Connectivity Reliability on an Urban Rail Transit Network from the Perspective of Passengers' Travel

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Abstract

In the context of the urbanization and the rapid development of Urban Rail Transit (URT). The reliability of URT network is getting attention. To measure it, three indicators are constructed from passengers' tolerable travel paths, passengers' travel efficiency and passengers' travel realization on URT network, respectively. The tolerable coefficient which is the ratio of passengers' tolerable travel time to shortest possible travel time is proposed and added to indicators. It can reflect passengers' travel paths choice behavior. The ratio of affected passenger volume (RPV) is proposed to identify the important stations. The Automatic Fare Collection (AFC) data, train running time data are used to calculate the passenger volume and the number of passengers' tolerable travel paths in Wuhan subway (China). Finally, the connectivity reliability of Wuhan subway network is analysed through simulating attack stations. The result shows that the important stations identification indicators of Degree Centrality (DC), Betweenness Centrality (BC) and ratio of affected passenger volume (RPV) can effectively identify the important stations on connectivity reliability of Wuhan subway. In particular, the important station identification indicator of RPV can identify the stations effectively which have great influence on passengers' travel realization. In addition, attacking stations has greater impact on passengers' tolerable travel paths than passengers' travel efficiency and passengers' travel realization.

Keywords

Urban Rail Transit network, Connectivity reliability, Passengers' travel, Tolerable travel paths, Travel efficiency, ravel realization

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1 Introduction

In recent years, China is experiencing rapid urbanization. Urban Rail Transit (URT) with large capacity, high speed and environmental protection is constructing rapidly. Now, China has longest URT operation lines in the world. However, some emergencies (such as natural disasters and operation accidents) always cause great harm to the operation of URT. Passengers' travel time will increase and passengers' travel paths will be disrupted because of these emergencies. In addition, URT network is a small world network. Most of the nodes are not connected to each other directly in small world network. It means that the connectivity of URT network is easily affected by emergencies. Therefore, how to evaluate and analyze the connectivity reliability of URT network is of great significance to improve the reliability of URT and ensure the normal operation of URT.

Connectivity reliability was proposed by Mine and Kaiwai (1982) at first [8]. In connectivity reliability research, researchers always studied connectivity reliability of transportation network based on graph theory and topology of traffic network. Such as Bell & Iida (1997), and Wakabayashi &Iida (1992) thought that connectivity reliability is the probability that there is still a connection between a pair of nodes when one or more links are removed [1,11]. Zhang et al. (2015) studied the resilience of seventeen principal networks in terms of connectivity. Some researchers researched the connectivity reliability of transportation network considering passengers [13]. Such as Mattsson and Jenelius (2015) summarized and reviewed the existing research on transportation reliability. They pointed out that it is necessary to consider traffic demand and transport supply to study the reliability of transportation network [7]. Zhang et al. (2009) proposed LOS-based connectivity reliability evaluation model to calculate connectivity reliability of a regional transportation network [12]. Liu et al. (2017) analyzed the connectivity reliability of rail transit with Monte Carlo simulation [5]. Guidotti et al. (2017) proposed two types of indicators to measure connectivity reliability of transport network. The indicators that consider weights of nodes and links are compared with indicators that did not consider weights of nodes and links [2]. Li et al. (2014) proposed four connectivity reliability indicators. Then, the indicators were weighted to establish one indicator. Finally, they analyzed the connectivity reliability of Beijing URT [4]. Reggiani et al. (2015) further deepen the analysis of how resilience and vulnerability can be framed, interpreted and measured, and their relationship with connectivity [9].

In these literatures on connectivity reliability of transport networks. Most researchers assumed that as long as there is at least one connected path between the two nodes, the two nodes are in a connected state. However, this assumption does not entirely consistent with passengers' travel path choice behavior. Actually, passengers inclined to choose the travel path whose travel time is shortest. If the shortest travel path (Corresponding to shortest travel time) can not be used, passengers will choose other path whose travel time is less than they could bear (tolerable travel time). If all connected paths' travel time is longer than their tolerable travel time, although the paths are connected, passengers will not use them. Therefore, tolerable coefficient is put forward to confirm whether a travel path is tolerable. In addition, researchers emphasized on studying the topological connectivity reliability of URT network. The passenger volume should be considered when analyzing the connectivity reliability of URT network.

2 Method

To research the connectivity reliability of URT network, the URT network is defined as a directed graph. Three indicators are put forward to measure the connectivity reliability of URT network from passengers' tolerable travel paths, passengers' travel efficiency and passengers' travel realization on URT network, respectively. Based on the maximum impact of passengers, a new indicator to identify the important stations is put forward. The connectivity reliability of URT network is analysed by simulating destroy stations.

2.1 Network Definition

The URT network can be defined by a directed graph $G(N,E)$. N represents the set of stations. $E \subseteq N \times N$, it represents the set of links between stations. There are multiple connected travel paths from one station to another. Assuming that $path_{od}$ is the set of connected path(s) from station o to station d. $path_{od}^{i}$ is the path i from station o to station d, $path_{od}^{i} \in path_{od}$. $path_{od}^{i}$ includes set of stations $path_{od}^{i,N}$ and set of links $path_{od}^{i,E}$. The set of stations $path_{od}^{i,N}$ in $path_{od}^{i}$ includes sets of transfer station(s) $path_{od}^{i,N_1}$ and non-transfer station(s) $path_{od}^{i,N_2}$.

2.2 Passengers' Tolerable Travel Paths

As mentioned above, not all connected paths is meaningful for passengers. Only the connected path is tolerable travel path, passengers use it. To calculate the number of tolerable travel path from one station to another, tolerable coefficient α is put forward. Assuming that $path_{od}^{i}$ is path i from station o to station d, equation (1) is used to confirm whether $path_{od}^{i}$ is a tolerable travel path.

$$n\left(t_{path_{od}^{i}}, \min(t_{path_{od}})\right) = \begin{cases} 1; t_{path_{od}^{i}} / \min(t_{path_{od}}) \le \alpha \\ 0; otherwise \end{cases} . \tag{1}$$

Where $n\left(t_{path_{od}^{i}}, \min(t_{path_{od}})\right)$ is a 0-1constant. If the path i is a tolerable travel path, then $n\left(t_{path_{od}^{i}}, \min(t_{path_{od}})\right) = 1$, otherwise $n\left(t_{path_{od}^{i}}, \min(t_{path_{od}})\right) = 0$. $path_{od}$ is the set of connected path(s) from station o to station d. $t_{path_{od}^{i}}$ and $t_{path_{od}}$ are the travel time of path i and the travel time set of connected path(s) from station o to station d, respectively. α is the tolerable coefficient. It reflects the relation between tolerable travel time and shortest possible travel time.

$t_{path_{od}^{i}}$ can be calculated with equation (2). It includes the train running time in links (including the train start and stop time), train dwell time, passengers' transfer time and passengers' waiting time.

$$t_{path_{od}^{i}} = \sum\nolimits_{e \in path_{od}^{i,E}} t_e + \sum\nolimits_{m \in path_{od}^{i,N_1}} t_m^{dwell} + \sum\nolimits_{n \in path_{od}^{i,N_2}} t_n + t_{wait}^{o} . \tag{2}$$

Where e and $path_{od}^{i,E}$ are link e and set of link(s) in path i from station o to station d, respectively. t_e is the train running time in link e. m and $path_{od}^{i,N_1}$ are non-transfer station m and set of non-transfer station(s) in path, respectively. t_m^{dwell} is the train dwell time at non-transfer station m. n and $path_{od}^{i,N_2}$ are transfer station n and set of transfer station(s) in path i, respectively. t_n is the transfer time at transfer station n. t_{wait}^{o} is passengers' waiting time at origin station o which can be estimated as half of the headway.

2.3 Evaluating Connectivity Reliability of URT Network

Most of researchers usually adopted the network efficiency (the mean of the reciprocal of shortest distance between all nodes), maximal connected subgraph and other similar indicators to measure the connectivity of URT network. These indicators lack of information on passenger volume and the number of tolerable travel paths. Therefore, three indicators which consider the passenger volume and tolerable travel paths are proposed to evaluate the connectivity reliability of URT network.

2.3.1 The Number of Tolerable Travel Paths in URT Network

The number of tolerable travel paths from one station to another can reflect the connectivity reliability from one station to another. Sometimes, operational accidents, terrorist attacks and natural disasters will destroy the URT network. They cause some tolerable paths become unavailable. Therefore, the higher the number of tolerable travel paths between station pairs, the higher the probability that passengers can travel between them. Before the URT network is destroyed, the average number of tolerable travel paths for every passenger is used to evaluate the connectivity reliability of URT network. It is represented as equation (3):

$$R_{path}^{0}=\frac{\sum_{o\in N}\sum_{d\in N,o\neq d}\sum_{path_{od}^{i}\in path_{od}} v_{od}\cdot n\left(t_{path_{od}^{i}},\min(t_{path_{od}})\right)}{V}. \qquad (3)$$

Where R_{path}^{0} is average number of tolerable travel paths for every passenger before the network is destroyed. N is the set of stations in URT network. v_{od} is the passenger volume from station o to station d . V is passenger volume in URT network.

After the URT network is damaged, the average number of tolerable travel paths for every passenger will decrease. Assuming that the network is suffered from damage event δ . It causes x number of stations lost their functions. In this situation, the average number of tolerable travel paths for every passenger is $R_{path}^{(\delta,x)}$. The relative number of tolerable travel paths is used to measure the connectivity reliability of URT network after the network is destroyed. It can be calculated with equation (4):

$$R_{path}=\frac{R_{path}^{(\delta,x)}}{R_{path}^{0}}. \qquad (4)$$

Where R_{path} is relative number of tolerable travel paths in URT network. It can reflect the connectivity reliability of URT network from passengers' tolerable travel paths when the network is destroyed.

2.3.2 Travel Efficiency of URT Network

The connectivity reliability of URT network is always measured from passengers' shortest travel time (network efficiency). The implicit assumption of network efficiency is that every station plays the same role on the network (the weight of stations is same). However, the functions of stations in URT network are different. The passenger volume between stations varies considerably. Therefore, in order to measure the travel efficiency of URT, the passenger volume is considered. Before the URT network is destroyed, the travel efficiency of URT network can be presented by equation (5):

$$E_{eff}^{0}=\frac{1}{V}\sum_{o\in N}\sum_{d\in N,o\neq d} v_{od}\cdot\frac{1}{\min(t_{path_{od}})}. \qquad (5)$$

Where E_{eff}^{0} is travel efficiency of URT network when URT network is not damaged.

The passenger volume is used to calculate the stations' weight. $\left(\sum_{d\in N,o\neq d} v_{od}\right)/V$ is the weight of station o . Assuming that the network is suffered from damage event δ . It causes x

number of stations lost their functions. In this situation, the travel efficiency will decrease to $E_{eff}^{(\delta,x)}$. The relative travel efficiency of URT network can be calculated with equation (6):

$$R_{eff} = \frac{E_{eff}^{(\delta,x)}}{E_{eff}^{0}}. \tag{6}$$

Where R_{eff} is relative travel efficiency of URT network, which can reflect the connectivity reliability of URT network when the network is destroyed.

2.3.3 The Rate of Passengers' Travel Realization on URT Network

In normal operations, most of passengers can travel on URT network successfully. Therefore, Passengers' travel realization rate in URT network is near to 100%. However, if the URT network is destroyed, then some passengers' tolerable travel paths are interrupted. passengers will give up travel on URT network because their tolerable travel paths are interrupted. The Passengers' travel realization rate will decrease. Therefore, the rate of passengers' travel realization on URT network is put forward to measure the connectivity reliability of URT network. Assuming that the network is suffered from damage event δ. It causes x number of stations lost their function. In this situation, the rate of passengers' travel realization on URT network is represented by equation (7):

$$R_{rate} = \frac{V^{(\delta,x)}}{V} = \frac{\sum_{o\in N}\sum_{d\in N, o\neq d} v_{od}\cdot n_{od}^{(\delta,x)}}{V}. \tag{7}$$

Where R_{rate} is the rate of passengers' travel realization on URT network when the network is destroyed. $V^{(\delta,x)}$ is passenger volume that can travel on URT network when URT network is damaged. $n_{od}^{(\delta,x)}$ is a 0-1 constant, if there is at least one tolerable travel path from station o to d ,then $n_{od}^{(\delta,x)}$ is 1, otherwise, $n_{od}^{(\delta,x)}$ is 0.

2.3.4 Identifying Important Stations

Many researchers have done some work on identifying important nodes (Liu et al., 2016; El-Rashidy and Grant-Muller, 2014; Hu et al., 2015) in complex network [6,10,3]. Some indicators had been used to evaluate the importance of nodes, such as Degree Centrality (DC), Betweenness Centrality (BC) and Closeness Centrality (CC). DC emphasizes the number of links linked to the node directly. BC describes the ratio of all shortest paths that passing through the node in the network. CC reflects distances between the node and other nodes. However, these indicators focus on identifying the important nodes from the topology of the network. The passenger volume has not been considered. Therefore, to reflect the importance of nodes to passengers' travel, the ratio of passenger volume (RPV) affected by the station to the total passenger volume is used to measure the importance of the station. Supposing that some passengers' travel is affected by station j. Then, station j can affect passengers whose origin station is j, whose destination station is j and whose travel path includes station j. The importance indicator of station j is represented by equation (8):

$$I_j = \frac{\sum_{o\in N, o\neq j}\sum_{d\in N, o\neq d} v_{od}^{j} + \sum_{d\in N, d\neq j}(v_{jd}+v_{dj})}{V}. \tag{8}$$

Where I_j is the importance indicator of station j. v_{od}^{j} is passenger flow travel from station o to station d via station j. v_{jd} and v_{dj} are passenger volume who travel from station j to station d and travel from station d to station j, respectively. V is passenger volume in URT network.

In equation (8), to calculate the importance indicator of station j, the passenger flow via station j need to be confirmed. A user stochastic equilibrium model is used to calculate the passenger flow in URT network. Then the importance indicators of all stations can be calculated.

3 Implementation

3.1 Data Preparation

Wuhan subway system in China is used to validate effectiveness of the method and indicators used. Figure 1 shows the operation lines, stations' name and numbers for Wuhan subway in September 2018.

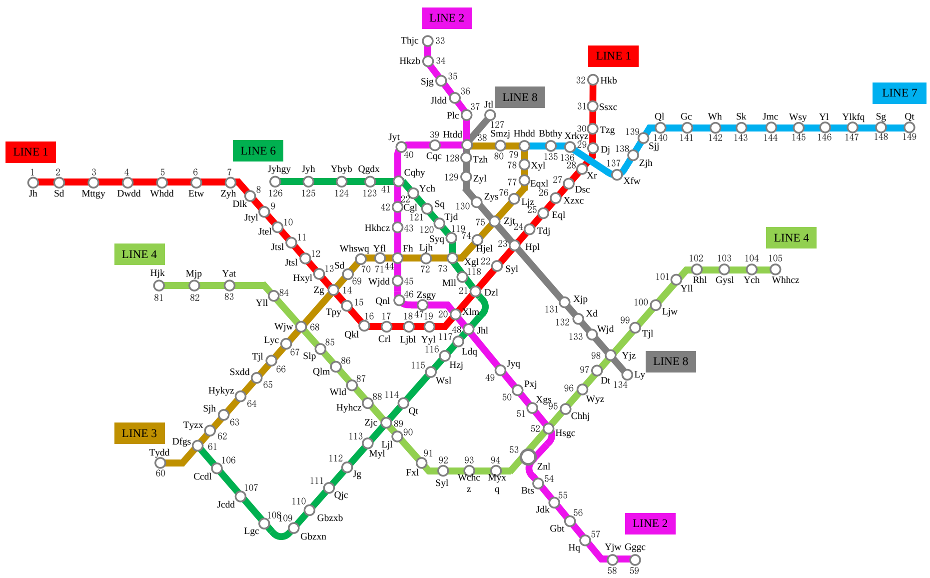

Figure 1: Network of Wuhan subway in September 2018

The Automatic Fare Collection (AFC) data record for 5 working days in September 2018 were obtained from Wuhan subway company. The data recorded the types of the tickets, tickets' number, entry and exit time at stations, stations' names and numbers. the tickets' number is matched to obtain the stations' entry and exit time for passengers. Passenger volume between stations is counted to construct Origin-Destination (OD) matrices during morning peak hours.

To calculate the travel time between stations, the travel time (including train dwell time and train running time) of links, headway of different lines and the transfer time at transfer stations are obtained from Wuhan subway company.

3.2 Important Stations

Degree Centrality (DC) of stations, Betweenness Centrality (BC) of stations, Closeness Centrality (CC) of stations and the ratio of passenger volume affected by the stations (RPV) during morning peak hours are calculated, respectively. The most important ten stations in Wuhan subway network are listed in Table 1.

Table 1: Ten important stations identified by four identification indicators

Stations' number	DC	Stations' number	BC	Stations' number	CC	Stations' number	RPV
38	5	75	0.026	21	11.05	21	0.030
14	4	73	0.025	73	11.12	48	0.026
20	4	21	0.023	20	11.16	20	0.024
21	4	14	0.021	118	11.29	23	0.022
23	4	48	0.020	22	11.32	73	0.022
41	4	74	0.019	74	11.35	75	0.022
44	4	68	0.019	48	11.43	14	0.020
48	4	23	0.017	72	11.50	44	0.016
68	4	89	0.016	23	11.51	118	0.016
73	4	76	0.015	75	11.54	89	0.016

Table 1 shows that four identification indicators have identified some same important stations. Such as, station 21, station 20, station 23 and station 48. These four stations are transfer stations in Wuhan subway network. To analyse the connectivity reliability of Wuhan subway network and demonstrate the effectiveness of different identification indicators, the influence of important stations' failure on connectivity reliability of Wuhan subway network is analysed.

3.3 Connectivity reliability of Wuhan Subway Network

To analyse the connectivity reliability of Wuhan subway network, MATLAB is used to simulate destroying stations. If the station is destroyed, then the station and the links which are connected to the station directly are removed from the network. The connectivity reliability of Wuhan subway network is reflected by calculating the three indicators (from equation (3) to (7)). Supported by National Key R & D Program of China (2017YFB1200700) and The National Natural Science Foundation of China (No. U1834209), we conducted an in-depth questionnaire survey on characteristics of passengers' travel path choices in Wuhan subway. It is found that passengers' tolerable coefficient in Wuhan subway is 1.38.

3.3.1 The Relative Number of Tolerable Travel Paths in Wuhan Subway Network

The connectivity reliability of Wuhan subway network can be reflected by relative number of tolerable travel paths R_{path} . Attacking stations randomly and attacking important stations deliberately are used to simulate destroying stations. The Figure 2 shows relative number of tolerable travel paths R_{path} simulation outputs under five attack modes. Before the stations are destroyed, the number of tolerable travel paths in Wuhan subway network is 3.49. It means that every passenger in Wuhan subway network has 3.49 tolerable travel paths on average. After the stations are destroyed, it is found that attacking important stations deliberately can make R_{path} decrease quickly. Attacking stations randomly leads to the decease of R_{path} moderately. In addition,

R_{path} is more sensitive to attacking important stations that are identified by identification indicators of CC, BC and RPV. R_{path} decease to 0.37 (average passenger has 1.29 tolerable travel paths) when one station (station 21) identified by indicators of CC and RPV is destroyed. Therefore, the station 21 plays an important role in connecting tolerable travel paths. It also shows that important stations have huge impact on the diversity of passengers' travel paths choices. Attacking one to three important stations that are identified by identification indicators of BC, CC and RPV make R_{path} decease much quickly. When over three important stations are destroyed, the identification indicators of DC, BC and RPV can identify the important stations effectively which can influence R_{path} heavily.

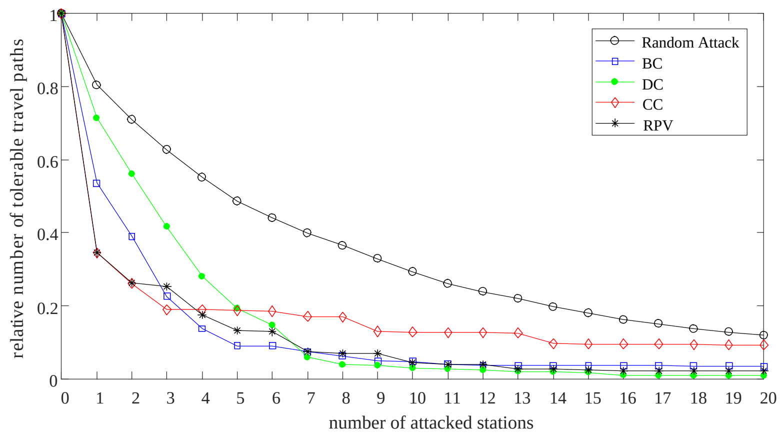

Figure 2: Relative number of tolerable travel paths R_{path} simulation outputs under five attack modes

3.3.2 Relative Travel Efficiency of Wuhan Subway

The shortest travel time between stations will increase when stations are destroyed. Therefore, the connectivity reliability of Wuhan subway network can be reflected by relative travel efficiency R_{eff} of Wuhan subway. The Figure 3 shows relative travel efficiency R_{eff} simulation outputs under five attack modes. Before the stations are destroyed, R_{eff} of Wuhan subway network is 0.037. When the stations are destroyed, attacking important stations deliberately can make R_{eff} decrease quickly. However, compared with relative number of tolerable travel paths (Figure 2), R_{eff} decreases slowly when stations are destroyed. It can bear attacking two stations before R_{eff} drops to 0.8. Therefore, attacking stations has greater impact on passengers' tolerable travel paths than passengers' travel efficiency. In addition, R_{eff} is more sensitive to attacking important stations which are identified by identification indicators of BC, DC and RPV. When less than four stations are destroyed, then the identification indicator of BC can identify the important stations which can influence R_{eff} heavily. When over five important stations are destroyed, identification indicator of DC can identify the important stations which can influence R_{eff} heavily.

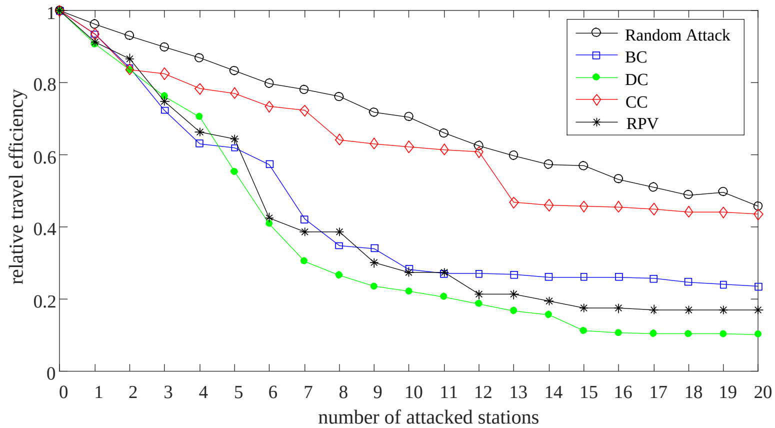

Figure 3: Relative travel efficiency of Wuhan subway simulation outputs under five attack modes

3.3.3 The Rate of Passengers' Travel Realization in Wuhan Subway Network

Some passengers' tolerable travel paths will be unconnected when stations are destroyed in URT network. It causes some passengers' travel can not be realized on URT network. Therefore, the connectivity reliability of Wuhan subway network can be reflected by the rate of passengers' travel realization R_{rate} . The Figure 4 shows the rate of passengers' travel realization R_{rate} simulation outputs under five attack modes. Attacking three important stations which are identified by indicators of BC, DC and RPV causes R_{rate} decrease to less than 0.8. It means that more than 20% of passengers will not travel on Wuhan subway network. Attacking six important stations which are identified by indicators of DC and RPV are destroyed causes R_{rate} decrease to nearly 0.4. Therefore, only 40% of passengers can get tolerable travel paths to travel on Wuhan subway network. The rate of passengers' travel realization decreases more slowly than relative number of tolerable travel paths when stations are destroyed (Figure 2). It also proved that the impact of attacking stations on passengers' tolerable travel paths is greater than passengers' travel realization. In addition, when attacked stations are over three, identification indicator of RPV can identify the important stations which have most influence on R_{rate} . It demonstrates that identification indicator of RPV is effective to identify the important stations that can influence R_{rate} heavily.

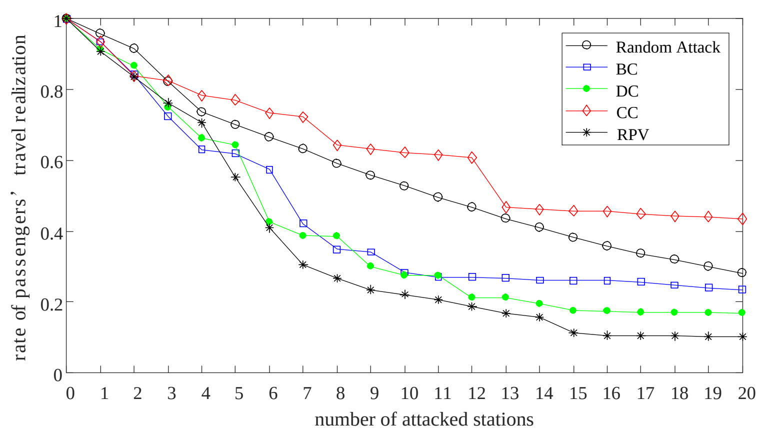

Figure 4: Passengers' travel realization simulation outputs under five attack modes

4 Conclusions

The connectivity reliability of URT network are measured from passengers' tolerable travel paths, passengers' travel efficiency and passengers' travel realization on URT network, respectively. Three indicators which considering passenger volume and passengers' tolerable coefficient are used to analyze the connectivity reliability of Wuhan subway network. A new indicator RPV which can maximize the number of affected passengers is proposed to identify the important stations. The important stations identification indicators of DC, BC, CC and RPV are used to identify the important stations in Wuhan subway network. Combining the above measures together, the connectivity reliability of Wuhan subway network is analyzed through attack stations simulation. Some findings and conclusions are summarized as below:

- The connectivity reliability of Wuhan subway is more sensitive to attacking important stations deliberately than attacking stations randomly. The indicators of DC, BC and RPV can effectively identify the important stations on connectivity reliability of Wuhan subway. The simulation result shows that the different important stations identification indicator can identify the important stations effectively when the connectivity indicators and the number of destroyed stations are different.
- Compared with relative number of tolerable travel paths, the relative travel efficiency and the rate of passengers' travel realization decrease slowly when the stations are destroyed. Before the relative travel efficiency and the rate of passengers' travel realization drop to 0.8, they can bear attacking two stations. Therefore, attacking stations has greater impact on the passengers' tolerable travel paths than passengers' travel efficiency and travel realization.
- The new indicator RPV can identify the important stations on connectivity reliability of Wuhan subway effectively. In particular, it can identify the important stations that can influence the passengers' travel realization on URT network most.

In URT network, identifying important stations effectively is of great importance on the operation of URT network. Since, the connectivity reliability of URT network can be improved by protecting important stations. The three indicators are used to measure the connectivity reliability of URT network comprehensively. Although one URT network is analyzed here, the same indicators and method can be used to other URT networks. Further studies will consider more factors, such as using historical data to confirm the probability of stations failure and considering the passengers' travel quality.

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