Computing Base Train Equivalents for Delay-Based Capacity Analysis with Multiple Types of Trains

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Abstract

Different types of trains may have substantially dissimilar characteristics, resulting in various capacity impacts. The concept of base train equivalent (BTE) was proposed to standardize different train types into a universal unit, namely, base train unit (BTU). However, the previously developed delay-based model suffers from consistency issue, and its application is limited to only two train types. Thus, this study proposes a new concept of delay-based BTE computation and corresponding BTE models. The dynamic BTE model considers volume and heterogeneity and aims to reflect fully the actual capacity impact of non-base trains. The fixed BTE model identifies the most appropriate BTE value at a particular traffic heterogeneity. Results from the case studies demonstrate that the proposed method can address scenarios with all types of traffic mixes and multiple train types. The unit of delay-based rail capacity can be converted into a standard unit using the proposed models. The effect of an additional train can be easily assessed, and the capacity measurements from different lines or systems can be compared and evaluated.

Keywords

Rail Transport, Capacity Analysis, Base Train Equivalent

1 Introduction

Multiple types of trains usually operate on a railroad line to accommodate different demands. Different train types may have substantially dissimilar characteristics, resulting in various capacity impacts. Lai et al. (2012) proposed the use of base train equivalent (BTE) to convert different train types into a universal unit, namely, base train unit (BTU) Delay-(Lai et al. (2012)) and headway-based approaches (Lai et al. (2015)) were developed to compute BTE depending on the types of capacity model.

Delay, which uses parametric and simulation models, is a common output of capacity analysis in North America (Confessore et al. (2009); Dingler et al. (2014); Krueger (1999); Lai et al. (2012); Lai and Barkan (2009); Pouryousef and Lautala. (2013); Prokopy and Rubin. (1975); Sogin et al. (2013) and Shih et al. (2015)). Although the delay-based BTE model was established by Lai et al. (2012), their model adopted the delay-based approach from highway research and defined BTE as the delay ratio of a marginal non-base train over a base train. A deficiency of this method is that the BTU converted from a mixed traffic through the BTE may be different from the number of base trains at the same delay level. In addition, the delay-based BTE model cannot handle scenarios with more than two train types (Lai et al. (2012)).

In the present study, we proposed a new concept and developed a set of corresponding delay-based BTE models. Furthermore, we extended the model framework to accommodate multiple types of trains. The unit of delay-based rail capacity could be converted into a standard unit through the proposed models. The capacity measurements from different lines or systems could be compared and evaluated.

2 Methodology

Figure 1 demonstrates the new concept proposed in this study for determining BTE. The two points from the mixed and base flows at the same delay level are used to compute the BTE for non-base trains. For example, the delay level of mixed traffic for 18 days in the mixed flow is equivalent to that of the homogeneous traffic with 52 base trains in the base blow. Therefore, if the non-base trains in the mixed flow are converted into base trains through BTE, then the total number of base trains after the conversion (30×1+10×BTE) should be 52, thereby resulting in a BTE value equal to 2.2. In this way, we can easily compare different traffic flows in the same standard and convert the mixed flow to the base flow meaningfully and consistently.

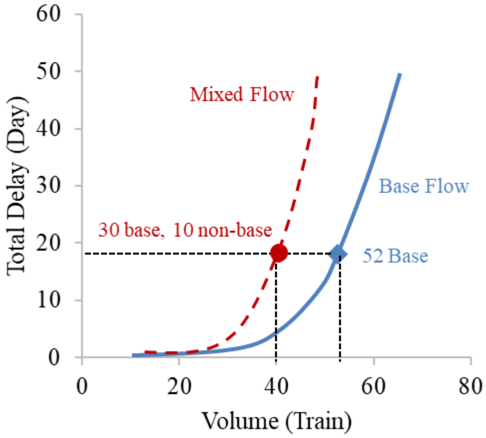

Figure 1: Concept for BTE computation

Several types of BTE model are developed on the basis of the new concept proposed in this study. In terms of a particular route, BTE will only vary with traffic volume and heterogeneity because most of the other factors are fixed. Therefore, this study initially develops dynamic BTE models with consideration of volume and heterogeneity. Furthermore, we develop a fixed BTE model with consideration of only heterogeneity because its influence to BTE value is considerably higher than that of traffic volume.

Another breakthrough of this study is enabling the possibility to compute BTEs for multiple types of train. If only two types of trains exist, BTE can be directly computed. However, the same model cannot be applied directly to scenarios with multiple types of train due to additional unknown BTEs. Therefore, this study also adopts the concept of projecting vector to identify a suitable BTE for each type of non-base trains.

2.1 Dynamic BTE Model

Given the new concept of the BTE computation, the computational process should first determine the number of base trains in the homogeneous flow that corresponds to the delay level of the mixed traffic in the heterogeneous flow. Equation (1) can be used to determine BTE to non-base trains by allocating impacts to non-base trains, where n_b is number of base type of trains in the mixed flow; n_i is number of *i*th type of non-base train in the mixed flow; n_B is number of base type of trains in the base flow; i is index for train type; I is total number of types of non-base train in the mixed flow; E_i is BTE of the *i*th type of non-base train; E_b is BTE of the base train in the mixed flow (= 1); E_B is BTE of the base train in the base flow (= 1).

$$n_b E_b + \sum_{i}^{I} n_i E_i = n_B E_B. \qquad (1)$$

If only one type of non-base train is found in mixed flow, then the BTE value for non-base trains (E_i) can be easily determined by Equation (1). However, if more than one type of non-base train is observed, then multiple unknown BTEs with only one equation exist. We build the coordinates in three-dimensional space to determine each relative position (Figure 2).

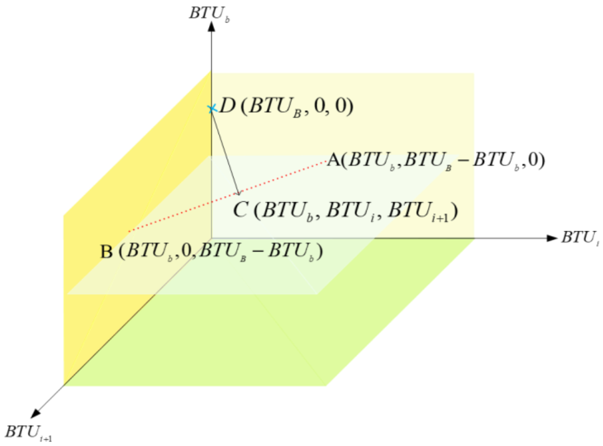

Figure 2: Schematic of BTUs for the three types of train

According to Equation (1), the BTU of each type of train can be summarized as Equation (2). The right-hand side is the BTU of the base flow, and the left-hand side is the BTU of the mixed flow.

$$BTU_b + \sum_{i=1}^{I} BTU_i = BTU_B. \qquad (2)$$

This example can be illustrated as a three-dimensional space in Figure 2. In this figure, point D demonstrates the number of BTUs in the base flow, that is, BTU_B, and the red dashed line ($\overrightarrow{AB}$) represents the feasible region for BTU_i and BTU_{i+1} (and the corresponding E_i and E_{i+1}) in the mixed flow. Each point in the feasible region ($\overrightarrow{AB}$) reflects the same delay and BTU with the base flow. To determine the appropriate values of E_i and E_{i+1}, we project point D to $\overrightarrow{AB}$ by setting the inner product of the direction vector [$\overrightarrow{AB}$ = (0, BTU_B - BTU_b, -(BTU_B - BTU_b))] and normal vector [$\overrightarrow{CD}$ = (BTU_B - BTU_b), -BTU_i, BTU_{i+1}] to zero, as described in Equation (3), where $\vec{u}$ is direction vector; $\vec{v}$ is normal vector. Equations (4) and (5) demonstrate the process of determining the BTEs (i.e., E_i and E_{i+1}) for the two types of non-base train. Equation (4) corresponds to the detailed process of the inner product. Finally, the BTEs (i.e., E_i and E_{i+1}) can be obtained by using Equation (5). Although we take three types of train as examples here, the proposed process can be easily applied to scenarios with four or more types of train.

$$\vec{u}\cdot\vec{v}=0. \tag{3}$$

$$\vec{u}\cdot\vec{v}=0=\overrightarrow{AB}\cdot\overrightarrow{CD}= \left[0, BTU_B-BTU_b, -(BTU_B-BTU_b)\right]\cdot\begin{bmatrix}BTU_B-BTU_b\\ -BTU_i\\ -BTU_{i+1}\end{bmatrix}. \tag{4}$$

$$E_i=\frac{\frac{(BTU_B-BTU_b)}{2}}{n_i},\quad E_{i+1}=\frac{\frac{(BTU_B-BTU_b)}{2}}{n_{i+1}}. \tag{5}$$

2.2 Fixed BTE Model

The fixed BTE model adopts the same concept used in the proposed delay-based BTE computational process in this study. However, the fixed BTE model aims to identify the most appropriate BTE value to represent a specific heterogeneity regardless of the traffic volume. In the fixed BTE model, the mixed flow is no longer only a point but a line with the same heterogeneity (red line in Figure 3).

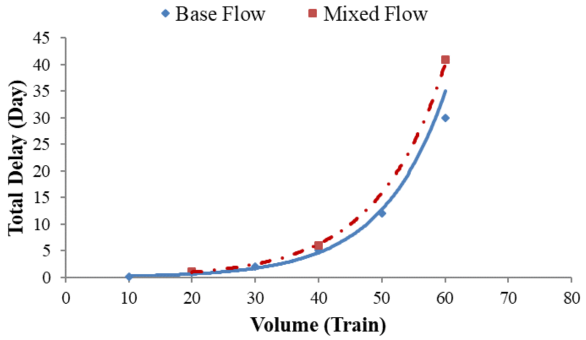

Figure 3: Delay–volume curve of the fixed model

As shown in Figure 3, the most appropriate BTE value should convert the mixed flow (red dashed line) to the base flow (blue solid line) at any delay level. Thus, the proposed process applies an iterative process to determine the most appropriate BTE value by minimizing the difference between the BTU in the base flow and that in the mixed flow (after conversion) with the given delay levels (*K*) (Equation (6), where *K* is number of selected delay levels; BTU_{mk} is BTU in the mixed flow at the *K*th delay level; BTU_{bk} is BTU in the base flow at the *K*th delay level; n_{ik} is number of type *i* non-base trains in the mixed flow of *K*th delay level; n_{bk} is number of base trains in the mixed flow of *K*th delay level.).

If three delay levels are selected in the model and we expanded Equation (6), and remove the squared and root denoting the difference between the base and mixed BTUs of each selected delay base as d_k, then we can move the number of base trains in the mixed flow (n_b) and the base flow (n_B)and dk can be moved to the right-hand side because they are all known values (Equation (7)). The fixed BTE model aims to identify only one BTE for a particular heterogeneity. E_i in each of the three equations in Equation (7) is the same. Therefore, it can be regarded as one equation. The right-hand side denotes constant *f*, and the general equation is Equation (8). If only one type of non-base train is found in the mixed flow, then the most appropriate BTE value (E_i) can be determined by Equation (8) with a given set of delay levels. However, if more than one type of non-base train is found, then more than one possible solution exists.

$$\begin{aligned} &\min \sum_{k=1}^{K} \sqrt{(BTU_{mk} - BTU_{Bk})^2} \\ &= \min \sum_{k=1}^{K} \sqrt{(\sum_{i=1}^{I} n_{ik} \times E_i + n_{bk} \times E_b - BTU_{Bk})^2}. \end{aligned} \tag{6}$$

$$\begin{aligned} \sum_{i=1}^{I} n_{i,1} \times E_i &= d_1 + n_{B,1} - n_{b,1}, \\ \sum_{i=1}^{I} n_{i,2} \times E_i &= d_2 + n_{B,2} - n_{b,2}, \\ \sum_{i=1}^{I} n_{i,3} \times E_i &= d_3 + n_{B,3} - n_{b,3}. \end{aligned} \tag{7}$$

$$\sum_{i}^{I} n_i \times E_i = f. \tag{8}$$

In Figure 4, we also take three types of train as example, which are easily presented in the three-dimensional space. From the figure, points D_1–D_3 demonstrate the number of BTUs in the base flow with different volumes but same heterogeneity, and the red dashed line ($\overrightarrow{A_1B_1} \sim \overrightarrow{A_3B_3}$) represents the feasible region for BTU_i and BTU_{i+1} in the mixed flow. Figure 4 shows three sub-spaces based on the three selected delay levels (*K*). The mixed flow distribution is proportional, and the base line is coordinated with the BTU_b axis. The aforementioned concept shows that only one delay base conduct projection can be selected.

The solution process for multiple types of train is almost the same as that in the dynamic BTE model for a similar case. Each point in the feasible region ($\overrightarrow{AB}$) reflects the same delay and BTU with base flow. To determine appropriate values of E_i and E_{i+1}, we project point *D* to $\overrightarrow{AB}$ by setting the inner product of the direction vector [$\overrightarrow{AB}$= (0, BTU_B - BTU_b, -(BTU_B- BTU_b))] and normal vector [$\overrightarrow{CD}$= (BTU_B - BTU_b), -BTU_i, BTU_{i+1}] to zero. Equations (4), (9), and (10) demonstrate the process to determine the BTEs (i.e., E_i and E_{i+1}) for the two types of non-base train. Equation (9) can be derived from Equations (4) and (8). Finally, the BTEs (i.e., E_i and E_{i+1}) can be obtained using Equation (10). Similarly, although we take three types of train as example here, the proposed process can be applied to scenarios with four or more types of train.

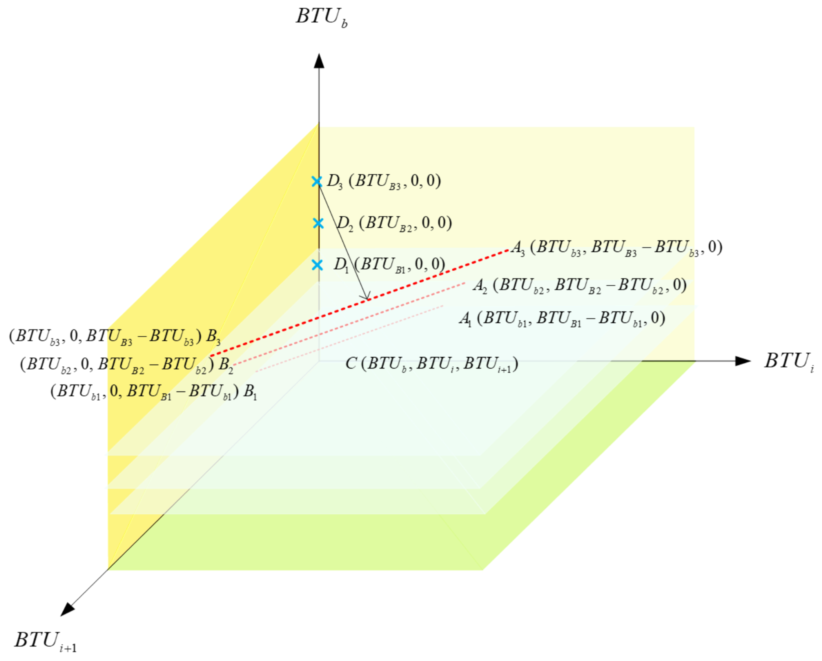

Figure 4: Schematic of BTUs of the three types of trains for the fixed BTU

$$BTU_i = BTU_{i+1} = \frac{f}{2}. \tag{9}$$

$$E_i = \frac{\frac{f}{2}}{n_i}, \quad E_{i+1} = \frac{\frac{f}{2}}{n_{i+1}}. \tag{10}$$

3 Case study

To demonstrate the use of the proposed model, dynamic and fixed BTE models are applied to scenarios with three train types. For the three train types, we add intermodal trains and regard it as a base train to understand the changes in the BTE values for coal and passenger trains. Table 1 shows the characteristics of all train types.

Table 1: Train Characteristics

Train	Passenger train	Intermodal	Coal train
Locomotive	P42-DC locomotives	5 SD70 locomotives	3 SD70 locomotives
No. of Cars	13 cars	93 cars	115 cars
Weight (tons)	500 tons	5,900 tons	16,445 tons
Train Length	500 feet	5,649 feet	6,325 feet
HP/TT	15.4	3.64	0.78
Max Speed	79 mph	70 mph	50 mph

RTC simulation software is used to obtain the delay data. This case study is based on a set of inputs that represent the typical characteristics of a Midwestern North American single-track main line. The route characteristics are as follows: (1) section length: 262.25 miles; (2) siding spacing: 2.75 miles; (3) signal spacing: 2.75 miles; (4) three-aspect signaling system; (5) sidings are evenly distributed in the section; (6) the number of bidirectional train departures is consistent; and (7) passenger train stops at three stations on the section are evenly distributed, and dwell time is 2 minutes (Dingler et al. (2013)). According to each different combinations of train type, we perform 30 different random seeds in RTC to acquire average delay. An alternative method is the use of other types of delay-based capacity model, such as the parametric capacity model. These delay data can then be used to compute BTE values by using the proposed computational process.

3.1 Analytical Results for Multiple Train Types

Dynamic BTE of Three Train Types

In the three train types, intermodal is added as a base train. The non-base trains are coal and passenger trains. We use 10% of train heterogeneity for the interval unit. A total of 36 heterogeneous groups are found in the three train types, and each heterogeneous group has three volumes, that is, 20, 40, and 60 trains. Therefore, 108 types of train combinations are found.

Figure 5 shows the BTEs of the three train types. For the case of 20 mixed trains (10% intermodal, 10% passenger, and 80% coal trains), the BTEs of these three train types are 1, 7.73, and 0.8. However, the BTEs of 20 mixed trains with 10% intermodal, 80% passenger train, and 10% coal train can also be considered 1, 0.5, and 4.02. In other words, when the percentage of a train type in the traffic mix is lower, its BTE is usually higher because these trains are more special than other trains that have a higher tendency to disturb the traffic flow and incur higher delay.

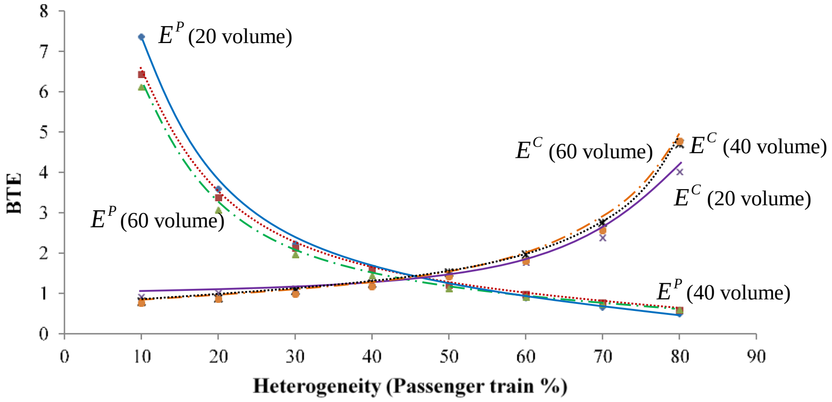

Figure 5: BTE of three train types for the dynamic model
(given the proportion of intermodal trains is 10%)

Fixed BTE of Three Train Types

In this case, the proportion of every type of train ranges from 10% to 80%, thereby resulting in 36 combinations. A fixed BTE should be the most appropriate one among the 36 combinations evaluated in the process.

Figure 6 shows the fixed BTE value of two non-base trains assuming that the proportion of intermodal trains is 10%. If the proportion of one type of train is lower, then its BTE is higher, and vice versa. This trend is the same as the previous case, in which a train type with lower percentage affects the flow more considerably.

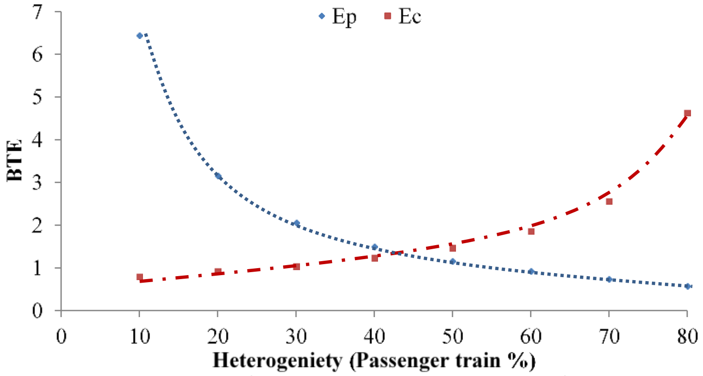

Figure 6: BTE of three train types based on the fixed BTE model
(given the proportion of intermodal trains is 10%)

4 Discussion: BTE Application

Capacity is usually defined as the maximum system throughput. We can further define the maximum throughput to the maximum base trains using BTE. Table 2a presents a capacity evaluation by using capacity and BTU for different traffic compositions among various dates. In terms of traffic volume, 40 trains exist for each day of the periods. However, if the traffic volume is converted into BTU, then they are all relatively different. Similarly, Table 2b shows three different sections. Capacity and BTE in different sections are dissimilar due to the difference in route characteristic. The comparison in BTU is considerably more meaningful than that in the number of trains.

Table 2: Capacity Evaluation Based on BTU

(a) Same Section

Date	n_P	n_I	n_C	E_P	E_I	E_C	N	BTU	C	V/C
3/1	4	32	4	1.72	1	1.72	40	45.76	55	0.832
3/2	20	8	12	1.04	1	1.72	40	49.44	55	0.899
3/3	20	16	4	0.7	1	3.49	40	43.96	55	0.799
3/4	12	12	16	1.67	1	1.24	40	51.88	55	0.943
3/5	8	8	24	2.92	1	0.97	40	54.64	55	0.993

(b) Different Sections

Section	n_P	n_I	n_C	E_P	E_I	E_C	N	BTU	C	V/C
1	12	12	16	1.67	1	1.24	40	51.88	55	0.943
2	32	4	4	0.48	1	3.87	40	34.84	66	0.528
3	4	16	20	4.41	1	0.88	40	51.24	51	1.005

Section 2：length = 161.75 miles, siding = 5.5 miles, signal = 2.75 miles

Section 3：length = 109.75 miles, siding = 16.5 miles, signal = 1.375 miles

5 Conclusions

This study proposes a new concept of delay-based BTE computation and the corresponding BTE models. The dynamic BTE model considers volume and heterogeneity and aims to reflect fully the actual capacity impact of non-base trains. The fixed BTE model identifies the most appropriate BTE value at a particular traffic heterogeneity. The results from the case studies demonstrate that the proposed method can address scenarios with all types of traffic mixes and multiple types of trains. The unit of delay-based rail capacity can be converted into a standard unit using the proposed models. The capacity measurements from different lines or systems can be compared and evaluated

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