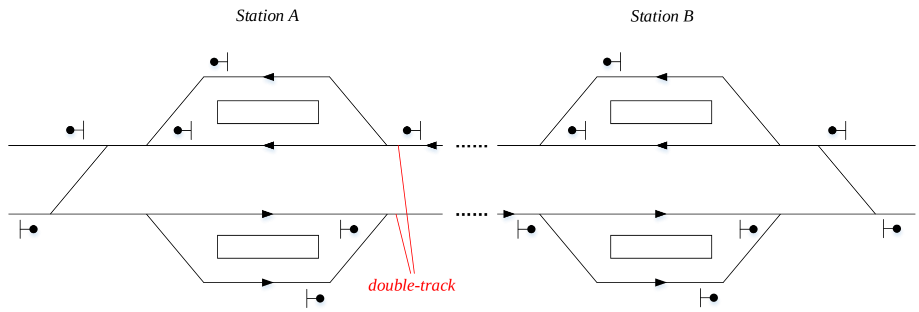

Figure 2: Railway physical network

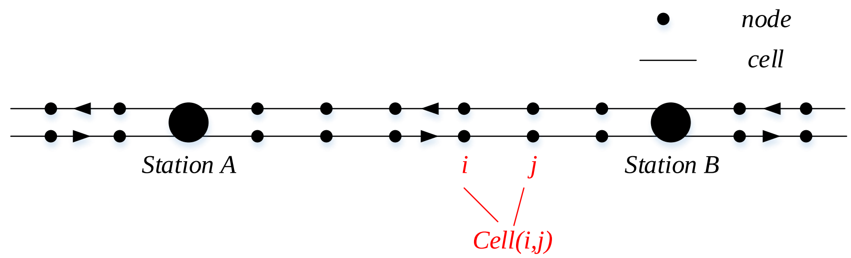

Figure 3: Modelling network

the minimum running unit for a train. The default of cell capacity in this paper is one at any given time, so that any of two trains cannot occupy one cell simultaneously.

4.2 Problem statement

In this optimization problem, the external inputs include:

(1) A high-speed railway (HSR) line given with stations and segments. Stations are simplified to a number of nodes, and the double-track segments are modelled as a sequence of directional cells, as illustrate in Fig.3.

(2) A set of existing trains with their origins, destinations, prescribed arrival and departure time at each cells, free flow running time at each segments, minimum dwelling time at each stations, loading quantity of passengers at each stations, and remaining seats between different origin and destination (OD) pairs.

(3) A set of candidate trains for inserting with their origins, destinations, earliest departure time at original station, free flow running time at each cells, minimum dwelling time at each stations, and capacity for transporting passengers.

(4) A set of ULPF with their origin and destination (OD), expected departure and arrival time at OD stations, and quantity of passengers.

The models proposed in this paper result in determining the arrival/departure time and train orders at each cell of all the trains, include new inserting train. Note that the granularity of time is one minute.

Six major assumptions are considered in the following formulations:

(1) A station is assumed as a node in this paper, since the routing and capacity of the station is not considered.

(2) The length of a train is assumed to be zero.

(3) Passengers' transfer in the intermediate station is not considered in this paper,

which means passengers can only take direct trains from origin to destinations.

(4) The value of 1)volume of ULPF 2)remaining seats in the existing trains 3)numbers of loading passengers at each station are all known before rescheduled.

(5) In the process of serving ULPF, other disruptions are not occurred for simplicity.

(6) We assumed that all the ULPF have the same origin and destination (OD), and cannot be divided furthermore.

4.3 Notation

Table 2-4 list the subscripts, input parameters and decision variables respectively.

Table 2: Subscripts

Symbol	Description
i, j, k	Node index, $i, j, k \in N$, N is the set of nodes, $N = N_s \cup N_r$, N_s is the set of station nodes and N_r is the set of segment nodes
e	Cell index, generated by two adjacent nodes i and j, $e = (i, j) \in E$, E is the set of cells
f	Train index, $f \in F$, F is the set of trains, $F = F_1 \cup F_2$, F_1 is the set of existing trains and F_2 is the set of candidate inserting trains
M	A sufficiently large positive number

Table 3 Input parameters

Symbol	Description
N_f	Set of station nodes train f need to stop for loading/unloading, $N_f \in N_s \in N$
E_f	Set of cells train f may use, $E_f \in E$
$w_f^{min}(i)$	Minimum dwell time for train f at station node i
$\vartheta_f(i, j)$	Free flow running time for train f to drive through the cell (i, j)
o_f	Origin node of train f
s_f	Destination node of train f
ε_f	Earliest departure time of train f from its origin node
ϵ_f	Latest arrival time of train f at its destination node
$\bar{a}_f(i, j)$	Predetermined arrival time of existing train f on cell (i, j), $f \in F_1$
$\bar{d}_f(i, j)$	Predetermined departure time of existing train f on cell (i, j), $f \in F_1$
o_p	Origin node of ULPF
s_p	Destination node of ULPF
ψ_o	Ideal departure time of ULPF from its origin node
ψ_d	Ideal arrival time of ULPF at its destination node
p	Passenger number of ULPF
$p_f(i, j)$	Remaining seats of train f between nodes i and j
$\overline{p}_f(i)$	Number of loading passengers on train f at node i
λ_a	Delay cost of each passenger on existing trains
λ_b	Loss cost of each passenger of ULPF failure to transfer to railway

Table 4 Decision variables

Symbol	Description
$x_f(i,j)$	0-1 binary routing variables, $x_f(i,j)=1$, if train f used cell (i,j) at some time, and otherwise $x_f(i,j)=0$
$a_f(i,j)$	Arrival time of train f on cell (i,j)
$d_f(i,j)$	Departure time of train f on cell (i,j)
$\theta(f,f',i,j)$	0-1 binary train ordering variables, $\theta(f,f',i,j)=1$, if train f' arrive at cell (i,j) after train f, and otherwise $\theta(f,f',i,j)=0$
$T_f(i,j)$	Running time for train f on cell (i,j)
$TT_f(i)$	Delay time of train f at station node i
$\delta_f(i,j)$	Passenger number of ULPF transport from origin node i to destination node j through train f

4.4 Mathematical model

A mathematical model, which formulizes inserting trains to serve ULPF by a set of constraints, is first presented. The objective is 1) to minimize the total delay costs of passengers in existing trains 2) and to maximize the revenues of increasing passengers transferred from ULPF simultaneously. Since the two objective is on the contrary, we transfer the revenues of increasing passengers transferred from ULPF to the loss costs of passengers failure to transfer from ULPF, as formulated in Eq.(1).

$$Min\ C=\sum_{f\in F_1}\sum_{i\in N_f\backslash\{o_f\}}\lambda_a\times\overline{p}_f(i)\times TT_f(i)+\sum_{f\in F}\lambda_b\times\left(p-\delta_f(o_p,s_p)\right) \qquad (1)$$

Subject to

Group I: Flow balance constraints

Flow balance constraints at origin node:

$$\sum_{j:(o_f,j)\in E_f}x_f(o_f,j)=1,\forall f\in F_1 \qquad (2)$$

$$\sum_{j:(o_f,j)\in E_f}x_f(o_f,j)\le 1,\forall f\in F_2$$

(3)

Flow balance constraints at intermediate nodes:

$$\sum_{i:(i,j)\in E_f}x_f(i,j)=\sum_{k:(j,k)\in E_f}x_f(j,k),\forall f\in F,j\in N\backslash\{o_f,s_f\}$$

(4)

Flow balance constraints at destination node:

$$\sum_{i:(i,s_f)\in E_f}x_f(i,s_f)=1,\forall f\in F_1$$

(5)

$$\sum_{i:(i,s_f)\in E_f}x_f(i,s_f)\le 1,\forall f\in F_2$$

(6)

Group II: Running and dwelling time constraints

Running time constraints:

$$T_f(i,j)=d_f(i,j)-a_f(i,j),\forall f\in F,(i,j)\in E_f$$

(7)

Minimum running time constraints:

$$T_f(i,j)\ge x_f(i,j)\times\vartheta_f(i,j),\forall f\in F,(i,j)\in E_f$$

(8)

Minimum dwelling time constraints:

$d_f(i,j) + w_f^{min}(j) \leq a_f(j,k), \forall f \in F, j \in N_f \backslash \{s_f\}, (i,j) \in E_f, (j,k) \in E_f$
(9)

Group III: Time-space network constraints

Starting time constraints at origin node:

$$a_f(o_f, j) + \left(1 - x_f(o_f, j)\right) \times M \geq \varepsilon_f, \forall f \in F, (o_f, j) \in E_f \quad (10)$$

Ending time constraints at destination node:

$$d_f(i, s_f) + \left(1 - x_f(i, s_f)\right) \times M \leq \epsilon_f, \forall f \in F, (i, s_f) \in E_f \quad (11)$$

Departure time constraints at intermediate node:

$$a_f(i,j) \geq \overline{a}_f(i,j), \forall f \in F_1, i \in N_f, (i,j) \in E_f \quad (12)$$

Cell transition constraints:

$$d_f(i,j) \geq a_f(i,j), \forall f \in F, (i,j) \in E_f \quad (13)$$

Cell-to-cell transition constraints at station nodes:

$$\sum_{i:(i,j) \in E_f} d_f(i,j) \leq \sum_{k:(j,k) \in E_f} a_f(j,k), \forall f \in F, j \in N_f \quad (14)$$

Cell-to-cell transition constraints at segment nodes:

$$\sum_{i:(i,j) \in E_f} d_f(i,j) = \sum_{k:(j,k) \in E_f} a_f(j,k), \forall f \in F, j \in N \backslash N_f \quad (15)$$

Mapping constraints between time-space network and physical network:

$$x_f(i,j) - 1 \leq a_f(i,j) \leq x_f(i,j) \times M, \forall f \in F, (i,j) \in E_f \quad (16)$$

$$x_f(i,j) - 1 \leq d_f(i,j) \leq x_f(i,j) \times M, \forall f \in F, (i,j) \in E_f \quad (17)$$

Group IV: Inserting trains constraints

Starting time constraints of inserting trains to serve ULPF:

$$a_f(o_f, j) + \left(1 - x_f(o_f, j)\right) \times M \geq \psi_o, \forall f \in F_2, (o_f, j) \in E_f \quad (18)$$

Number constraints of inserting trains

$$\sum_{f \in F_2} \sum_{j:(o_f, j) \in E_f} x_f(o_f, j) \leq 1 \quad (19)$$

Group V: Mapping constraints between two types of decision variables

Mapping constraints between train orders and cell usage:

$$x_f(i,j) + x_{f'}(i,j) - 1 \leq \theta(f, f', i, j) + \theta(f', f, i, j) \leq 3 - x_f(i,j) - x_{f'}(i,j), \forall f \in F, f' \in F, f \neq f', (i,j) \in E_f \cap E_{f'} \quad (20)$$

$$\theta(f, f', i, j) \leq x_f(i,j), \forall f \in F, f' \in F, f \neq f', (i,j) \in E_f \cap E_{f'} \quad (21)$$

$$\theta(f, f', i, j) \leq x_{f'}(i,j), \forall f \in F, f' \in F, f \neq f', (i,j) \in E_f \cap E_{f'} \quad (22)$$

Mapping constraints between passengers transportation and cell usage

$$x_f(o_f, j) - 1 \leq \delta_f(o_p, s_p) \leq x_f(o_f, j) \times M, \forall f \in F_2, o_p \in N_f, s_p \in N_f, (o_f, j) \in E_f \quad (23)$$

Group VI: Capacity constraints on the same cell

$$a_{f'}(i,j) + \left(3 - x_f(i,j) - x_{f'}(i,j) - \theta(f, f', i, j)\right) \times M \geq d_f(i,j), \forall f \in F, f' \in F, f \neq f', (i,j) \in E_f \cap E_{f'} \quad (24)$$

Group VII: Delay time constraints

$$TT_f(j) \geq d_f(i,j) - \overline{d}_f(i,j), \forall f \in F, j \in N_f, (i,j) \in E_f \quad (25)$$

$$TT_f(j) \leq \left| d_f(i,j) - \overline{d}_f(i,j) \right|, \forall f \in F, j \in N_f, (i,j) \in E_f \quad (26)$$

$$TT_f(j) \geq 0, \forall f \in F, j \in N_f, (i,j) \in E_f \quad (27)$$

Group VIII: ULPF constraints

Passenger volume constraints:

$$0 \leq \delta_f(o_p, s_p) \leq p, \forall f \in F, o_p \in N_f, s_p \in N_f \quad (28)$$

$$\delta_f(o_p, s_p) \leq 0, \forall f \in F: \varepsilon_f < \psi_o, o_p \in N_f, s_p \in N_f \quad (29)$$

$\delta_f(o_p, s_p) \le p \times \left(1 - 5\% \times \left(d_f(i, s_p) - \psi_d\right)\right), \forall f \in F, (i, s_p) \in E_f, o_p \in N_f, s_p \in N_f$ (30)

$\delta_f(o_p, s_p) \le p_f(o_p, s_p), \forall f \in F, o_p \in N_f, s_p \in N_f$ (31)

$\sum_{f \in F, o_p \in N_f, s_p \in N_f} \delta_f(o_p, s_p) \le p$ (32)

In Group I, constraints (2)-(6) ensure the consistency of trains' movement in the network at their origin, intermediate and destination nodes respectively. Note that the flow of trains at their origin and destination nodes in Eq.(3) and Eq.(6) is not identical equal to one, as not all the trains in candidate inserting set need to put into operation necessarily.

In Group II, constraint (7) defines the required running time on cells. Constraints (8) and (9) force the minimum running time on cells and minimum dwelling time at station nodes respectively.

In Group III, constraints (10) and (11) guarantee that trains do not leave their origin nodes before earliest departure time and not reach their destination nodes after latest arrival time respectively. Constraint (12) make sure that existing trains do not leave intermediate station nodes before the prescribed departure time, so as the passengers predetermined can boarding successfully. Constraints (13) and (14) enforce the sequential time orders between departure time and arrival time on the cells and at the station nodes respectively. Constraint (15) further makes sure that all trains cannot stop at segment nodes. Constraints (16) and (17) are imposed to map the arrival and departure time in time-space network to the cell usage variables in physical network, so as to describe the relationship between cells selection of a train and its timetables.

In Group IV, constraint (18) further guarantees that the departure time of inserting trains cannot be early than ideal departure time of ULPF at their origin nodes, so as the strategy of inserting is effective for serving ULPF. Constraint (19) denotes the total quantity of inserting trains.

In Group V, constraints (20)-(22) link train orders variables and cell usage variables. Additionally, if and only if both train f and train f' use cell (i, j), the two trains have the sequential order $\theta(f, f', i, j) = 1$ or $\theta(f', f, i, j) = 1$, that is either train f arrives at cell after train f' or the opposite condition. If $x_f(i, j) = 0, x_{f'}(i, j) = 1$ or $x_f(i, j) = 1$, $x_{f'}(i, j) = 0$ or $x_f(i, j) = 0$, $x_{f'}(i, j) = 0$, constraints (20) reduce to non-active inequalities. Constrain (23) link passengers' transportation variables and cell usage variables. If and only if the inserting train f use cell (i, j), i.e., $x_f(i, j) = 1$, the number of passengers served by train f is greater than or equal to zero, else $\delta_f(o_p, s_p) = 0$.

In Group VI, constraint (24) explicitly makes sure that any of two trains cannot occupy the same cell simultaneously at any given time. Note that for train f and f' traversing on cell (i, j), i.e., $x_f(i, j) = x_{f'}(i, j) = 1$, constraint (24) can be reduced to common if-then conditions as follow: If train f arrives at cell after train f', i.e., $\theta(f, f', i, j) = 1$, then the arrival time of train f' should be no earlier than the departure time of train f on cell (i, j); else the constraint reduce to non-active inequality.

In Group VII, constraints (25)-(27) define the delay time of existing trains at each station nodes. If train f arrive at the station node j before its predetermined time point, i.e., $d_f(i, j) \le \overline{d}_f(i, j)$, then the delay time $TT_f(j) = 0$; else delay time $TT_f(j) = d_f(i, j) - \overline{d}_f(i, j)$.

In Group VIII, constraints (28) and (32) restrict range of the numbers of passengers from ULPF. Constraint (29) expresses that an existing train with its departure time earlier than the ideal departure time of ULPF cannot be used to serveULPF. Constraint (30) indicate the function relationship between arrival time and numbers of served passengers of each train f, in which the reduction is assumed as 5% per minute for simplicity. Constrain (31) make sure that train f has enough seats for serving ULPF.

5 Numerical experiments

5.1 Experimental setup

The optimization model, which proposed for serving Unpredicted Large Passengers Flow (ULPF), is implemented as an integer programming model through a commercial solver ILOG CPLEX by IBM with version number 12.3. All the following experiments are performed on a Lenovo PC with 2.3GHz Intel i5-6200U CPU and 8 GB memory.

Due to the protection of commercial data, we couldn't get detailed block sections data of the Beijing-Shanghai high-speed railway (HSR) line. Therefore, we use an assumed line with the background of Beijing-Shanghai HSR line as the test bed. The total length of this line is 180 km with 4 stations and 30 block sections, as illustrate in Fig.4. There are totally 9 trains being dispatched in our case study, including 6 existing trains and 3 candidate inserting trains. The time horizon of this case is 52 minutes. The minimum running time in the segments and the minimum dwelling time at stations are 12 min and 1 min respectively. The OD station of ULPF are Sta_A and Sta_D respectively, and the quantity of ULPF is assumed as 1000 in this paper, which is the same as the capacity of a train.

Table 5: Timetables of existing trains (unit: min)

Train ID	Station A	Station B		Station C		Station D
	Departure time	Arrival time	Departure time	Arrival time	Departure time	Arrival time
1	0	12	14	26	28	41
2	4	16	16	28	30	43
3	6	18	20	32	32	45
4	10	22	22	34	34	47
5	12	24	24	36	36	49
6	14	26	26	38	38	51

Table 6 Loading volume of existing passengers

Train ID	Station A	Station B	Station C	Station D
1	——	200	400	900
2	——	——	400	900
3	——	400	——	900
4	——	——	——	900
5	——	——	——	900
6	——	——	——	900

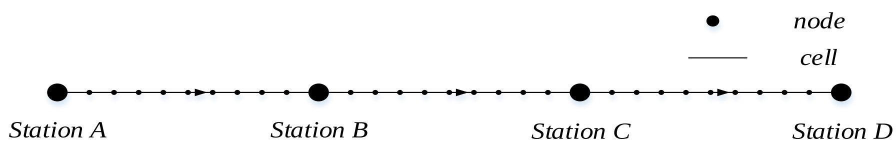

Figure 4: A sample network for case study

Table 7 Remaining seats between each OD pair

Train ID		Station B	Station C	Station D
1	Station A	50	50	100
	Station B	——	50	50
	Station C	——	——	50
2	Station A	——	50	100
	Station B	——	——	——
	Station C	——	——	50
3	Station A	50	——	100
	Station B	——	——	50
	Station C	——	——	——
4	Station A	——	——	100
	Station B	——	——	——
	Station C	——	——	——
5	Station A	——	——	100
	Station B	——	——	——
	Station C	——	——	——
6	Station A	——	——	100
	Station B	——	——	——
	Station C	——	——	——

The detailed information of arrival/departure time of existing trains is shown in Table 5, and the detailed information of loading passenger number at each station and remaining seats between each OD pair are illustrated in Table 6 and 7.

5.2 Experimental results

There are 6 existing trains and 3 inserting trains calculating in the study case. The number of variables and constraints are 2199 and 6406 respectively. The computational time is about 5.87 seconds on the platform stated above. The result of this case study is illustrated in Table 8, Table 9 and Fig.5.

As list in Table 8, one of the trains in candidate set, i.e., train ID9, is inserted from station A to station D to serve ULPF with the minimum travel time of 36min and the mode of nonstop at intermediate stations. The timetables before and after inserting are depicted intuitively in Fig.5 for convenient exhibition.

Due to the inserting of new train ID9, all of the existing trains are affected at their intermediate stations and/or destination stations. Owing much to the recovery time predetermined in the running and dwelling time of existing trains, all the passengers on train 3-6 are not affected at their destinations. However, a part of passengers on train 1 and 2 are not so lucky due to the delay of trains with totally about 1600 person-time as a trade-off for inserting new train. Due to the shorter travel time of inserting train, all the passengers causing by stop of air transportation are willing to transfer to this train to their destinations, as illustrated in Table 9. The reason why passengers do not take the existing trains is not uniform. Train 1 do not take part in serving ULPF only because of its earlier departure time than the ideal time of ULPF. And the reason of train 2-6 is that their arrival time at destination are later than the inserting new train.

Table 8 Computational result of all trains (unit: min)

Train ID	Inserting or not (I/N)	Station A	Station B		Station C		Station D
		Departure time	Arrival time	Departure time	Arrival time	Departure time	Arrival time
1	——	0	12	16	28	29	41
2	——	4	16	18	30	31	43
3	——	6	18	20	32	33	45
4	——	10	22	22	34	35	47
5	——	12	24	24	36	37	49
6	——	14	26	26	38	39	51
7	N	——	——	——	——	——	——
8	N	——	——	——	——	——	——
9	Y	2	14	14	26	26	38

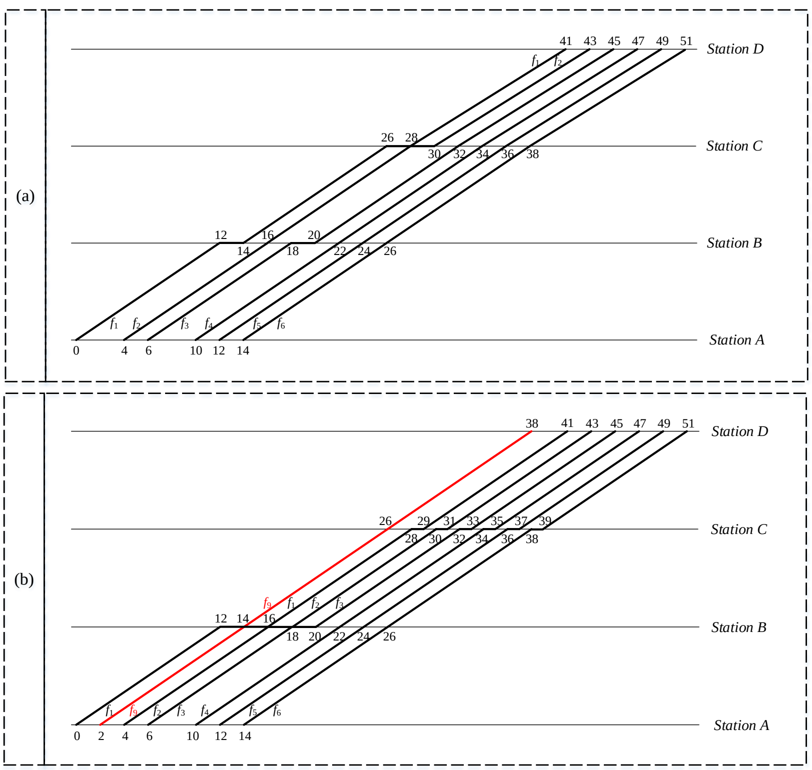

Figure 5: Timetables before and after inserting

Table 9 Computational result of passenger from ULPF transported in each train

Train ID	1	2	3	4	5	6	7	8	9
Passengers volume	0	0	0	0	0	0	—	—	1000

6 Conclusion and Future Research

This paper concentrates on solving the TTP for serving ULPF causing by the stop of air traffic in collaborating with air transportation companies. We address this problem through an optimization approach to explicitly consider ULPF as a stochastic perturbation in normal dispatching. Two strategies of organizing remaining seats and inserting new trains are adopted to formulate the integer programming model to serve ULPF. The proposed model is solved by a standard CPLEX solver and test through a study case. The effectiveness of the proposed model is demonstrated in the study case and both two strategies take part in serving ULPF.

Our future research would address the following main extensions.

(1) Station capacity and routing are not considered in this paper. Our future research is to develop a model incorporating these constraints.

(2) We assume that passengers take a direct train to arrive at destinations in the proposed model. The next step, we are going to relax this assumption and to consider the transfer of passengers.

(3) In this paper, the schedule of rolling stock is ignored for simplicity. One may take rolling stock schedule, even the crew schedule into account to better represent the realistic conditions.

(4) In the computational experiments, we use a line with only 4 station and 3 segments. In the future study, one can enlarge the scale of the network and solve the model using heuristic algorithm.

(5) The weight of delay cost of existing passengers and increased revenue of ULPF is assumed in this paper. One can concentrate on the study of influence factors of the weight and calculate the value more close to realistic condition for the further study.

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