The Comparison of Three Strategies in Capacity-oriented Cyclic Timetabling for High-speed Railway

Xin Zhang[a], Lei Nie[a,1], Yu Ke[a]
[a] School of Traffic and Transportation, Beijing Jiaotong University, Beijing, China
[1] E-mail: lnie@bjtu.edu.cn, lnie8509@yahoo.com, Phone: + 86 010 51688173

Abstract
The expansion of the scale of high-speed railway networks and the growth of passenger demand imply a high frequency of high-speed trains in China, i.e. higher railway capacity utilization. Based on given infrastructures and train line plans, there are some timetabling strategies which affect the capacity utilization, e.g. changing train departure sequence at origin stations, overtakings between trains, and adding new train stop at stations. Nowadays, managers of high-speed railway in China are eager to find out that what kind of impact these strategies have on the capacity utilization. In this study, new variables of train stops and constraints of overtakings are proposed with an extended cyclic timetabling model based on the periodic event scheduling problem (PESP). Minimum cycle time, train travel time and the total number of train stops are calculated as objectives to measure the differences between the strategies. The effectiveness of the three timetabling strategies are compared and presented by a series of experiments based on one real-world rail line in China. According to our results, with flexible train departure sequence at the origin stations and train overtakings, the possibility of acquiring good capacity utilization can be higher, but too many overtakings will have negative effect on the quality of timetable. The effectiveness of adding new stops on the capacity utilization depends on the ways of adding stops, i.e. which train is allowed to be added new stops and which stations can be selected to stop at.

Keywords
Cyclic timetabling, Capacity utilization, Train sequence, Train stop, Overtaking

1 Introduction

With the expansion of the scale of high-speed railway networks in China, the exchange among the different regional areas causes the passenger flow volumes to expand, which implies more high-speed trains, i.e. better capacity utilization. In general, there are two kinds of ways to improve the capacity utilization, i.e. upgrade railway infrastructure and equipment, and increase the efficiency of transportation. Compared with the former, improving train operating plans for the efficiency of transportation, e.g. improving line plans and timetables, can be low-cost, since upgrading infrastructure/equipment always needs more time and money. Therefore, capacity-oriented timetabling is necessary for improving railway capacity utilization and transport management.

Railway timetabling and railway capacity analysis has been deeply studied in recent years. Based on given infrastructures and train line plans, there are some timetabling strategies which affect the capacity utilization, e.g. changing the train departure sequence in the origin stations, overtakings between trains and adding new train stops at intermediate stations. Nowadays, the railway company of China is eager for higher capacity utilization, i.e. operating more trains in limited time period. However, what kind of strategies is better or easier to improve the capacity utilization with acceptable cost is not studied deeply. Sparing and Goverde (2013) discussed the capacity utilization as follows:

"*The relationship between the nominal and the minimum cycle time describes the capacity utilization of the timetable (Hansen and Pachl (2008)): the timetable is stable exactly if the minimum cycle time T is less than the nominal cycle time T_0, i.e. $T<T_0$, and the larger T_0 -T is, the more time reserve there is available.*"

In this paper, the minimum cycle time of operating a series of trains are considered as one index of the capacity utilization, i.e. the smaller the minimum cycle time is, the more opportunities that we have to design smaller time period to operate the trains. In other words, based on given operating time period, if the minimum cycle time of a series of trains is small, more other trains can be operated in the remaining time of the given operating time period, i.e. it is possible to operate more trains in the given operating time period and the capacity utilization can be increased. In this study, with given train line plan, a cyclic timetabling model based on the periodic event scheduling problem (PESP) is built. New variables and constraints to modified train stop plans and describe train overtakings at stations are introduced. On the one hand, the three timetabling strategies, i.e. train departure sequence at the origin stations, overtakings at stations and new train stops, are described in different constraints, and each strategy can be considered by using the corresponding constraint in the model. On the other hand, the minimum cycle time, train travel time and the number of new train stops are used as objectives to measure the differences between the strategies.

This study is a further study of our previous paper, i.e. Zhang and Nie (2016). Literature review is presented in Section 2. The cyclic timetabling problem and the model are displayed in Section 3 and 4, respectively. New variables and the constraints of the three timetabling strategies are included. Experiments based on one real-world case of high-speed rail corridor in China and detailed conclusions are presented in Section 5. Finally, conclusions are included in Section 6.

2 Literature Review

In recent years, many remarkable studies have been devoted to train timetabling (e.g., Caprara et al. (2006); Zhou and Zhong (2007); Salido and Barber (2009); Goverde (2010); Cacchiani and Toth (2012); Harrod (2012); Schmidt and Schöbel (2015)). Among the research performed on cyclic train timetabling, models based on the periodic event scheduling problem (PESP), which was introduced by Serafini and Ukovich in 1989 (Peeters (2003)), have demonstrated great power in periodic railway timetabling. A PESP-based model for the cyclic railway timetabling problem (CRTP) was first considered in 1993, and a stronger model, the cycle periodicity formulation (CPF) was introduced. The PESP and the CPF are based on the construction of an auxiliary graph, whose nodes correspond to events (train departures and arrivals) and whose arcs model the constraints acting on the time separations between those events (Cordone and Redaelli (2011)). This auxiliary graph, known as the event-activity network (EAN), which is also used in this paper, has been widely applied in the literatures on train timetabling (e.g., Kroon and Peeters (2003); Schöbel (2007); Liebchen et al. (2010); Schachtebeck and Schöbel(2010)).

Many extended models and effective algorithms based on the PESP have been studied in depth in recent years (e.g., Kroon and Peeters (2003); Liebchen (2004); Mathias (2008); Xie and Nie (2009); Caimi et al. (2011); Cordone and Redaelli (2011); Kroon et al.

(2013)). With regard to operating rule constraints, Peeters (2003) and Caimi et al. (2011) discussed a non-collision constraint with variable trip time to prevent overtaking between successive stations. With regard to the objective function, an objective for the PESP based on the minimum cycle time T (i.e., the minimum period length of one regular timetable) was presented by Sparing and Goverde (2013, 2017), where the stability of the timetable is considered. Regarding applicable algorithms, Siebert and Goerigk (2013) studied a series of experimental comparisons of various extended PESP models (the Origin Destination aware PESP (ODPESP) and the Extended PESP (EPESP)) and three different methods based on the modulo simplex algorithm proposed by Nachtigall and Opitz (2008), which is a powerful heuristic for solving the PESP (Goerigk and Schöbel (2013)). For an in-depth overview of the PESP, the CRTP, and the CPF, we refer to Peeters (2003) as well as Liebchen (2006), Liebchen and Möhring (2007) and Liebchen et al. (2010). In particular, based on Heydar et al. (2013), Petering et al. (2015) presented an innovative mixed-integer linear programming model, which falls outside the framework of the PESP, of a cyclic train timetabling and platforming problem. The new model and their pre-processing techniques have great potential to analyse the railway capacity utilization based on various factors and the computation time is reasonable.

In many capacity analysis studies of cyclic timetables which have the same setting as ours, influencing factors such as train speed, line plan specifications (train stop plans), overtaking and train heterogeneity have been discussed (e.g., Burdett and Kozan (2006); Abril et al. (2008); Landex et al (2008); Zhu et al. (2009); Dicembre and Ricci (2011); Lindfeldt (2011); Lai and Wang (2012); Petering et al. (2015)). However, to our knowledge, this paper is the first study to build one cyclic timetabling model based on the PESP which includes new variables of adding train stops. Based on the model, it is possible to modify train stops while cyclic timetabling.

3 The Cyclic Timetabling Problem Defined

We now formally introduce the problem. Stations are presented by nodes in our cyclic timetabling problem. There is only one rail line for one direction and no sidings in block sections, so it is impossible for trains to overtake each other between two successive stations. In order to define the cyclic train timetabling problem, the event-activity network is presented first.

3.1 Event-Activity Network and sets

In cyclic timetabling based on the PESP, mathematical formulations are typically constructed in terms of events and activities. Before introducing these formulations, we assume that a public transportation network (PTN) and a *line* have been determined a priori.

Notation 1. A public transport network PTN=($\boldsymbol{S}$, $\boldsymbol{T}$) (where $\boldsymbol{S}$ is the set of nodes and $\boldsymbol{T}$ is the set of edges) is a simple, undirected graph in which the nodes represent stations and the edges represent connections between them. A line $\boldsymbol{l}$ is a path in the PTN, and $\boldsymbol{f}$ is the corresponding frequency of the line (Siebert and Goerigk, 2013). For cyclic timetables, the time horizon on which trains are scheduled, such as one hour or two hours, is usually considered to be the ***cycle time***.

The goal of our model is to determine the departure times and arrival times such that the cycle time, the number of new stops or total train travel time can be minimized.

Assume that a line plan is known, i.e., the stop plans of the lines (sequences of stations at which trains stop) and their corresponding frequencies are given. Then, a given line $\boldsymbol{l_*}$ can be transformed into its individual trains according to its frequency $\boldsymbol{f}$ (i.e. $\boldsymbol{l_{*,1}}$, $\boldsymbol{l_{*,2}}$,..., $\boldsymbol{l_{*,f}}$), and the PTN is thus transformed into EAN $= (\varepsilon, A)$, where ε is the set of events and A is the set of activities. Events can be arrivals at or departs from stations (define ε_{dep} as the set of departure event and ε_{arr} as the set of arrival event), i.e., $\varepsilon = (\varepsilon_{dep}, \varepsilon_{arr})$, and activities are the transitions between pairs of events. To distinguish different types of train operating behavior, the corresponding activity sets can be described as follows (see Table 1). Moreover, Figure 1 presents an example of the EAN.

Table 1: Sets in the EAN

Symbol	Definition
ε	Set of events (nodes)
ε_{dep}	Set of departure events, $\varepsilon_{dep} \subset \varepsilon$
ε_{arr}	Set of arrival events, $\varepsilon_{arr} \subset \varepsilon$
A	Set of activities (arcs)
A_{run}	Set of running activities, $A_{run} \subset A$
A_{run-o}	Set of running activities of trains which depart from their origin stations to intermediate stations (e.g. running activities from station A to B in Fig. 1), $A_{run-o} \subset A_{run}$
A_{run-d}	Set of running activities of trains which depart from intermediate stations to their destination stations (e.g. running activities from station C to D in Fig. 1), $A_{run-d} \subset A_{run}$
A_{run-n}	Set of running activities between intermediate stations of trains (e.g. running activities from station B to C in Fig. 1), $A_{run-n} \subset A_{run}$
A_{dwell}	Set of all dwelling activities at stations (i.e. one dwelling activity is from one arrival event to one departure event, and the train may stop at the stations), $A_{dwell} \subset A$
$A_{dwell-a}$	Set of alternative dwelling activities at stations (i.e. trains may stop at the stations or not), $A_{dwell-a} \subset A_{dwell}$
$A_{dwell-c}$	Set of common dwelling activities at stations (i.e. trains have to stop at the stations), $A_{dwell-c} \subset A_{dwell}$, $A_{dwell} = A_{dwell-a} \cup A_{dwell-c}$
A^r_{dwell}	Set of all dwelling activities of the same train r, $A^r_{dwell} \in SA_{dwell}$, $r = 1,2,\ldots$
A_{pass}	Set of passing activities at stations (i.e. one passing activity is from one arrival event to one departure event, but the times of the arrival and the departure events must be the same since the train passes the station), $A_{pass} \subset A$
A_{safe}	Set of safety activities between trains (i.e. connections between any two arrival events or departure events that interact with each other because they occupy the same physical infrastructure at minimum headway times), $A_{safe} \subset A$
$A_{regular}$	Set of regularity activities between two trains at their origin stations (i.e. connecting two departure events between successive trains of the same line), $A_{regular} \subset A$

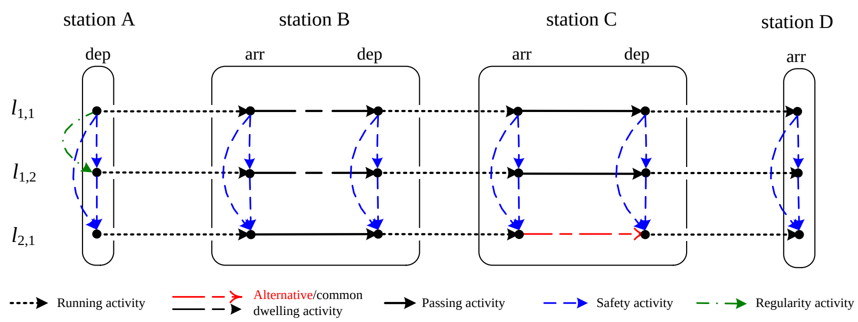

Figure 1: an example of the EAN

3.2 Parameters and Variables

Based on the assumptions and the EAN above, our problem is the train cyclic timetabling problem with stop planning (CTP-SP) of one rail corridor. Based on the structure of the PESP model, new variables of train stops are introduced and the CTP-SP model can be considered to be an extended model of the traditional PESP model. However, train stop plans are not allowed to be "regenerate" when timetabling, but modified according to the line plan, i.e. we are only adding limited number of train stops in this study. Meanwhile, the number of new train stops will be restricted in the model. It is also assumed that all trains will depart from the same station (i.e. the first station of the corridor according to one operation direction), such that the strategy of "train departure sequence" can be analysed. Table 2 and Table 3 present the subscripts, parameters and decision variables in the CTP-SP model, respectively. Mathematical formulations are presented in Section 4.

Table 2: Subscripts and parameters in the CTP-SP model

Symbol	**Definition**
i, j, i', j'	Indexes for the events
a	Activity, $a \in A$, $a_{ij} = (i, j)$, $i, j \in \varepsilon$
l_a	Lower duration bound of activity a, $l_a \in \mathbb{N}$, $0 \leq l_a \leq T - 1$
u_a	Upper duration bound of activity a, $u_a \in \mathbb{N}$, $0 \leq u_a \leq T - 1$ (different from the traditional PESP model, since our model can be linear only in this way)
$h_a(h'_a)$	Minimum headway time of activity a for two trains at the same station, $a \in A_{safe}$, $h_a \in \mathbb{N}$
f_a	Frequency of the line to which activity a belongs, $a \in A_{regular}$, $f_a \in \mathbb{N}^+$
δ	A nonnegative integer describing the relaxation level of regularity activity constraints, $\delta \in \mathbb{N}$
DE_a	Deceleration time loss of activity a, $a \in A_{run}$
AC_a	Acceleration time loss of activity a, $a \in A_{run}$
O	An index of overtaking, equals to 0 when overtakings are prevented, and a (very large) constant when overtakings are allowed
P^i	The maximum number of stops of train i
M_{order}	Sequence matrix of (some) departure events of trains at their origin stations, e.g. $M_{order} = [\pi_{16}, \pi_{10}, \ldots]$
T_{min}	Minimum value of the cycle time T, $T_{min} \in \mathbb{N}$
T_{max}	Maximum value of the cycle time T, $T_{max} \in \mathbb{N}$

Table 3: Decision variables in the CTP-SP model

Symbol	Definition
T	The cycle time, $T \in \mathbb{N}$
π_i	Planned time for event i, $0 \leq \pi_i \leq T-1, \pi_i \in \mathbb{N}$
x_a	Planned duration for activity a, $a \in A$, $0 \leq x_a \leq T-1, x_a \in \mathbb{N}$
z_a	A binary variable that is equal to 1 when $\pi_i > \pi_j$ and equal to 0 otherwise (i.e., the modulo variable of activity a)
y_a	The value of $z_a \times T$ for activity a, equals to T when $\pi_i > \pi_j$ and equals to 0 otherwise, $a \in A$
p_a	A binary variable that is equal to 1 when the trains of activity a stops at the station, and equal to 0 otherwise, $a \in A_{dwell} \cup A_{pass}$
$w_{ii'jj'}$	An auxiliary integer variable which takes values of 0, 1 or 2 for activities a_{ij}, $a_{i'j'}$, $a_{ii'}$, and $a_{jj'}$. a_{ij}, $a_{i'j'} \in A_{run}$ and belong to the same section, $a_{ii'}$, $a_{jj'} \in A_{safe}$ (see Zhang and Nie (2016), Yan and Goverde (2018) for further explanations)
$o_{ii'jj'}$	A binary variable which takes the value of 0 when overtakings are prevented at the stations, and equal to 1 otherwise. a_{ij}, $a_{i'j'} \in A_{dwell} \cup A_{pass}$ and belong to the station, $a_{ii'}$, $a_{jj'} \in A_{safe}$(see Yan and Goverde (2018) for further explanations)
$v_{ii'jj'}$	An auxiliary integer variable which takes values of 0, 1 or 2 for activities a_{ij}, $a_{i'j'}$, $a_{ii'}$, and $a_{jj'}$. a_{ij}, $a_{i'j'} \in A_{dwell} \cup A_{pass}$ and belong to the same station, $a_{ii'}$, $a_{jj'} \in A_{safe}$ (see Yan and Goverde (2018) for further explanations)

Decision variables in our problems are defined as integers measured in minutes. In fact, this assumption is based on common operating parameters, and integers measured in seconds are also feasible for our model which may increase the computation time.

4 Mathematical Formulation of the Cyclic Timetabling Model

In this section, the mathematical formulations of the CTP-SP model are presented, and the objectives and the constraints are explained in detail.

(1) Objective functions:

O1: Minimize T, (1)

O2: Minimize $\sum p_a$, $\quad a \in A_{dwell-a}$, (2)

O3: Minimize $\sum x_a$, $\quad a \in A_{run} \cup A_{dwell}$. (3)

Objective function (1) strives to minimize the cycle time. The number of the new train stops and the total train travel time are minimized in Objective function (2) and (3). Each objective function can be calculated individually and iteratively, such that the model can be considered as single-objective and easier to be calculated.

(2) Constraints of events and activities:

$$x_a = \pi_j - \pi_i + y_a, \quad \forall a \in A_{run} \cup A_{dwell} \cup A_{pass} \cup A_{safe} \cup A_{regular}, \tag{4}$$

$$l_{a_{ij}} + p_{a_{jj'}} \times AC_{a_{ij}} \leq x_{a_{ij}} \leq u_{a_{ij}} + p_{a_{jj'}} \times AC_{a_{ij}}, \quad \forall a_{ij} \in A_{run-o}, a_{jj'} \in A_{dwell} \cup A_{pass}, \tag{5}$$

$$l_{a_{ij}} + p_{a_{i'i}} \times DE_{a_{ij}} \leq x_{a_{ij}} \leq u_{a_{ij}} + p_{a_{i'i}} \times DE_{a_{ij}}, \quad \forall a_{ij} \in A_{run-d}, a_{i'i} \in A_{dwell} \cup A_{pass}, \tag{6}$$

$$l_{a_{ij}} + (p_{a_{i'i}} \times DE_{a_{ij}} + p_{a_{jj'}} \times AC_{a_{ij}}) \le x_{a_{ij}} \le u_{a_{ij}} + (p_{a_{i'i}} \times DE_{a_{ij}} + p_{a_{jj'}} \times AC_{a_{ij}}),$$
$$\forall a_{ij} \in A_{run-n},\ a_{i'i}, a_{jj'} \in A_{dwell} \cup A_{pass}, \quad (7)$$
$$l_a \le x_a \le u_a, \quad \forall a \in A_{run} \backslash (A_{run-o} \cup A_{run-d} \cup A_{run-n}), \quad (8)$$
$$h_a \le x_a \le T - h'_a, \quad \forall a \in A_{safe}, \quad (9)$$
$$l_a \times p_a \le x_a \le u_a \times p_a, \forall a \in A_{dwell} \cup A_{pass}, \quad (10)$$
$$\frac{T}{f_a} - \delta \le x_a \le \frac{T}{f_a} + \delta, \forall a \in A_{regular}, \quad (11)$$
$$z_{a_{ij}} + z_{a_{i'j'}} + z_{a_{ii'}} + z_{a_{jj'}} = 2 \times w_{ii'jj'}, \forall a_{ij}, a_{i'j'} \in A_{run}, a_{ii'}, a_{jj'} \in A_{safe}. \quad (12)$$

Constraint (4) defines the relationship between event times and activity durations. In the original PESP model, Constraints (4) are typically formulated as $x_{ij} = \pi_j - \pi_i + z_{ij} \times T$. However, T and z_{ij} are decision variables in our model, and the use of this equation will cause the model to be non-linear. To prevent the model from violating linear programming conditions, the new variables $y_{ij} = z_{ij} \times T$ are proposed by Sparing and Goverde (2013, 2017). The usage of the new variable requires that $0 \le u_a \le T - 1$, which is different from the traditional PESP models. Constraints (5)-(8) describe the lower and upper bounds of running activities and the relationship between the planned duration of activities and the variables of stops. A binary variable p_a is generated for each dwelling and passing activity since it will be easier to build these constraints. It is clear that one train needs time to decelerate and accelerate when it plans to stop at one station and the related constraints of running time should be modified. Safety operation of two trains using the same infrastructure (station) is guaranteed in Constraint (9). In Constraint (10) and (11), bounds of dwelling, passing and regularity activities are restricted. And Constraint (12) can prevent illegal overtakings between two successive stations in sections (see Zhang and Nie (2016), Yan and Goverde (2018) for further explanations).

(3) Constraints of the timetabling strategies:

$$z_{a_{ij}} + z_{a_{i'j'}} + z_{a_{ii'}} + z_{a_{jj'}} = 2 \times v_{ii'jj'} + o_{ii'jj'},$$
$$\forall a_{ij}, a_{i'j'} \in A_{dwell} \cup A_{pass}, a_{ii'}, a_{jj'} \in A_{safe}, \quad (13)$$
$$\sum o_{ii'jj'} \le O, \quad a_{ij}, a_{i'j'} \in A_{dwell} \cup A_{pass}, a_{ii'}, a_{jj'} \in A_{safe}, \quad (14)$$
$$\pi_{m_i} \le \pi_{m_j}, \quad \forall m_i, m_j \in M_{order}, i < j, \quad (15)$$
$$\sum_{a \in A^r_{dwell}} p_a \le P^r, \quad \forall A^r_{dwell} \in SA_{dwell}. \quad (16)$$

Overtakings at stations can be described in Constraints (13) and (14) by changing the value of parameter O, i.e. O equals zero when overtakings are prevented, and a very large constant when overtakings are allowed (see Yan et al. (2018) for further explanations). In fact, these constraints can be used to restrict the number of overtakings, but we will not extend this topic in this paper. In Constraint (15), the departure sequence of trains at the origin stations can be restricted. Clearly, it is possible that M_{order} is an empty set, such that the order of trains at the origin stations is flexible. As mentioned, train stop plans can be only modified by adding a limited number of stops of trains in this study. Therefore, the maximum number of stops of each train is restricted in Constraint (16).

(3) Logic constraints:

$$y_a \le T_{max} \times z_a, \quad \forall a \in A_{run} \cup A_{dwell} \cup A_{safe} \cup A_{regular}, \quad (17)$$
$$y_a \le T, \quad \forall a \in A_{run} \cup A_{dwell} \cup A_{safe} \cup A_{regular}, \quad (18)$$
$$y_a \ge T - T_{max} \times (1 - z_a),$$
$$\forall a \in A_{run} \cup A_{dwell} \cup A_{safe} \cup A_{regular}, \quad (19)$$
$$y_a \ge 0, \quad \forall a \in A_{run} \cup A_{dwell} \cup A_{safe} \cup A_{regular}, \quad (20)$$
$$x_a = 0, \quad \forall a \in A_{pass}, \quad (21)$$

$p_a = 0, \quad \forall a \in A_{pass},$ (22)
$p_a = 1, \quad \forall a \in A_{dwell-c},$ (23)
$T_{min} \leq T \leq T_{max},$ (24)
$z_a \in \{0,1\}, \quad \forall a \in A_{run} \cup A_{dwell} \cup A_{safe} \cup A_{regular},$ (25)
$p_a \in \{0,1\}, \quad \forall a \in A_{dwell} \cup A_{pass},$ (26)
$0 \leq \pi_i \leq T-1, \quad \forall i \in \varepsilon,$ (27)
$0 \leq x_a \leq T-1, \quad \forall a \in A.$ (28)

Constraints (17)-(28) are logic constraints. Constraints (17)-(20) are used to linearize the model (see Sparing and Goverde (2013) for more details). In Constraint (24), it will be better if T_{min} is known since this parameter can reduce the solution space of the model. Otherwise, $T_{min} = 0$ can be accepted.

5 Experiments and Results

In this section, the comparison results of the three timetabling strategies are presented based on a series of experiments of the Beijing-Shanghai High-speed Railway in China (see Figure 2). There are 23 stations in the rail corridor. All trains in the experiments are chosen from one practical line plan, which run from Beijing South station to Shanghai Hongqiao station (see Table 4, Figure 3 and Table 5). Parameters including minimum headway time at stations, accelerating and decelerating time loss of trains refer to the practical data. When using the strategy of adding new stops, it is assumed that one new stop can be added for each train at most. Trains of type A are not allowed to be added new stops, except for the experiments in Section 5.3. Due to the requirements of service, trains of type B in Case2240 and Case2204 depart from their origin stations exactly every *T*/2, i.e. half of the minimum cycle time. Trains of type A run at speed of 350km/h, and trains of other types run at speed of 300km/h. In our opinion, Case0008 has higher train homogeneity compared to the other two cases since the trains have the same train speed and the number of stops at least. The model was coded by MATLAB R2012a and solved by Cplex 12.5. The calculations were performed on a PC with an Intel E7 2.0-GHz processor, 28 CPU cores and 256 GB of RAM. In general, the computation time is always about several seconds/minutes (average computation time of all presented cases is 47 minutes). Nevertheless, the computation times of those cases with “more flexible” strategies will be much longer, i.e. may cost several hours (12 hours at most). Some iterative ideas are used in our experiments to reduce the computation time (e.g. the method in Zhang and Nie (2016)). All of the solutions are optimal.

For all cases, Objective (1) will be used first (O1), then the value of the minimum cycle time is transformed into a constraint (i.e. to guarantee that the T equals to the minimum cycle time) and Objective (2) will be used (“O1”+O2). After that, both of the values of the minimum cycle time and the minimum number of train stops are transformed into constraints, and Objective (3) will be calculated (“O1+O2”+O3).

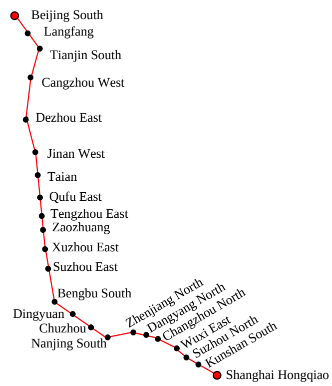

Figure 2: Schematic map of the Beijing-Shanghai high-speed railway (1318km)

Table 4: Number of trains in the cases*

Number of trains	**Case 2240**	**Case 2204**	**Case 0008**
type **A**	2	2	0
type **B**	2	2	0
type **C**	4	0	0
type **D**	0	4	8
Total	8	8	8

Notice*: names of the cases represent the number of different train types.

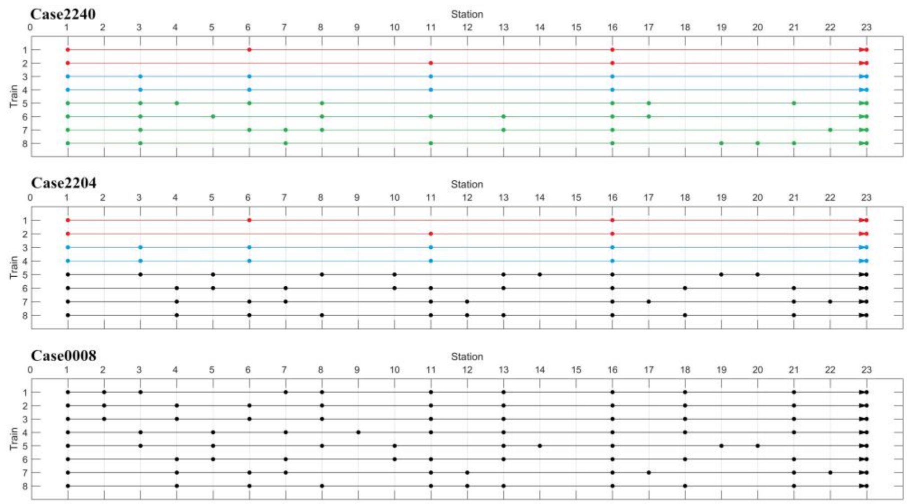

Figure 3: Stop plans of trains of type A, B, C and D in the cases
(red, blue, green and black lines for each train types, respectively)

Table 5: Abbreviation of the three strategies in the experiments

Abbreviation	Strategies
FT/IT	**F**lexible/g**i**ven **T**rain departure sequence at the origin stations
FO/IO	**F**lexible/forb**i**dden **O**vertakings at stations
FS/IS	**F**lexible/forb**i**dden new train **S**tops compared to the original line plan

5.1 Train sequence at origin stations and overtakings

In general, the strategies of train departure sequence at the origin stations and overtakings are widely used in timetabling, and both of them will not change the original line plan. Based on the given line plan, the impact of these two strategies on the capacity utilization and total train travel time are presented in Table 6. As expected, flexible train departure sequence and train overtakings lead to higher capacity utilization when train homogeneity is lower (i.e. in Case2240 and Case2204). However, too many overtakings always cause longer train travel time since trains have to wait at stations, and decrease the robustness of timetables because of the closer relationship of trains. If train homogeneity is high, i.e. in Case0008, train departure sequence will play a more important role than overtaking. In our opinion, finding a “good” train departure sequence at the origin stations is an important way to optimize capacity utilization of rail corridors, and corresponding “sacrifices” can be small. Hence, good train departure sequence and appropriate overtakings can be jointly considered since these two strategies can guarantee the quality of timetables with good capacity utilization, i.e. balance demand and supply.

Table 6: Experimental results of different strategies: train sequence and overtakings

No.	Train sequence	Over-taking	New stops	O1 (min)	"O1" +O3 (min)	Number of overtakings
Case 2240	FT	FO	IS	74	2750	12
	FT	IO	IS	110	2711	/
	IT	FO	IS	100	2734	8
	IT	IO	IS	194	2690	/
Case 2204	FT	FO	IS	86	2815	13
	FT	IO	IS	122	2763	/
	IT	FO	IS	104	2803	9
	IT	IO	IS	208	2752	/
Case 0008	FT	FO	IS	51	2990	0
	FT	IO	IS	51	2990	/
	IT	FO	IS	68	2988	0
	IT	IO	IS	68	2988	/

5.2 Overtakings and adding new stops

In practice, trains may have their "ideal departure time window" according to passenger demand or operation requirements. And new stops will be added at one station when one overtaking is needed at the station in practice sometimes. Therefore, it is necessary to analyse the impact of overtakings and new stops with given/fixed train departure sequence (see Table 7). In this section, train departure sequences are given beforehand and different from the results of the "flexible" sequence strategy. It is obvious that overtakings have more impact on the capacity utilization compared to adding new stops when train homogeneity is lower (i.e. in Case2240 and Case2204). And when overtakings are allowed, adding new stops will be better for the capacity utilization compared to the results with no overtakings. Further discussions of adding new stops are presented in Section 5.3. When train homogeneity is higher (i.e. in Case0008), the impact of overtakings are weaker, while the capacity utilization can be higher by adding new stops with longer train travel time. In our opinion, this may be the result of "balanced" stops of trains after adding new stops. For example, one train can be more "similar" (i.e. have the same stops at stations) to the neighbouring trains by adding new stops (e.g. in Figure 4).

Table 7: Experimental results of different strategies: overtakings and new stops

No.	Train sequence	Over-taking	New stops	O1 (min)	"O1" +O2 (min)	"O1+O2" +O3 (min)
Case 2240	IT	FO	FS	76	4	2793
	IT	FO	IS	100	/	2734
	IT	IO	FS	194	0	2690
	IT	IO	IS	194	/	2690
Case 2204	IT	FO	FS	74	5	2881
	IT	FO	IS	104	/	2803
	IT	IO	FS	208	0	2752
	IT	IO	IS	208	/	2752
Case 0008	IT	FO	FS	49	7	3103
	IT	FO	IS	68	/	2988
	IT	IO	FS	58	7	3101
	IT	IO	IS	68	/	2988

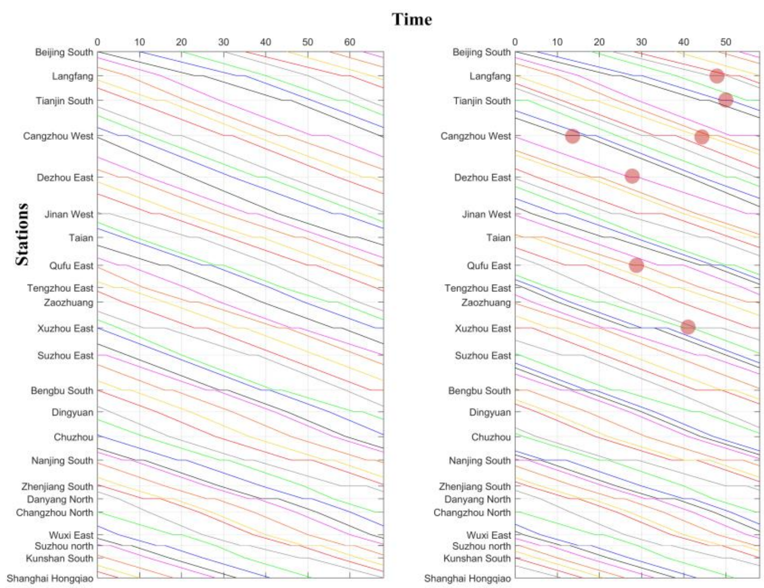

Figure 4: An example of timetables of adding new stops
(circles show the locations of new stops compared to the original timetable (left),
different trains have different colors)

5.3 Trains offering the "fastest" transport service

In Section 5.2, we conclude that adding new stops have little impact on the capacity utilization when train homogeneity is lower. However, trains of type A are not allowed to be added new stops in the above cases since this kind of trains offer the "fastest" transport service (i.e. *highest train technical speed and least number of stops*). In this section, we relax this assumption in Case2240 to present the impressive impact of adding new stops for the "fastest" trains (see Table 8 and Figure 5). "*" means trains of type A are allowed to be added new stops and each train can be added at most one new stop. Obviously, capacity utilization can be higher if new stops of the "fastest" trains are allowed (i.e. FT-IO-FS* versus FT-IO-FS (yellow lines), and IT-IO-IS* versus IT-IO-FI (pink lines) in Figure 5), and better effectiveness of new stops are further presented. In order to obtain higher capacity utilization, new stops prefer to be added to the "fastest trains", i.e. trains of type A (the last volume in Table 8). When overtakings are allowed, the total number of overtakings of case* are less than that of the original case (i.e. FT-FO-FS* versus FT-FO-FS (purple nodes), and IT-FO-FS* versus IT-FO-FS (blue nodes)). In other words, overtakings can be more useful with adding new stops for the "fastest" trains.

Table 8: Experimental results of the "fastest" trains in Case2240

Train sequence	Over-taking	New stops	O1 (min)	"O1" +O2 (min)	"O1+O2" +O3 (min)	Number of new stops of different types of trains			
						A	B	C	D
FT	FO	IS	74	/	2750	/	/	/	/
FT	FO	FS	68	5	2798	0	1	4	0
FT	FO	FS*	60	6	2810	2	2	2	0
FT	IO	IS	110	/	2711	/	/	/	/
FT	IO	FS	108	2	2716	0	0	2	0
FT	IO	FS*	94	5	2756	2	2	1	0
IT	FO	IS	100	/	2734	/	/	/	/
IT	FO	FS	76	4	2793	0	2	2	0
IT	FO	FS*	70	6	2808	2	2	2	0
IT	IO	IS	194	/	2690	/	/	/	/
IT	IO	FS	194	0	2690	0	0	0	0
IT	IO	FS*	170	1	2708	1	0	0	0

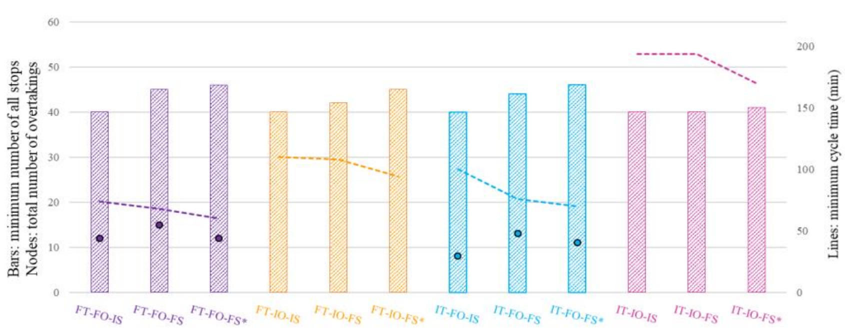

Figure 5: Impact of the "fastest" trains in Case2240

In sum, with flexible train departure sequence at the origin stations, the possibility of acquiring good capacity utilization can be higher and the impact on the quality of timetables can be little. Overtakings are very beneficial to the capacity utilization when train homogeneity is low, but train travel time and the robustness of timetables will be affected. Adding new stops changes the original line plans, and the impact on the capacity utilization depends on the usage of this strategy, i.e. which train is allowed to be added new stops and which stations can be selected to stop at. In our opinion, the capacity utilization and the service level of transportation should be balanced and jointly optimized by using the three timetabling strategies properly according to the characteristic of trains and passenger demand.

6 Conclusion

In this study, we propose a cyclic timetabling model based on the PESP with new variables which describe whether trains stop at the intermediate stations, and analyse the impact of the three timetabling strategies (i.e. train departure sequence at the origin stations, overtakings at stations and new train stops) on the capacity utilization by a series of experimental results. Flexible train departure sequence at the origin stations leads to higher possibility of acquiring good capacity utilization, and requires small sacrifices of the quality of timetables. For trains of low homogeneity, overtakings are also very beneficial to good capacity utilization. However, train travel time is always long and the robustness of timetables will be affected. The effectiveness of adding new stops depends on the ways of adding stops. Trains with higher technical speed and few stops should be mainly focused on, and integrating overtakings with new stops can be beneficial to the capacity utilization. Further research includes the analyses of the impact of the number of overtakings on the minimum cycle time.

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