Study on Station Buffer Time Allocation According to Delay Expectation

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Abstract

Trains are inevitably subject to interference from the external environment and internal systems during operation, leading to delays and conflicts. In this regard, there are usually buffer times allocated at (in) the station (section) in the train timetable, to recover delays. Most of the existing methods that deal with the buffer time allocation mainly consider the length of the section and the traffic density. These methods usually fail to consider the impact of the actual delay of trains, and the buffer time allocation (BTA) is unreasonable. The integration of the actual delay effects into the BTA needs to be resolved. Based on this, in this work, a delay time distribution model was established, and the models were compared according to the standard error of each parameter in the model. Subsequently, based on the delay distribution, a BTA model with weighted average delay expectation time as the objective function was constructed in which the weight coefficients were determined based on the delay strength, and the model was solved by a mathematical analysis method. Different allocation models were designed for different ranges of the total buffer time values. Finally, taking the Dutch railway network trunk section Maarssen–Utrecht Centraal (Mas–Ut) as an example, the results show that the buffer time after redistribution of the BTA model reduces the expected delay time in the segment by 5.25% compared with the original buffer time of the station, indicating that the BTA is reasonable.

Keywords

Buffer time, Delay distributions model, Delay strength, Mathematical analysis

1 Introduction

Trains are inevitably subject to interference from the external environment or internal systems during operation. When the disturbance intensity is high, the train is delayed. The buffer time set in the train timetable is usually used to eliminate or reduce delays. To make the timetable have enough strain capacity and ensure the punctuality of the train, when the train is in disorder, it can restore normal operation order as soon as possible and make the timetable more flexible. It is often necessary to reserve a certain "buffer" time between the train running lines, which is called the buffer time between the train running lines. The buffer time set in the train timetable is usually used to eliminate or reduce delays.

Zhang et al. (1997) collected a large number of data about the average delay time and buffer time of trains for statistical analysis, and they obtained the change law of the average delay time of trains with the buffer time shown in Figure 1. On the one hand, I in Figure 1 is the train-tracking interval, while the minimum train-tracking interval is $I_{\min} = 5$ min, and the buffer time of each train is $I - I_{\min}$; On the other hand, the horizontal axis shows the redundant parking time of the train station. The figure shows that the average delay time of trains with various train interval (I = 6, 7, and 10 min) and different stopping buffer times tends to decrease with the increase of buffer time. When the station stop buffer time is 6 min, the average train delay time is 10 min when the tracking interval buffer time is 5 min and 20 min when the tracking interval buffer time is 1 min. Buffer time plays an active role in alleviating the fluctuation of the train interval running time and train delays caused by various random factors during train operation. The setting of the buffer time is conducive to improving the stability of the train timetable and enhancing the anti-interference ability of the train timetable.

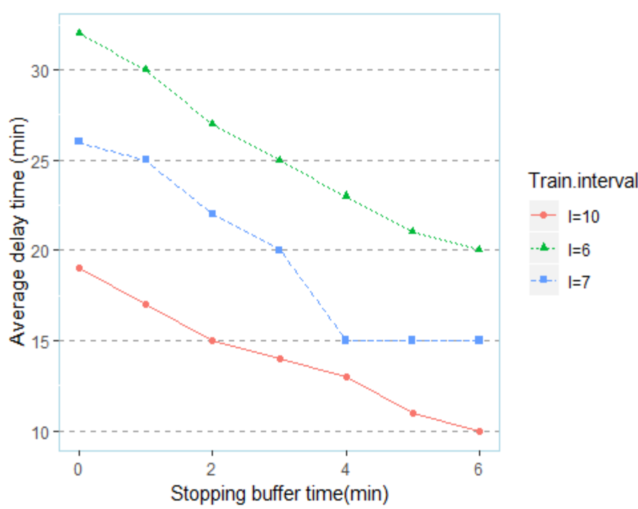

Figure 1: Variation of average train delay time with buffer time

The buffer time has incomplete accumulation, which means that the buffer time is limited to the use of a given station and section. It is shown that the buffer time is used only when the train is disturbed, deviates from the operation plan, and needs to be adjusted. When the train operation adjustment is performed in the current section and station, the buffer time of the previous section and station cannot be stored in the current section and station and has no effect on the train operation adjustment. Similarly, the buffer time that is not fully utilized in the current section and station cannot be accumulated in the station and section ahead of the train operation. Therefore, the excessive buffer time in the train timetable affects the capacity of the section and reduces the efficiency of the transportation organization.

According to the above analysis, to make full use of the buffer time and not waste the capacity, the buffer time allocation (BTA) should consider the actual demand of train delay recovery. In this study, based on the operational performance data, a model of delay time distribution was constructed. Based on this model and considering the impact of actual delay, a BTA model was established with the objective function of minimizing weighted average delay expectation time. In the process of solving the model, the corresponding allocation models were solved by using a mathematical analytic algorithm, aiming at different value ranges of the total buffer time. Finally, the model is validated by a case study. The results show that the established BTA model can reduce the delay expectation time by 5.25%.

The remaining sections of this work are arranged as follows. Section 2 gives an analysis of the current research on buffer time. In Section 3, the relationship between buffer time utilization and delay recovery is discussed, and the rules for buffer time are summarized. In Section 4, a BTA model is established with the objective function of minimizing the weighted average expected delay time, based on the established delayed distribution model and combined with the buffer time, and the model is tested by an example. The conclusions and direction of future research are described in Section 5.

2 Literature Review

Delays seriously affect the order of railway operation. To eliminate or reduce delays, many experts and scholars have done corresponding research. Buffer time is considered the main resource of delay recovery and is closely related to delay recovery. Abril et al. (2008) took Spanish railway infrastructure as an example to analyze the main indicators affecting railway capacity. The results show that railway capacity varies with train speed, train stopping point, distance between railway signals, and robustness of the train timetable. The concept of elasticity was proposed to measure the ability of a railway system to absorb interference and recover interference (Adjetey-Bahun et al., 2016). The train timetable must be designed with appropriate travel time and be able to withstand delays, disturbances, and changes in operating conditions to achieve a higher level of service during operation (Goverde et al., 2013). Yuan et al. (2007) proposed a new stochastic model for train delay propagation analysis at stations. The model was validated by the example of the Hague Holland Spoor in the Netherlands. The study found that when the planned buffer time between trains at level crossings decreases, the average knock-on delay of all trains increases exponentially. It was pointed out that buffer time in a train timetable has a significant effect on solving and reducing train interference, and the allocation scheme of buffer time affects the possibility of interference (Yuan et al., 2008).

Statistical methods and computational theory have become the main research methods in studying the effect of buffer time on delay recovery. Liebchen et al. (2009) introduced restorable robustness into the study of delay recovery and optimized recovery plans and strategies under resource constraints. In this study, it was assumed that the uncertainty of the time required for train operation and stopping can be obtained from historical data. The proposed method was applied to the Palermo Central Station, and the results show that delay propagation can be largely reduced. Khadilkar et al. (2016) proposed a data-based stochastic model to evaluate the robustness of train timetables that considers delayed recovery. Buffer time and station running time are often used to absorb delay, and the efficiency of delay recovery can be estimated statistically based on empirical data. The average recovery rate obtained from the arrival and departure records of more than 38,000 trains in the Indian Railway Network was 0.13 min/km. However, the number of data in this study was too small — only 15 days of empirical data were available, and it was difficult for the fixed average recovery rate to reflect the real recovery capacity of different sections and stations.

The BTA has become a research hotspot in recent years. In terms of BTA, relevant literature has been studied and some conclusions have been drawn. The buffer time allocated for a single train is generally considered proportional to the section distance of the train, and the average weighted distance was proposed as the basis for BTA (Vromans, 2005; Fischetti et al., 2009). According to the guide “UIC CODE 451-1 OR” (2000), the BTA needs to be calculated according to the train running distance or the average travel time, and the [min/km] or [%] is used to determine the BTA at (in) stations (sections). However, this kind of statistical method does not allocate buffer time by trains, stations, and sections in accordance with specific conditions. Kroon et al. (2008) distributed the buffer time by establishing a stochastic optimization model to increase the robustness of the train timetable. The model was tested and verified with the Dutch train passenger train timetable. Vansteenwegen et al. (2007) calculated the ideal BTA in sections by using negative exponential distributions. They constructed the delay loss equation on this basis and optimized the timetable by using a linear programming method. Carey et al. (2007) applied probability theory to determine the reasonable buffer time under the condition of train operation performance, but they did not consider the impacts of delay. Krasemann et al. (2012) used the depth-first greedy algorithm to assist with train operation adjustment planning. The buffer time is a tool to eliminate random interference of train operation, but there is no in-depth study of the BTA. Carey (2007) and Dewilde (2013) have made a series of studies on how to allocate buffer time in the process of compiling train timetables and achieved certain results.

Because of the difficulty in acquiring and processing operational performance data, the above literature seldom addressed the BTA based on operational performance data. In recent years, more and more researchers have used machine-learning methods to study the BTA. Huang et al. (2018) established a data-driven BTA model based on the Wuhan–Guangzhou high-speed railway. Based on the utilization of buffer time, the model redistributes buffer time, which provides a new research method for BTA. Wen et al. (2016) proposed a data-driven method based on a multiple linear regression model and stochastic forest model to solve the problem of delay recovery of high-speed rail trains after initial delays. In addition, under the same explanatory variables and datasets, the stochastic forest regression proposed is superior to the over-limit learning machine and stochastic gradient descent methods (Bottou, 2010; Huang et al., 2004). Therefore, on the premise of data availability, it has become an inevitable trend to discover rules from data and construct models to study the BTA.

However, the existing literature on BTA mainly considers the length of the interval and the driving density, and rarely considers the influence of the actual delay strength. It is especially important to integrate the delay effects into the BTA, and the delay distributions can effectively evaluate the delay effects, which can be used as an entry point for the BTA. Therefore, it is of great significance to study the BTA based on the delay distributions.

3 Relationship between Buffer Time Utilization and Delay Recovery

The BTA needs to consider various factors comprehensively to achieve the scientific and rational selection of buffer time. The International Railway Union standardized the selection of train operation buffer time. In terms of operating mileage, it is 1.5 min for every 100 km of single-engine passenger trains. For multimachine traction, it is compensated for 1 min per 100 km. In terms of travel time, the buffer time needs to be based on the running speed of the train, which ranges from 3% to 7% of the total travel time.

Generally, delay recovery mainly depends on buffer time, which can be used to restore the train to the planned train timetable as soon as possible. As shown in Figure 2, t_j^i represents the minimum stop operation time of train i at station s_j ; $t_{j,j+1}^i$ represents the minimum running time of train i between station s_j and station s_{j+1} ; b_j^i and $b_{j,j+1}^i$ represent

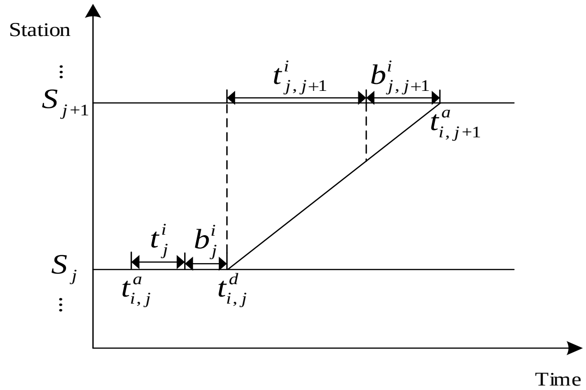

Figure 2: Schematic diagram of buffer time utilization in stations (sections)

the buffer time of the station and interval, respectively; $t_{i,j}^a$ represents the actual arrival time of train i at station s_j ; and $t_{i,j}^d$ represents the actual departure time of train i at station s_j .Then, there is

$$b_j^i = t_{i,j}^d - t_{i,j}^a - t_j^i. \tag{1}$$

$$b_{j,j+1}^i = t_{i,j+1}^a - t_{i,j}^d - t_{j,j+1}^i. \tag{2}$$

Based on buffer time, train delay recovery can be described as follows.

(1) If the delay time of the train at station s_j is $d_j^i \le b_j^i$, it indicates that the delay time can be absorbed by the buffer time of s_j , thus achieving the effect of delay recovery.

(2) If $b_j^i < d_j^i \le b_j^i + b_{j,j+1}^i$, it shows that the delay cannot be absorbed completely by the station buffer time, but the part that is not absorbed completely can be absorbed by the interval buffer time, so as not to affect the station arrival time, thus achieving the delay recovery effect.

(3) If $d_j^i > b_j^i + b_{j,j+1}^i$, it indicates that the delay cannot be absorbed by the station buffer time and interval buffer time, and the delay is propagated at station s_{j+1} .

The analysis of three cases of train delay recovery clearly shows that the buffer time has the effect of delay recovery, but the effect is closely related to the length of the specific delay time. Considering the buffer time separately from the delay situation either wastes the buffer time or makes the delay recovery effect not obvious. Therefore, in this work, the BTA was studied by comprehensively considering the actual impact of delay. First, the buffer time was combined into the delay time distribution model, and the expected delay time was calculated based on the distribution model. Then, taking the delay strength as the weight coefficient of the expected delay time of each station, a BTA model with the minimum expected total delay time as the objective function was established. Finally, the buffer time after reallocation could be obtained by solving the model.

4 BTA Model

4.1 Model establishment

Delays in the section will show up at the station. For example, when the train runs in the section, it is delayed 2 min. If the buffer time in the section is not considered, the delay will be expressed as the train arrival delay at the station, and the arrival delay time is also 2 min. Therefore, the delay of the section can be analyzed by the station, and the buffer time of the sections can be summarized as the station buffer time — that is, the running time of the train in all the sections is assumed to be the minimum running time, and the train is assumed to be on time at the originating station.

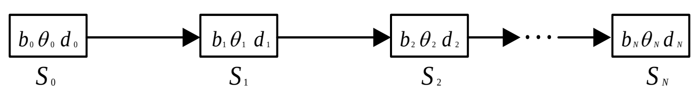

Figure 3: Schematic diagram of train operation

Figure 3 is a schematic diagram of a train operating at N stations, using the collection T record of the station where the train arrives, the delay time distributions at the station, and the buffer time at the station, which is $T_i = \{S_i, b_i, \theta_i, d_i \mid i = 0,1,2,\cdots N\}$. Here, S_i indicates the i-th station, b_i shows the buffer time assigned to station i, and θ_i shows the delay distributions in the i-th station, and d_i is the delay time at the i-th station ($d_i \geq 0$).

In the BTA model, delays in the interval are generalized to delays at the station, thus simplifying the BTA model. The problem of BTA at (in) the station (section) is transformed into a whole for research and analysis, which has no effect on the BTA result.

Assuming that the total amount of buffer time is constant, there are:

$$\sum_{i=1}^{N} b_i = b. \;\; and \;\; b_i \geq 0. \tag{3}$$

where b represents the total buffer time.

Making $f_i(\theta)$ indicate the delay distributions density function of station i, then the probability that train departures from S_0 are on time to S_1 with a delay time d_1 that less than or equal to x is:

$$P\{d_1 \le x\} = \int_0^{x+b_1} f_1(\theta)d\theta. \tag{4}$$

The delay mathematics expectation at S_0 is:

$$E(d_1) = \int_0^{+\infty} \theta f_1(\theta + b_1)d\theta. \tag{5}$$

When the train is running, if the delay time d_{i-1} is generated at S_{i-1} , and $d_{i-1} > b_{i-1}$, then the delay will spread to S_i . Therefore, the probability that the delay time of the train at S_i is less than or equal to x is:

$$P\{d_i \le x\} = \int_0^{x+b_i} \int_0^{b_{i-1}+x+b_i-\theta} f_i(\theta) f_{i-1}(\eta) d\theta d\eta. \tag{6}$$

The delay mathematics expectation at S_i is:

$$E(d_i) = \int_0^{+\infty} \int_0^{+\infty} \theta f_i(\theta + b_i) \eta f_{i-1}(\eta + b_{i-1}) d\eta d\theta. \tag{7}$$

Therefore, the average delay expectation of the train during the entire operation can be calculated as:

$$E(d) = \frac{1}{N} \sum_{i=1}^{N} E(d_i). \tag{8}$$

Because the delay strength can be used to evaluate the frequency and severity of the delay, different weights are given to the delay mathematical expectation of each station according to the delay strength. Then, Eq. (8) is amended to the Eq. (9):

$$\begin{cases} E(\overline{d}) = \sum_{i=1}^{N} w_i E(d_i) \\ \sum_{i=1}^{N} w_i = 1 \\ w_i > 0 \end{cases} . \tag{9}$$

In Eq. (9), w_i is the expected weight coefficient of the delay at S_i ,which is determined based on the delay strength. Therefore, if $E(\overline{d})$ in Eq. (9) is minimized, the BTA function can be obtained as follows:

$$\min E(\overline{d}). \tag{10}$$

In summary, Eq. (10) is a BTA function, and Eq. (3) to Eq. (9) are constraints.

4.2 Delay distribution model

In the construction of the BTA model, the key is to solve the problem of the delay time distribution. This part focuses on the construction of the delay time distribution model. The research idea is to select the common data distribution model to fit the delay time based on the delay time data and take the standard error of each parameter in the distribution model as the model comparison criterion, to select the optimal delay time distribution model.

Based on the train operation performance data in Maarssen–Utrecht Centraal (Mas–Ut) of the Dutch railway network trunk section, the BTA under the condition of continuity was studied. This section contains three stations: Maarssen (Mas), Utrecht Zuilen (Utzl), and Utrecht Centraal (Ut). The time span of operational performance data in the segment was three months, and the data volume was 122,480, of which there were 27,728 delay records. After the screening and noise reduction of the delay data, the delay time distribution model at the station was established based on this. The lognormal distribution, exponential distribution, and Weber distribution models were selected to study the delay distributions. Based on the station delay data, the above models were used to fit the station delay data.

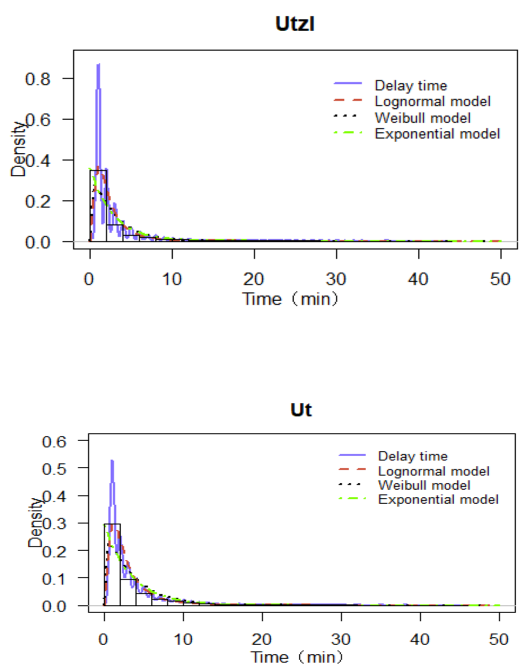

Figure 4: Fitting diagram of station delay distributions

Figure 4 is a schematic diagram of the lognormal distribution, Weber distribution, and exponential distribution used to fit the probability density of the station delay time. To compare the above models, the optimal delay distribution model was determined by comparing the standard error of parameters in each model as the criterion (Maas, 2004). The standard error of each model parameter was calculated, and the results are shown in Table 1.

According to the results in Table 1, compared with other models, the standard error of the model parameters of the exponential distribution model is the smallest, so the exponential distribution model was selected as the station delay distribution model.

Table 1: Standard error of model parameters

Station	Distribution Model	Parameters	Standard error
Utzl	Exponential distribution	rate	0.0038
	Lognormal distribution	meanlog	0.0081
		sdlog	0.0058
	Weber distribution	shape	0.0076
		scale	0.0301
Ut	Exponential distribution	rate	0.0025
	Lognormal distribution	meanlog	0.0071
		sdlog	0.0050
	Weber distribution	shape	0.0063
		scale	0.0295

After determining the station delay distribution model, the maximum-likelihood algorithm was used to solve the parameters of the exponential distribution model, and the station delay distribution model was obtained, as shown below.

$$f_1(\theta) = \begin{cases} \lambda_1 e^{-\lambda\theta}, \theta \geq 0 \\ 0 \qquad, \theta < 0 \end{cases} = \begin{cases} 0.293 e^{-0.293\theta}, \theta \geq 0 \\ 0 \qquad\qquad, \theta < 0 \end{cases}. \tag{11}$$

$$f_2(\theta) = \begin{cases} \lambda_2 e^{-\lambda_2\theta}, \theta \geq 0 \\ 0 \qquad, \theta < 0 \end{cases} = \begin{cases} 0.316 e^{-0.316\theta}, \theta \geq 0 \\ 0 \qquad\qquad, \theta < 0 \end{cases}. \tag{12}$$

where $f_1(\theta)$ and $f_2(\theta)$, respectively, represent the delay distribution density function of Utzl and Ut. $\lambda_1 = 0.293$ and $\lambda_2 = 0.316$ are, respectively, parameters of the delay distribution density function.

4.3 Delay expectation time model based on buffer time optimization

A delay expectation model considering buffer time optimization was built based on the delay time distribution model. The redistributed buffer time can be obtained by solving the model. For the convenience of the following statement, stations Utzl and Ut are replaced with S_1 and S_2, respectively. The buffer time allocated by S_1 and S_2 is represented by b_1 and b_2, respectively. From Eq. (3), there are $b = b_1 + b_2$ and $b_1 \geq 0$, $b_2 \geq 0$.

The probability that the delay time d_1 of train at S_1 is less than or equal to x is:

$$P\{d_1 \leq x\} = \int_0^{x+b_1} \lambda_1 e^{-\lambda_1 \theta} d\theta = 1 - e^{-\lambda_1 (b_1 + x)}. \tag{13}$$

Then, after increasing the buffer time b_1, the delay probability density function of S_1 is:

$$g_1(x) = \frac{dP\{d_1 \leq x\}}{dx} = \lambda_1 e^{-\lambda_1 (b_1 + x)}. \tag{14}$$

According to Eq. (14), the expected delay time of the train at S_1 is:

$$\begin{aligned} E(d_1) &= \int_0^{+\infty} x g_1(x) dx = \int_0^{+\infty} x \lambda_1 e^{-\lambda_1 (b_1 + x)} dx \\ &= \frac{1}{\lambda_1} e^{-\lambda_1 b_1}. \end{aligned} \tag{15}$$

For S_2, it is necessary to consider the delay time generated on S_1. Figure 2 shows that delays generated on S_1 can be absorbed through the buffer time of S_1 and S_2, while delays on S_2 can only be absorbed through the buffer time by S_2. Therefore, the probability of the train at S_2 with a delay time $d_2 \leq x$ is:

$$\begin{aligned} P\{d_2 \leq x\} &= \int_0^{x+b_2} \int_0^{b_1 + x + b_2 - \theta} f_2(\theta) f_1(\eta) d\eta d\theta \\ &= \int_0^{x+b-b_1} \int_0^{x+b-\theta} \lambda_2 \mathrm{e}^{-\lambda_2 \theta} \lambda_1 \mathrm{e}^{-\lambda_1 \eta} d\eta d\theta \\ &= 1 - e^{-\lambda_2 (b - b_1 + x)} - \frac{\lambda_2}{\lambda_1 - \lambda_2} e^{-\lambda_1 (b + x)} [e^{(\lambda_1 - \lambda_2)(b - b_1 + x)} - 1]. \end{aligned} \tag{16}$$

The delay probability density function of S_2 is:

$$\begin{aligned} g_2(x) &= \frac{dP\{d_2 \leq x\}}{dx} \\ &= \lambda_2 e^{-\lambda_2 (b - b_1)} e^{-\lambda_2 x} + \frac{\lambda_2{}^2}{\lambda_1 - \lambda_2} e^{-\lambda_1 b_1 - \lambda_2 (b - b_1)} e^{-\lambda_2 x} - \frac{\lambda_1 \lambda_2}{\lambda_1 - \lambda_2} e^{-\lambda_1 (b + x)}. \end{aligned} \tag{17}$$

According to Eq. (17), the expected delay time of the train at S_2 can be obtained as follows:

$$\begin{aligned} E(d_2) &= \int_0^{+\infty} x g_2(x) dx \\ &= \frac{1}{\lambda_2} e^{-\lambda_2 b} e^{\lambda_2 b_1} + \frac{1}{\lambda_1 - \lambda_2} e^{-\lambda_2 b} e^{(\lambda_1 - \lambda_2) b_1} - \frac{\lambda_2 e^{-\lambda_1 b}}{\lambda_1 (\lambda_1 - \lambda_2)}. \end{aligned} \tag{18}$$

After $E(d_1)$ and $E(d_2)$ are obtained, the expected delay time weighting coefficients of station S_1 and S_2 are determined according to the delay strength of station. The calculation formula for the delay strength is shown in Eq. (19).

$$q = \frac{m * k}{c * l * z}. \tag{19}$$

In Eq. (19), *q* is the delay strength, which is an indicator of the influence of the delayed train number; *m* indicates that delays affect the number of trains; *c* represents the traffic volume; *l* represents the length of the sections; *z* represents the effective working day; and *k* is a constant, and its role is to convert the value of *q* to (0, 1). Based on the delay strength and combined with Eq. (20) , the weight of delay expectation of S_1 and S_2 are determined to be w_1 and w_2 .

$$w_i = \frac{q_i}{\sum_{i=1}^{n} q_i}. \tag{20}$$

To sum up, the expected weighted delay time of trains in this trunk line section is:

$$E(\overline{d}) = w_1 E(d_1) + w_2 E(d_2). \tag{21}$$

Substitute Eqs. (15) and (18) into Eq. (21) to obtain:

$$E(\overline{d}) = w_1 \frac{1}{\lambda_1} e^{-\lambda_1 b_1} + w_2 [\frac{1}{\lambda_2} e^{-\lambda_2 b} e^{\lambda_2 b_1} + \frac{1}{\lambda_1 - \lambda_2} e^{-\lambda_2 b} e^{(\lambda_1 - \lambda_2) b_1} - \frac{\lambda_2 e^{-\lambda_1 b}}{\lambda_1 (\lambda_1 - \lambda_2)}]. \tag{22}$$

Then, one can solve Eq. (22) to obtain the minimum value of b_1^* , that is, the optimal buffer time at S_1 , and the optimal buffer time on S_2 is $b - b_1^*$. Take the derivative of b_1 in Eq. (22) and set the derivative result equal to 0, which is:

$$w_2 (\mathrm{e}^{\lambda_1 b_1} - 1) = w_1 e^{\lambda_2 (b - b_1)}. \tag{23}$$

Eq. (23) shows that the function on the left side of the equation increases as b_1 increases, and the function on the right side of the equation decreases as b_1 increases. Then, in $0 \le b_1 \le b$, there is an optimal solution, that is, Eq. (23) is solvable, but it is not easy to solve Eq. (23) directly, and it can be solved by the approximate estimation method.

(1) When $0 \le b < 1$ is equal to $0 \le b_1 < 1$, the Taylor formula is used to expand and simplify the exponential function to obtain:

$$\lambda_1 \lambda_2 w_2 b_1^2 + \lambda_1 w_2 b_1 - w_1 e^{\lambda_2 b} = 0. \tag{24}$$

By solving Eq. (24), one can obtain:

$$\begin{cases} b_1^* = \dfrac{-\lambda_1 w_2 + \sqrt{(\lambda_1 w_2)^2 + 4\lambda_1 \lambda_2 w_2 w_1 e^{\lambda_2 b}}}{2\lambda_1 \lambda_2 w_2} \\ b_2^* = b - b_1^* \end{cases}. \tag{25}$$

(2) When $b \geq 1$, the approximate estimation of Eq. (23) is:

$$\begin{cases} w_1 = e^{\lambda_1 b_1} - 1 \\ e^{\lambda_2 (b - b_1)} = w_2 \end{cases}. \tag{26}$$

By solving Eq. (26), one can obtain:

$$\begin{cases} b_1^* = \dfrac{\lambda_2 \ln w_1 + \lambda_1 (b\lambda_2 - \ln w_2)}{2\lambda_1 \lambda_2} \\ b_2^* = b - b_1^* \end{cases}. \tag{27}$$

4.4 Case study

The BTA was studied by taking the main line section Mas–Ut as an example. The buffer times allocated by the stations S_1 and S_2 were 3 and 2 min, respectively, and the buffer time allocated in the section was 5 min. That is, $b_1 = 3$, $b_2 = 2$, and $b = 5$, which can be used to calculate $E(\overline{d})$=2.170 .

The expected weight coefficients of delay of S_1 and S_2 were determined to be 0.58 and 0.42, respectively, through Eq. (19), that is, w_1=0.58 and w_2=0.42 . With the established BTA model, $b_1^* = 2.943$ and $b_2^* = 2.057$ can be obtained; then, $E(d_1) = 1.441$ and $E(d_2) = 2.905$ can be calculated, and, finally, $E(\overline{d^*}) = 2.056$.

Figure 5 shows that the delay expectation $E(\overline{d^*})$ after the BTA model is 0.114 min lower than the delay expectation $E(\overline{d})$ without the model — it was 5.25% lower. Therefore, the BTA model is effective. What is more, the buffer time focuses on the allocation of S_2. This measure can effectively reduce the expected delay time in the segment, provide a relevant basis for scheduling decisions, and help improve the efficiency of the work organization at (in) the stations (sections).

In conclusion, the BTA model established can consider the actual impact of delays. It provides a relevant research idea for the research of buffer time allocation based on operational performance data. Although the model only analyzes the BTA of several stations, the application of multiple stations remains to be studied. However, the results of the case study show that the model is reasonable and can be used to allocate buffer time between main stations in the trunk section.

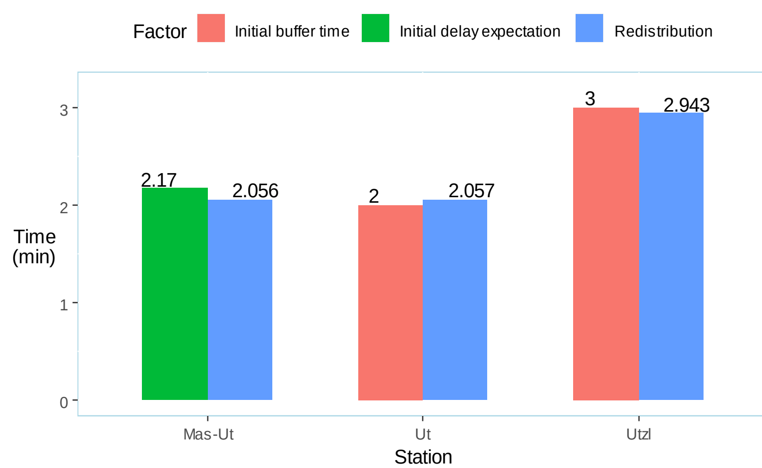

Figure 5: Comparison chart after BTA model

5 Conclusions

According to the delay distributions, a BTA model was established with the expected delay time as the objective function, realizing the redistribution of station buffer time, and the BTA model was verified by the Mas–Ut trunk section of the Dutch railway network. The results indicate the following.

(1) For the case where the total buffer time of the trunk section was different, the formula for assigning the station buffer time is given in Eqs. (24) and (26). The BTA formula shows that the delay distributions and delay strength have a certain influence on the BTA.

(2) The BTA model based on the delay distributions has a good effect on the redistribution of buffer time. By redistributing the buffer time of the stations Utzl and Ut, compared with the buffer time allocated before the station, after the BTA model, the total delay expectation time of the trunk segment decreased by 5.25%.

In conclusion, the dispatcher can adjust the work organization of the station according to the buffer time after BTA, to reduce the occurrence of station delays and improve the work efficiency of the station. Planned future work is the study of the BTA of the operation route and local network based on the BTA model of the trunk section. It is expected that the redistribution of buffer time can effectively reduce the delay of the operation route and local network and improve the delay recovery ability of the operation route and local road network.

Acknowledgments

This work was supported by the National Nature Science Foundation of China [grant number 71871188] and the Science & Technology Department of Sichuan Province [grant number 2018JY0567]. We are grateful for the contributions made by our project partners.

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