Energy-efficient Metro Train Operation Considering the Regenerative Energy: A Discrete Differential Dynamic Programming Approach

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Abstract
With the increase of the operating mileage, a large amount of energy consumption generated by metro systems needs to be taken seriously. One of the effective ways to reduce the energy consumption is to collaboratively optimize the driving strategy and train timetable by considering the regenerative energy (RE). However, the dimensionality and computational time will increase accordingly in optimization as the number of operating trains rises. With the intention of tackling this problem by efficiently reducing dimensionality, the energy-efficient metro train operation problem is optimized in this paper by applying the discrete differential dynamic programming (DDDP) approach. Firstly, the model calculating the net energy consumption that takes into account the RE is formulated. Then, the optimization model will be transformed to a discrete decision problem by using Space-Time-Speed (STS) network methodology, and the corresponding solution will be obtained through the DDDP based algorithm. Finally, two case studies will be conducted in a metro network to illustrate the effectiveness of the proposed approach.

Keywords
Energy reduction, Regenerative energy, Space-Time-Speed network methodology, Discrete differential dynamic programming

1 Introduction

Due to the advantages of high efficiency, large capacity and energy-efficient, metro systems are developing rapidly worldwide. However, with the increase of the operating mileage, a large amount of energy consumption generated by metro systems needs to be taken seriously. Furthermore, the traction energy accounts for the most important part in the energy consumption of the system. What is more, utilization of the regenerative energy (RE) provides us a good opportunity to reduce the energy consumption in metro systems. As a result, it is necessary to carry out the research on the optimization and control of the metro train operation by considering the RE.

The optimization and control of train operation are divided into driving strategy optimization and train timetable optimization. These two methods determine the energy consumption of the train operation by influencing the traction energy consumption and the reused RE. On the one hand, optimization of the driving strategy aims to find the energy-efficient control strategy so that the traction energy consumption is minimized by optimizing the regime sequences and the switching points. Literature on driving strategy optimization can date back to 1960s: Ishikawa (1968) proposed an optimal control model on the assumption that the train runs on a flat track with constant gradient and traction efficiency. Later, Khmelnitsky (2000) presented a complete study on the optimal train control problem, in which variable gradients, variable traction efficiency and arbitrary speed limits were all considered. Liu and Golovitcher (2003) gave an analytical solution to the problem with variable gradients for finding driving strategies for each part of the route. Chang and Sim (2008) applied a genetic algorithm on the train control problem to generate an optimal coast control based on evaluation of the punctuality, riding comfort and energy consumption together. Keskin and Karamancioglu (2017) developed the optimal train operation strategies by using three nature-inspired metaheuristic algorithms: Genetic Simulated Annealing, Firefly, and Big Bang-Big Crunch.

On the other hand, optimization of the train timetable can greatly reduce the traction energy consumption as well as efficiently utilize the RE from the macroscopic views, such as Su et al. (2013) provided an analytical formulation to calculate the optimal speed profile with fixed trip time for each section. He also designed a numerical algorithm to distribute the total trip time among different sections and prove the optimality of the distribution algorithm. Furthermore, Su et al. (2015) proposed a bisection method to solve the optimal departure time for an accelerating train. Rodrigo et al. (2013) used a semi analytical solution that leads to a discretization and to the application of the Langrange multipliers method to solve the optimization of n-tuples of speed. Tian et al. (2017) proposed a multi-train traction power network modelling method to determine the system energy flow of the railway system with regenerating braking trains. Yin et al. (2016) developed a stochastic programming model for metro train rescheduling problem in order to jointly reduce the time delay of affected passengers, their total traveling time and operational costs of trains.

However, in this field, previous research mainly carried on optimization by considering the driving strategy or the train timetable, which can only achieve the local optimization of the energy consumption. In order to get the global optimal solution, some experts proposed a kind of collaborative optimization which focuses on both aspects. In this way, the energy consumption of the metro system will be reduced from the perspective of the system and the performance of optimization will be significantly improved. For example, Bocharnikov et al. (2010) presented a single train speed profile optimization model considering both tractive energy consumption and utilization of RE. Furthermore, the authors performed a multi-train simulation to estimate the benefits and effects of the optimal speed profile on minimizing the net energy consumption. Li and Lo (2014) gave the quantitative analysis of tractive energy consumption, RE utilization and net energy consumption, then they proposed an energy-efficient scheduling and speed control model to minimize the net energy consumption, which assuming all trains run with maximum acceleration, coasting and maximum deceleration in each segment. Ning et al. (2018) proposed a two-stage urban rail transit operation planning approach comprising running time allocation and RE utilization to save energy consumption. Bu et al. (2018) set up a ’time slot and energy grid’ model, which can effectively reduce the complexity of analyzing the usage of RE among multiple bidirectional running trains. Based on the model, they designed the energy saving method. Zhou et al. (2018) proposed an integrated optimization model on train control and timetable to minimize the net energy consumption, in which the proposed train control is based on finding the optimal switching points among the control modes of maximum acceleration, cruising, coasting, and maximum braking to minimize the net energy consumption, while cruising and coasting regimes might be adopted for more than one time.

Nevertheless, among the existing research about integrated optimization, researchers mainly adopted two kind of ways to obtain the optimal solution: Some authors adopted the two-stage method to optimize the driving strategy and the train timetable respectively, this kind of hierarchy optimization can not make full use of real-time RE; Other authors achieved the simultaneous optimization, but they need to specify the transition sequence of control modes artificially. The solution obtained by above methods is still not optimal. As a result, it is necessary to simultaneous optimize the driving strategy and train timetable when the control strategy is uncertain. This paper proposes a integrated optimization approach of the driving strategy and the train timetable. In the approach, the control mode of trains can be arbitrary at each time.

Another challenge encountered in related research is that the problem of multi-train RE utilization is a highly dimensional problem. Because the states of multiple trains need to be considered at the same time during the optimization, "the curse of dimensionality" often exists with the number of trains increasing. Many researchers have tried to overcome similar multi-variable optimization problems by using improved DP algorithms, in which the discrete differential dynamic programming (DDDP) approach is an effective way. It is an iterative method firstly proposed by Heidari et al. (1971) when optimizing the operating policies of multiple unit and multiple purpose water resources systems. It can sharply reduce the computing time as well as the required computer's memory space by decreasing the dimension of the problem. A relative coefficient based on maximum output capacity and an adaptive bias corridor technology are proposed by Li et al. (2014) to improve the DDDP approach in order to get more power generation and enhance the convergence speed. Feng et al. (2017) optimized the operation of hydropower system by proposing a algorithm which combining the merits of DDDP and orthogonal experiment design. In this way, the computing amount can be sharply reduced when the quality of the result is influenced a bit. Tospornsampan et al. (2016) proposed a general operating policy for a multiple reservoir system operation which using the combination of a DDDP and a neural network (NN). The result shows the combination model performs satisfactorily. The previous studies show that DDDP approach is a suitable means to solve the high-dimensional problem and this algorithm has not been applied in the energy conservation optimization of metro systems yet. Therefore, the application of DDDP method in the simultaneous optimization of driving strategy and timetable motivates the study of this paper.

In this paper, we will propose an integrated energy-efficiency optimization model for multi trains which combines the driving strategy and the train timetable. Then a DDDP based algorithm will be designed to solve the proposed model so as to get the global optimal solution with low calculating time and the computer memory requirement. In this way, the energy consumption of the urban rail systems will be reduced from the perspective of the system. The optimal result can be more accurate and the calculation time can be shorter by comparing with the traditional dynamic programming algorithm.

The rest of this paper is organized as follows. In Section 2, the problem is formulated with a coordination optimization model. In Section 3, the solution approach consisting of the STS network methodology and DDDP approach is proposed. In Section 4, the effectiveness and efficiency of the proposed approach are demonstrated in a metro network by comparing with dynamic programming. In Section 5, the main contributions of this paper are summarized and some future research is discussed.

2 Mathematical Formulations

This paper aims to reduce the energy consumption of train operation by optimizing the train timetable and the driving strategy at the same time. Before introducing the solution approach of this problem, we will create the mathematical model and show the formulations in this section.

2.1 Key parameters

Firstly, for a better understanding of the paper, the key parameters of the model are illustrated in Table 1 and Table 2.

Table 1: Sets and decision variables

Symbol	Description
	Set
N	Set of time sequence
I	Set of trains
K	Set of stations
	Decision variable
$v_i(t_n)$	Speed of train i at time t_n (m/s)
$s_i(t_n)$	Space of train i at time t_n (m)

2.2 Energy consumption without considering RE

In this section, the energy consumption calculation model with considering RE is introduced, in which the net traction energy that defined as the difference between the traction energy and the reused RE is the objective function of this optimization problem.

Specifically, the operating time of trains is divided into many small parts and N is the set of time sequence. By using this method, we will make the calculation of RE transmission process more precise. The amount of utilised RE can be obtained after fixed time to approximate simulate the transmission process of RE. What is more, the total traction energy can also be obtained in this way.

Firstly, for a certain train i, the resistance can be divided into the basic resistance and line resistance. $RB(v_i(t_n))$ is the basic resistance including roll resistance and air resistance, which can be described as

$$RB(v_i(t_n)) = m(a_1 v_i(t_n)^2 + a_2 v_i(t_n) + a_3) \tag{1}$$

What is more, $RC(s_i(t_n))$ is the line resistance caused by track grade, curves and tunnels, which is related to the position of train. Resistance caused by tunnels is always too small compared with resistance caused by track grade and curves, so it can be ignored in the optimization. As a result, $RC(s_i(t_n))$ can be calculated by following equation, in which $\alpha(s_i(t_n))$ and $R(s_i(t_n))$ are the angle of gradient and radius of turning circle, respectively.

$$RC(s_i(t_n)) = mgsin\alpha(s_i(t_n)) + \frac{600}{R(s_i(t_n))} \tag{2}$$

Table 2: Notation

Symbol	Description
n	Time sequence index, $n \in N$
i	Train index, $i \in I$
k	Station index, $k \in K$
t_n	Time stamp
$u_i(t_n)$	Acceleration/deceleration rate of train i at time t_n (m/s^2).
m	Mass of train (kg)
$RB(v_i(t_n))$	Basic resistance of train i at time t_n (N)
$RC(s_i(t_n))$	Line resistance of train i at time t_n (N)
$fr(v_i(t_n), s_i(t_n))$	Total resistance of train i at time t_n
$F_i(t_n)$	Force of train i at time t_n (N)
$FT_i(t_n)$	Tractive effort of train i at time t_n (N)
$ET_i(t_n)$	Traction energy consumption of train i between t_n and t_{n+1} (J)
ET	Total traction energy consumed by trains in whole time range (J)
$FB_{i'}(t_n)$	Braking effort of train i' at time t_n (N)
$EB_{i'}(t_n)$	RE produced by train i' between t_n and t_{n+1} (J)
$w_{i',i}(t_n)$	Factor to measure how much RE transferred from i' to i (J)
$ER_{i',i}(t_n)$	RE allocated from train i' to train i at time t_n(J)
$EL_{i',i}(t_n)$	RE loss during transmission from train $i'$to train i at time t_n (J)
e	RE loss per distance unit (J/m)
$EU_{i',i}(t_n)$	RE absorbed by train i form train i' at time t_n (J)
$EU_i(t_n)$	RE actually utilized by train i (J)
EU	Total RE utilized in whole time range (J)
E	Net energy consumption in whole time range (J)
$d_i(k)$	Departure time for train i at station k (s)
$a_i(k)$	Arrival time for train i at station k (s)
$[\varpi^{min}(k), \varpi^{max}(k)]$	Dwell time threshold for trains at station k (s)
$[\xi^{min}(k), \xi^{max}(k)]$	Trip time threshold between station k and $k+1$ (s)
h^{min}	Minimum headway (s)
$[TC^{min}, TC^{max}]$	Cycle time threshold for trains (s)
v^{max}	Maximum speed (m/s)

In this way, the total resistance can be calculated as

$$fr(v_i(t_n), s_i(t_n)) = RB(v_i(t_n)) + RC(s_i(t_n)) \tag{3}$$

Then, the accelerate rate of train will be solved.

$$u_i(t_n) = \frac{F_i(t_n) - fr(v_i(t_n), s_i(t_n))}{m} \tag{4}$$

The total traction energy between two adjacent time t_n and t_{n+1} can be obtained by Equation (5) and (7), in which the traction force $FT_i(t_n)$ is the larger value between the actual force applied to the train $F_i(t_n)$ and 0.

$$FT_i(t_n) = \max\{u_i \cdot m + fr(v_i(t_n), s_i(t_n)), 0\} \tag{5}$$

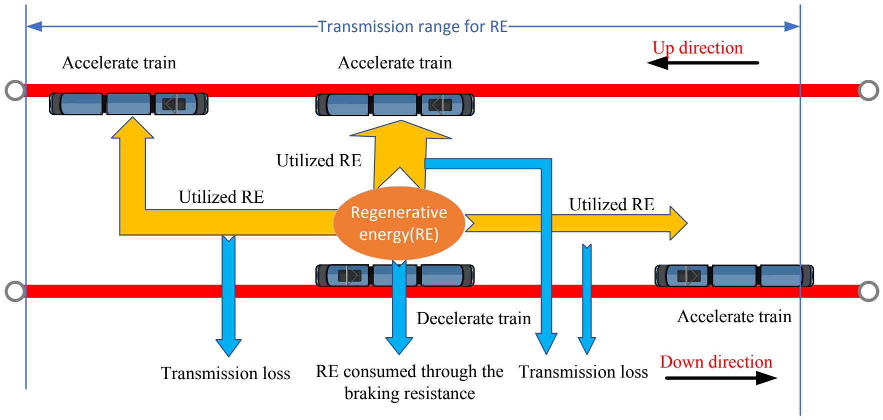


Figure 1: RE transmission schematic diagram

$$ET_i(t_n) = \int_{t_n}^{t_{n+1}} FT_i(t) \cdot v_i(t)dt \tag{6}$$

By adding up the traction energy of all trains during each periods of time, the total traction energy can be obtained by Equation (7).

$$ET = \sum_{i=1}^{I} \sum_{n=1}^{N-1} ET_i(t_n) \tag{7}$$

2.3 Energy consumption with considering RE

In addition to the traction energy during operation, RE produced by braking trains also need to be considered to realize energy efficient operation. Figure 1 is the schematic diagram of the RE transmission principle. It shows the RE produced by decelerate trains will be divided into all the accelerate trains within the energy transfer range. The amount of RE absorbed by each train is related to the distance between two trains as well as the voltage difference in the traction power grid. The energy loss during transmission and rest energy consumed by braking resistance should also be considered. The corresponding model is listed as follows:

The RE produced by train i' can be calculated by following equations. Some of the braking power will be consumed and cannot be transformed to RE. φ is the energy conversion rate in this process.

$$FB_{i'}(t_n) = -\min\{u_{i'} \cdot m + fr(v_{i'}(t_n), s_{i'}(t_n)), 0\} \tag{8}$$

$$EB_{i'}(t_n) = \int_{t=t_n}^{t_{n+1}} \varphi \cdot FB_{i'}(t) \cdot v_{i'}(t)dt \tag{9}$$

According to the actual principle, the RE produced by train i' can be used by all nearby accelerating trains, whether running in the same direction or negative direction. In this paper, the formulas of RE transmission are assumed as Equation (10), (11), (12). Factor $w_{i',i}(t_n)$ is set to measure how much RE transferred to train i from i' at time t_n. If the distance between two trains is farther than transmission range z, the value of $w_{i',i}(t_n)$ is 0. Otherwise, the value is related to distance and voltage difference between two trains. The larger the distance or the smaller the voltage difference is, the smaller the value of $w_{i',i}(t_n)$ is. Equation (10) is used to model the RE utilization, in which s_0 and E_0 are fixed values.

$$w_{i',i}(t_n) = \begin{cases} 1/\left(\frac{|s_i(t_n)-s_{i'}(t_n)|}{s_0} - \frac{max\{0,\Delta ET_i(t_n)-\Delta EB_{i'}(t)\}}{E_0}\right), & |s_{i'}(t_n)-s_i(t_n)| \leq z; \\ 0, & otherwise. \end{cases} \tag{10}$$

By considering all trains in the line, the RE allocated from train i' to train i can be calculated as

$$ER_{i',i}(t_n) = EB_{i'}(t_n) \times \frac{w_{i',i}(t_n)}{\sum_{j=1}^{I} w_{i',j}(t_n)} \tag{11}$$

What is more, the energy loss during transmission can be calculated with the Equation (12), which is related to the distance between trains. Then, the energy absorbed by train i can be obtained as Equation (13).

$$EL_{i',i}(t_n) = e \times |s_i(t_n) - s_{i'}(t_n)| \tag{12}$$

$$EU_{i',i}(t_n) = ER_{i',i}(t_n) - EL_{i',i}(t_n) \tag{13}$$

Further more, it is possible that not all the RE will be utilized by train i, because the energy absorbed may be more than traction energy required. In this condition, the redundant energy will be consumed by the braking resistance. As a result, the RE actually utilized by train i can be calculated as

$$EU_i(t_n) = \min\{\sum_{j=1}^{I} EU_{j,i}(t_n), ET_i(t_n)\} \tag{14}$$

By adding up the RE utilized by all trains during each periods of time, the total RE utilized during the whole time can be obtained by Equation (15).

$$EU = \sum_{i=1}^{I}\sum_{n=1}^{N-1} EU_i(t_n) \tag{15}$$

Finally, the objective function of this problem can be obtained by following formula, which is the result of total traction power minus total RE.

$$E = ET - EU \tag{16}$$

2.4 Optimization model

The optimization model is formulated as below, which includes the objective function of this problem and two kind of constraints.

Minimize

$$E = ET - EU$$

Subject to

$$\varpi^{min}(k) \leq d_i(k) - a_i(k) \leq \varpi^{max}(k); \quad \forall 1 \leq i \leq I, 1 \leq k \leq K \tag{17}$$

$$\xi^{min}(k) \leq a_i(k+1) - d_i(k) \leq \xi^{max}(k); \quad \forall 1 \leq i \leq I, 1 \leq k \leq K-1 \tag{18}$$

$$d_i(k) - d_{i-1}(k) \geq h^{min}; \quad \forall 2 \leq i \leq I, 1 \leq k \leq K \tag{19}$$

$$a_i(k) - a_{i-1}(k) \geq h^{min}; \quad \forall 2 \leq i \leq I, 1 \leq k \leq K \tag{20}$$

$$TC^{min} \leq d_i(K) - a_i(1) \leq TC^{max}; \quad \forall 1 \leq i \leq I \tag{21}$$

$$FT_i(t_n) \leq FT^{max}; \quad \forall 1 \leq i \leq I, 1 \leq n \leq N-1 \tag{22}$$

$$FB_i(t_n) \leq FB^{max}; \quad \forall 1 \leq i \leq I, 1 \leq n \leq N-1 \tag{23}$$

$$0 \leq v_i(t_n) \leq v^{max}; \quad \forall 1 \leq i \leq I, 1 \leq n \leq N \tag{24}$$

Among above constraints, Formula (17) and Formula (18) restrict the dwell time and travel time of trains. In Formula (17), $\varpi^{min}(k)$ and $\varpi^{max}(k)$ are both determined by the station's condition and the number of passengers. In Formula (18), $\xi^{min}(k)$ is determined by the accelerating and decelerating ability of trains, the length of segments and the speed limits of segments. $\xi^{max}(k)$ is determined by the planned train timetable. As for headway limits, the headway between adjacent trains should be within the given range, which are shown in Formula (19) and Formula (20). What is more, Formula (21) is the constraint for the cycle time of trains during operation. In summary, Formula (17)- (21) are the constraints of the train timetable.

On the other hand, Formula (22) and (23) restrict the maximum traction and braking force of trains, which are represented by FT^{max} and FB^{max}. Formula (24) is the speed limit constraint in order to ensure the safe operation of trains. These three inequations are the constraints of the driving strategy.

3 Solution approach

In order to obtain the optimal value of objective function when all the constraints are satisfied, the STS network methodology is applied to discretize the state variables and the DDDP approach is used to solve this problem. This section will simply introduce the theory of the STS and DDDP method and show how to realize energy efficient optimization by using this method.

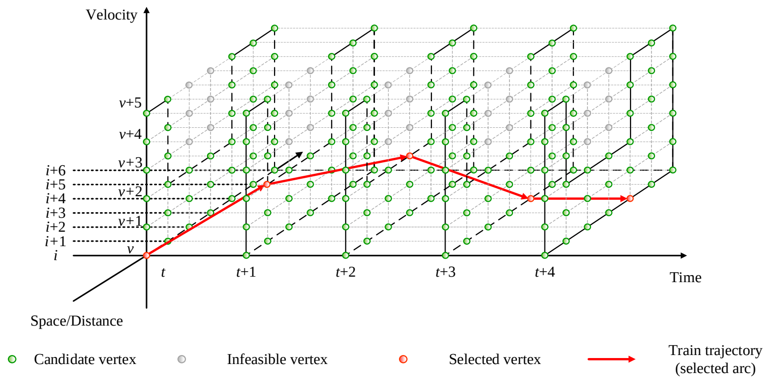


Figure 2: STS network for the integrated train operation problem

3.1 An overview of STS

The theory of STS network methodology is discretizing the space, velocity and operating time of trains to construct a large number of cells as shown in Figure 2, then the optimal operating route of trains can be selected flexibly in the network such as the red curve in the figure(Zhou et al. (2017)). STS network methodology mainly has following advantages: Firstly, it discretizes the problem into a multi-step decision process, which is suitable for solving the problem by dynamic programming and its improved algorithm, such as the DDDP approach in this paper. Secondly, in the STS networks, the shadow of the train route in the Space-Time side is the driving curve of trains and the shadow in the Space-Time side is the train timetable. Finally, the integrated optimization to reduce energy consumption can be realized by systematically incorporating the Space-Time (train timetable) model and the Space-Speed (driving strategy) model into the STS network.

As a result, the STS network methodology has been widely used in the transportation route optimization, various scheduling applications and general dynamic network flow modeling.

3.2 An overview of DDDP

DDDP is an improved DP method to overcome the "curse of dimensionality" by reducing the computational dimension. The principle of DDDP approach is dividing the solution space of problem into several subspaces and obtain the best local optimal solution in each iteration. By repeating this process, the global optimal solution can be solved. The schematic diagram of DDDP is shown as Figure 3 and the general procedures of DDDP are presented as follows (Heidari et al. (1971), Li et al. (2014), Feng et al. (2017)):

- The initial test trajectory which satisfies the constraints can be obtained by experience or other methods, shown as the red curve in Figure 3.
- In the neighborhood of the test trajectory, the solution space of problem at each stage can be separated into several subspaces and combined to form the corridor,

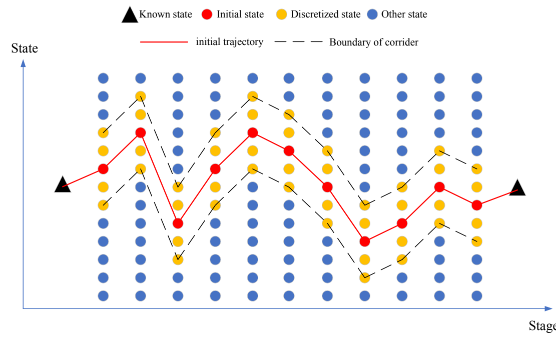


Figure 3: Schematic diagram of DDDP calculating process

- DP recursive equation is used to find an improved trajectory in the corridor,
- The optimal trajectory in the last iteration will be taken as the initial trajectory of the next iteration,
- Repeat iteration until the convergence condition is met.

It can be seen that, compared with the DP, DDDP method does not need to optimize in the entire feasible region of state variables, but only find the optimal solution within the corridor each time, which is a small range compared with the former. In this way, by using DDDP method we can effectively reducing the computational storage and time.

For example, we assume that in a optimization problem there are τ stages, κ state variables and ρ states of each variables in each stage. If we solve the problem by using DP method, the space and time complexity are $\tau\kappa^{\rho}$ and $\tau\kappa^{2\rho}$, respectively. If the number of variables or states is quite some, the computational amount will be too large to calculate. However, by using DDDP method, the space and time complexity can only reach $\tau\lambda^{\rho}$ and $\tau\lambda^{2\rho}$ by setting the number of states in corridor is λ in each stage. In this way, the dimension of problem will be reduced and the computational storage and time can decrease a lot by choosing λ as a small number. Energy efficient problem for multi-trains will have more state variables with the increase of train number, so it can get better effect of dimensionality reduction by using DDDP method.

3.3 Procedures to solve energy-efficient train operation problem

In this section, we will solve the multi-trains energy-efficient problem by using a combination of STS and DDDP method. The solving thought is discretizing the state variables of problem to create a multi-step decision process firstly, then obtain the global optimal solution by repeating solving the best solution in corridor selected. The detail procedures are presented as follows:

(i) Step 1: Discretization-Before optimizing, state variables of train should be discretized and STS network should be constructed firstly by using STS method. In this problem, operating time is the stage which can be represented by (t_a), a is the index of stage. What is more, the state variables in each stage conclude the speed and space of each train. If there are I trains operating in metro line at the same time, there will be $2I$ state variables in each stage, which can be represented by $\Psi_A(v_I, s_I)$. Meanwhile, for a certain train i, $\Psi_a(v_i, s_i)$ is the speed and space variables in stage a. In this way, we can obtain the best operating trajectory for each train by choosing optimal states in each stage.

(ii) Step 2: Parameters intialization-Set the basic parameters of DDDP method, including time interval between stages, initial speed and space interval in each stage, terminal condition and so on.

(iii) Step 3: Establish the initial trajectory-Randomly generate a feasible solution of this problem which satisfies all the constraints. The initial trajectory can be listed by $\Psi_A(v_I^0, s_I^0)$.

(iv) Step 4: Create optimizing corridor for initial trajectory-Firstly, we use Δv^0 and Δs^0 to represent initial speed interval and space interval. L is the width of corridor which can only be even number. In this way, there will be $L+1$ kinds for each state variable. Then, the corridor for initial trajectory can be constructed as Formula (25) for $i \in I, a \in A$. The best local solution in the first iteration will be selected from the states in this corridor.

$$
\begin{gathered}
\Psi_a(v_i^0 - \tfrac{L}{2}\Delta v_0, s_i^0 - \tfrac{L}{2}\Delta s_0) \\
\Psi_a(v_i^0 - (\tfrac{L}{2}-1)\Delta v_0, s_i^0 - (\tfrac{L}{2}-1)\Delta v_0) \\
\cdots \\
\Psi_a(v_i^0, s_i^0) \\
\cdots \\
\Psi_a(v_i^0 + (\tfrac{L}{2}-1)\Delta v_0, s_i^0 + (\tfrac{L}{2}-1)\Delta v_0) \\
\Psi_a(v_i^0 + \tfrac{L}{2}\Delta v_0, s_i^0 + \tfrac{L}{2}\Delta s_0)
\end{gathered} \tag{25}
$$

(v) Step 5: Calculation-Search the best local solution in the current corridor by using traditional DP method as following process:

- Determine the driving strategy of arcs. Given the starting point $\Psi_a(v_i, s_i)$ and the end point $\Psi_{a+1}(v_i, s_i)$ of a arc of train i in STS networks, we can determine the driving strategy of the train between these two points by the following means. Firstly, three reference states $\Psi_{a+1}(v_i^{ma}, s_i^{ma})$, $\Psi_{a+1}(v_i^{co}, s_i^{co})$, $\Psi_{a+1}(v_i^{mb}, s_i^{mb})$ in stage $a+1$ which represent the train operates with maximum acceleration, coasting and maximum deceleration between stage a and $a+1$ need to be calculated according to $\Psi_a(v_i, s_i)$. Next, by comparing $\Psi_{a+1}(v_i, s_i)$ and reference states, the driving strategy of arc will be chosen on the basis of Table 3. As shown in the table, there are 5 feasible driving strategy and other conditions will be eliminated. In this way, the equation of each train's curve between adjacent stages will be solved.
- Calculate the cost of each set of arcs. In this problem, the cost of each set of arcs represents the net energy consumption of trains during this process. By given

Table 3: Driving strategy for each kind of arcs

Condition	Driving strategy
$v_i = v_i^{ma}, s_i = s_i^{ma}$	Maximum acceleration
$v_i^{co} < v_i < v_i^{ma}, s_i^{co} < s_i < s_i^{ma}$	Partial acceleration-coasting
$v_i = v_i^{co}, s_i = s_i^{co}$	Coasting
$v_i^{mb} < v_i < v_i^{co}, s_i^{mb} < s_i < s_i^{co}$	Partial deceleration-coasting
$v_i = v_i^{mb}, s_i = s_i^{mb}$	Maximum deceleration

the driving curve of each operating train between continuous stage a and $a+1$ by using above the means, the cost of certain set of arcs $U(\Psi_{a,a+1}(v_I, s_I))$ can be calculated by Formulas (1)-(16). If there are one or more arc is not attainable, the cost of this set will be ∞.

- Choose the best set of arcs and eliminate others. After calculating the cost of the whole sets of arcs between two stages, it is necessary to choose the best set for each states in stage $a+1$ according to Bellman Equation to decrease the calculated amount. Up to stage a, the total cost of all the previous steps is $J(\Psi_a(v_I, s_I))$. Then, $J(\Psi_{a+1}(v_I, s_I))$ can be calculated by Equation (26).

$$J(\Psi_{a+1}(v_I, s_I)) = J(\Psi_a(v_I, s_I)) + U(\Psi_{a,a+1}(v_I, s_I)) \tag{26}$$

Finally, the best set of arcs with lowest cost for each states in stage $a+1$ will be selected according to Equation (27), in which $\Psi_a(v_I^*, s_I^*)$ is the chosen state to match $\Psi_{a+1}(v_I, s_I)$. In addition, other sets of arcs should be eliminated.

$$\Psi_a(v_I^*, s_I^*) = J(\Psi_a(v_I^*, s_I^*)) + U(\Psi_{a,a+1}(v_I, s_I)) \tag{27}$$

- Repeat above steps of DP until all the possible states in corridor have been selected and obtain the best local optimal solution in corridor which can be represented by Υ^ρ for the ρth iteration.

(vi) Judge the best local solution and adjust parameters-Compare Υ^ρ with $\Upsilon^{\rho-1}$ starting with the second iteration and adjust the speed and space intervals in $\rho+1$th iteration according to difference value of costs as Formula (28).

$$\begin{cases} \Delta v^{\rho+1} = \Delta v^\rho - \delta(v), \Delta s^{\rho+1} = \Delta s^\rho - \delta(s); & \Upsilon^\rho - \Upsilon^{\rho-1} \geq \delta(\Upsilon)\ and\ \rho \geq 2 \\ \Delta v^{\rho+1} = \Delta v^\rho, \Delta s^{\rho+1} = \Delta s^\rho; & \Upsilon^\rho - \Upsilon^{\rho-1} < \delta(\Upsilon)\ or\ \rho = 1 \end{cases} \tag{28}$$

(vii) Create new corridor for improved the trajectory. The improved trajectory in the ρth iteration will be the initial trajectory in the $\rho+1$th iteration. Then the corridor in $\rho+1$th iteration can be constructed as Formula (25).

(viii) DDDP iteration. Repeat step (v)-step (vii) until the terminal condition which is listed as Formula (29) is met. $\underline{v}$ is a fixed valve which means the minimal speed interval.

$$\Delta v^{\rho+1} < \underline{v} \tag{29}$$

4 Case study

In order to illustrate the effectiveness of the proposed model and numerical algorithm, two numerical examples are conducted based on a small metro network (3 stations, 2 segments, and 2 turn-back stations). The operation requirement and basic infrastructure data about this metro network is described in Table 4. The trains operate in the network are composed of six cars and three of them are traction units. The length of trains is 114m and the net train mass is 192000kg. As the important parameters to calculate the energy consumption, the mass of trains with passengers is set as 250000kg, energy loss per distance unit e is 100J/m, the conversion rate from braking energy to RE φ is 0.5 and the transmission range of RE z is 1500m. As for the parameters for DDDP method, the initial speed interval Δv^0 and space interval Δs^0 are 20m/s and 20m, respectively. The time interval between two adjacent stages is 15s and the minimal speed interval $\underline{v}$ is 10^6J. In the aspect of computer configuration, the algorithms described in this paper were implemented in MATLAB R2014a. What is more, the operating system is Windows 7 professional, the CPU consists of one Intel Core i7-7700@3.6GHz and the memory size 16 GB.

Table 4: Operation requirement and basic infrastructure data of metro network

Destination	**Distance**	T_{min}	T_{max}
Turn-back station 1	0m	-	-
Station 1	667m	72s	102s
Station 2	1700m	67s	97s
Station 3	3020m	80s	110s
Turn-back station 2	3687m	72s	102s

In Example 1, we apply DDDP approach to optimize the energy-efficient driving strategy as well as train timetable by considering there are 2 trains operate in the metro network and compare the result with DP approach. In Example 2, we apply DDDP approach to the same problem with considering 3 running trains and contrast the solution with DP approach.

4.1 Example 1

In example 1, there are 2 trains whose original locations and directions are shown in Figure 4 operate in the line. In order to test the performance of DDDP approach, we optimize the problem by using traditional DP approach firstly as control group, in which the speed interval and space interval are 10 m/s and 10m, respectively. The Space-Time-Speed diagram after optimization is shown as Figure 5 and the result data is shown in Table 5. Then the energy conservation issue is solved by DDDP approach: The Space-Time-Speed diagram, Space-Time diagram and Speed-Time diagram are presented in Figure 6 and Figure 7, respectively. In the figures, curves with different colors represent operating trajectories of different trains. Space-Time diagram can be regarded as the train timetable and Speed-Time diagram can be seen as the driving strategy of trains. Although the driving strategy of each train as shown in Figure 7 may not be the optimal when only one train is taken into consideration, the actual energy consumption of all trains will be optimal with considering the regenerative energy. What is more, the simulating data by using DDDP method is listed in Table 5.

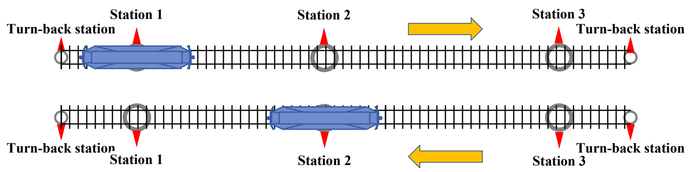


Figure 4: Original location and direction of 2 trains

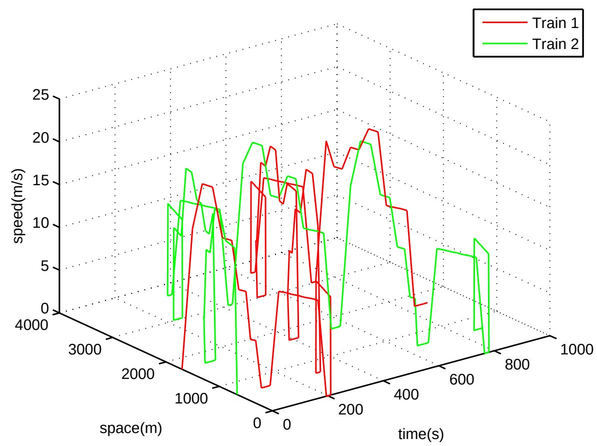


Figure 5: Space-Time-Speed diagram of 2 trains by using DP method

By comparing the data in Table 5, the total energy consumption and net energy consumption are reduced by 7.8% and 12.2%. What is more, the reused RE utilized by trains increases from 9.62 kW·h to 15.55 kW·h, in which the increase rate is 61.6 %. It is because the integrated optimum design can decrease the tractive energy needed and improve the utilization rate of RE at the same time. By using DDDP method, the obtained solution will be more accurate and the energy-saving effect is better. As for computing time, it only costs 95 seconds to calculate the problem by using DDDP approach, which is reduced by 73.9 % compared with DP's time.

More specifically, the contrast picture of the result of DP and DDDP method is shown as Figure 8, in which the computing time is plotted on the horizontal axis, the net energy consumption and variable interval are plotted on the primary vertical axis, the speed and space interval of DDDP are plotted on the secondary vertical axis. From Figure 8, the quality of DDDP's solution is not better than DP's in the first few times iterations. However, the net energy consumption after optimization by using DDDP will be less than DP's since the 16th iteration and the difference will be more and more with the passage of computing time. As a result, we can get better optimal solution in shorter time by applying DDDP approach.

Table 5: Data of energy consumption and computing time of 2 trains

Method	Total consumption	Utilized RE	losing RE	Net consumption	Computing time
DP method	162.91 kW·h	9.62 kW·h	1.82 kW·h	153.29 kW·h	364 s
DDDP method	150.18 kW·h	15.55 kW·h	1.81 kW·h	134.63 kW·h	95 s

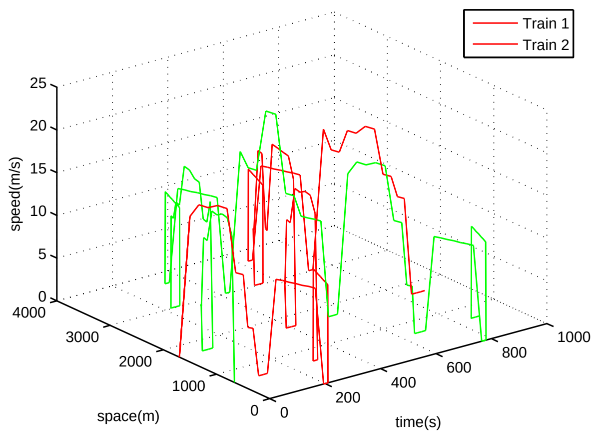


Figure 6: Space-Time-Speed diagram of 2 trains by using DDDP method

4.2 Example 2

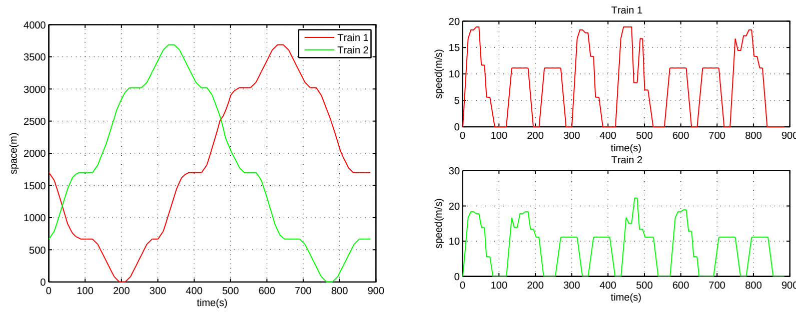
In this case study, DP and DDDP are applied to the same metro network with considering 3 running trains as shown in Figure 9. By using DP approach, the Space-Time-Speed diagram after optimization is shown as Figure 10 and the result data is shown in Table 6. As for the result of solving by DDDP approach, Figure 11 and Figure 12 show the Space-Time-Speed diagram, Space-Time diagram and Speed-Time diagram. The simulating data is also listed in Table 6.

On the one hand, by calculating the total energy consumption and net energy consumption are reduced by 11.8% and 20.3%. Also, the reused RE utilized by trains increases from 14.12 kW·h to 32.96 kW·h, in which the growth rate is 133.4 %. On the other hand, the computing time decreases from 69779 seconds to 14625 seconds. The droop rate is 79.0 %.

Table 6: Data of energy consumption and computing time of 3 trains

Method	Total consumption	Utilized RE	losing RE	Net consumption	Computing time
DP method	257.51 kW·h	14.12 kW·h	12.25 kW·h	243.39 kW·h	69779 s
DDDP method	227.02 kW·h	32.96 kW·h	1.26 kW·h	194.06 kW·h	14625 s

The result contrast picture of DP and DDDP method when considering 3 trains is shown as Figure 13. From the figure, we can find the net energy consumption after optimization by using DDDP will be less than DP's since the 16th iteration and the gap of difference will increase in the later iterations.



(a) Space-Time diagram

(b) Speed-Time diagram

Figure 7: Space-Time and Speed-Time diagram of 2 trains by using DDDP method

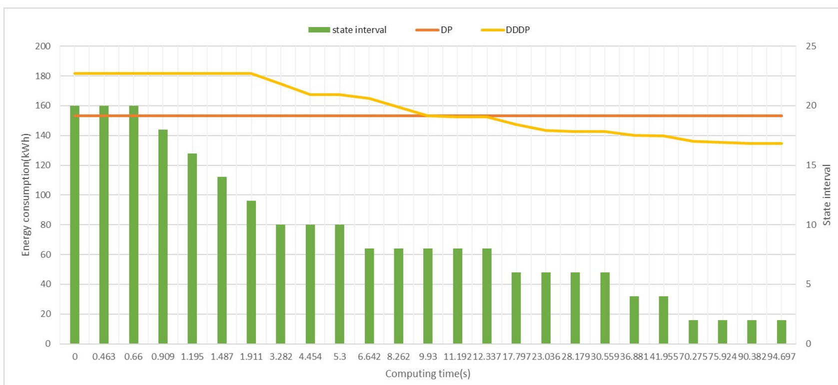


Figure 8: Result contrast picture of DP and DDDP method with considering 2 trains

4.3 Summary of experiment results

According to the above result data, the solution of DDDP can produce the lower energy consumption and make the better use of the RE then DP no matter when 3 trains or 2 trains operate in the line. What is more, the computing time is also much lower because DDDP method reduces the number of feasible states for each variables per phase. In summary, it is accurate as well as efficient to use this method in multi-train energy efficient problem.

We can also discover some information by comparing the results of two examples. Firstly, by using DDDP method, the energy-saving effect and RE utilization effect are more obvious with the increase of train number. It is because DDDP's state intervals of each train are more and more precise with the iterations. In this way, each additional train will increase the overall search accuracy to a greater extent compared to other algorithm. In addition, the decreasing amplitude of computing time compared to DP will be larger when more trains operate in the line. The reason is the calculated amount of each train is fewer by using DDDP approach and the amplitude reduction ratio of the whole computation is much less with the increase of exponent.

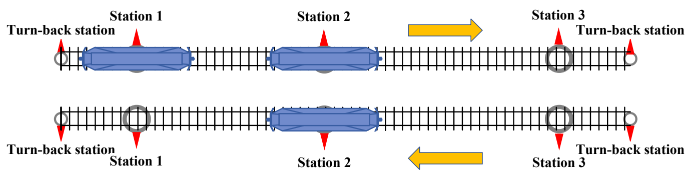


Figure 9: Original location and direction of 3 trains

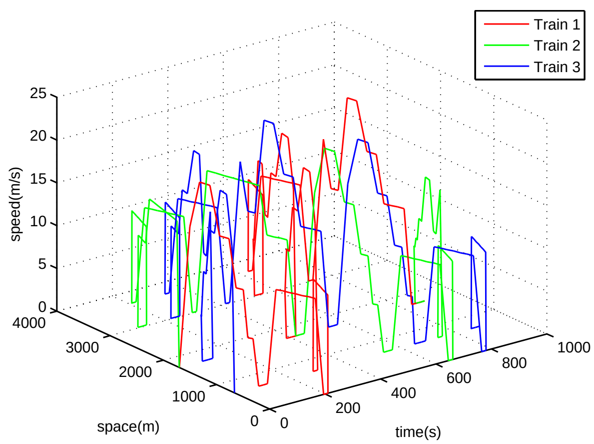


Figure 10: Space-Time-Speed diagram of 3 trains by using DP method

In summary, multi-trains optimization for energy efficiency by using DDDP approach is high-efficiency in terms of the energy-saving effect and the computing time. What is more, the optimizing performance will be better when more trains taken into considered.

5 Conclusions

In this study, a new efficient technique employing DDDP is presented to address the energy-efficient metro train operation problem with considering the RE. Firstly, the objective function and constraints are formulated to construct the mathematical model of the problem. Then, the solution approach is proposed, in which the state variables of trains in the proposed model are discretized by STS network methodology and optimized by DDDP approach. Finally, two numerical experiments are simulated to test the potential ability of DDDP when multi trains operate in the small network. The simulation results show that, the net energy consumption of trains by using DDDP is 12.2 % and 20.3 % lower compared with DP when 2 and 3 trains operate in the line, respectively. Besides, the use ratio of RE increases by 61.6 % and 133.4 %. The computing time of simulation also decreases by 73.9 % and 79.0 % in two cases. As a result, DDDP approach applied in this paper is an effective way to obtain more exact solution in shorter time. What is more, the effect of dimension-

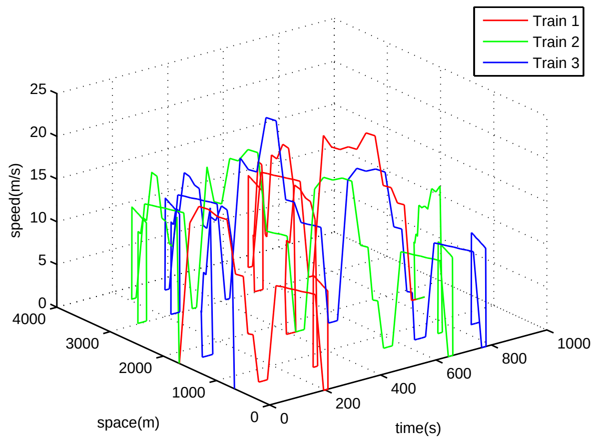
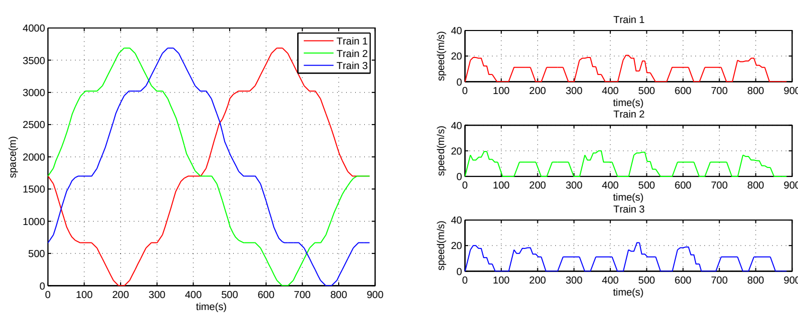


Figure 11: Space-Time-Speed diagram of 3 trains by using DDDP method

ality reduction by using DDDP can be more obvious compared to DP when more trains are considered in the optimization according to the result data, which will further prove the great performance of DDDP in the integrated optimization of the driving strategy and the train timetable.

However. the ability of DDDP approach remains limited in the optimization when large number of trains operate in the metro network and further design of algorithm is still necessary. In the future, we will focus on the reseach of parallel algorithms, in order to redouble reduce the computing time of optimization.



(a) Space-Time diagram

(b) Speed-Time diagram

Figure 12: Space-Time and Speed-Time diagram of 3 trains by using DDDP method

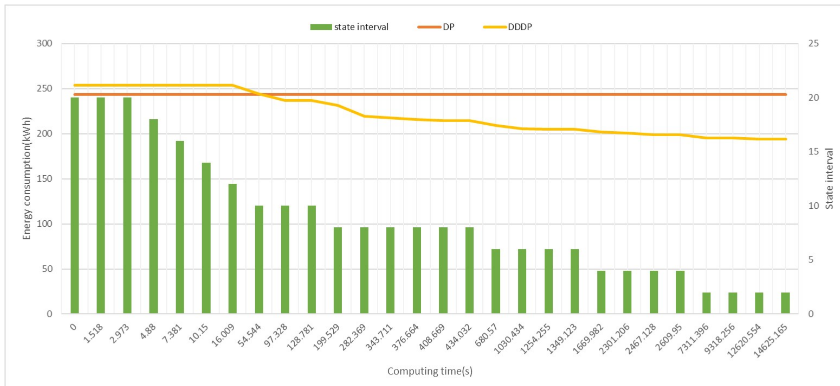


Figure 13: Result contrast picture of DP and DDDP method with considering 3 trains

6 Acknowledgements

This work was supported by Beijing Laboratory of Urban Rail Transit, Beijing Key Laboratory of Urban Rail Transit Automation and Control, by the National Natural Science Foundation of China (No. 61803021, U1734210) and the Beijing Natural Science Foundation "The Joint Rail Transit" (No. L171007).

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