Train Unit Shunting : Integrating rolling stock maintenance and capacity management in passenger railway stations

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Abstract
In passenger railway stations, train units preparation is crucial for service quality. This preparation includes maintenance check, cleaning, coupling and uncoupling. Such operations require parking train units on shunting yards located close to platforms. Therefore trains have to be moved between platform and shunting tracks. Taking over train units between their arrival and their departure in a station constitutes shunting. The Generalized Train Unit Shunting problem (G-TUSP) is the problem of shunting operations planning. The problem is to assign arriving train units to departing train units, shunting tracks and paths, to schedule shunting movements and to assign crews to maintenance operations. The aim of the paper is to provide an optimization approach for the G-TUSP. The contribution presents an integrated problem with a mixed-integer linear programming (MILP) formulation. The formulation is based on a microscopic model of the infrastructure and formal train units in order to consider coupling and uncoupling. The model is solved exactly using the commercial solver CPLEX. It is tested on instances based on Metz-Ville station in France. The results are promising and show the suitability of the model.

Keywords
Train Unit Shunting, Train Maintenance Scheduling, Track Allocation, Routing, Railway Station Capacity

1 Introduction

Rolling stock planning must manage *train units* between an arriving trip and a departure trip in a station. This specific part of rolling-stock management is called *shunting* Inside stations, train units are prepared for departure and possibly stored for several hours if they are not needed immediately. More precisely, they are cleaned and have maintenance checks. Moreover, train units can be coupled or uncoupled to match train configuration required for departure. This is done on siding tracks located around platform tracks. Parallel siding tracks form shunting *yards*. Some of these tracks have specific amenities such as train-wash for external cleaning or pits for maintenance checks. To be stored in yards, train units need first of all to be moved from their arrival platform. Then, they can possibly need to be moved there from one yard to another. Finally they need to be moved to their departure platform. Movements arriving or departing from a yard are called *shunting movements* and must respect traffic safety rules imposed by signalling system and by ground-agents instructions. Indeed, shunting movements must not create conflicts with the rest of train traffic in the station.

Shunting operations planning includes several decisions. First, arriving train units must be assigned to departures, which constitutes a matching decision. This matching must take into account rolling stock features required for departures. Another decision concerns train units location: they must be parked at one or several shunting tracks depending on amenities required by maintenance operations. Similarly, movements are set to achieve the parking locations. For these movements, route planning decisions are to be made, since paths are assigned to train units and movements are scheduled based on running times and potential conflicts. Finally, depending on maintenance crews availability, maintenance operations must be scheduled. Although all these decisions are often taken separately, they are usually strongly interdependent. For instance, some matching plans make train units parking or maintenance scheduling impossible.

The Generalized Train Unit Shunting problem (G-TUSP) is the problem of shunting operations planning. It integrates four sub-problems:

- The Train Matching Problem (TMP), the problem of matching arriving and departing train units.
- The Track Allocation Problem (TAP), the problem of choosing train units location.
- The Shunting Routing Problem (SRP), the problem of determining train units routing during shunting movement.
- The Shunting Maintenance Problem (SMP), the problem of defining train units maintenance scheduling.

The G-TUSP considers a station and a timetable with arriving and departing trains that need to be shunted. It is a pre-operational problem, it is solved from 6 days to 4 hours before operations. The problem aims to minimize departure delays and cancellations if timetable perturbations are expected, as well as maintenance call off. Moreover, the minimization of the number of coupling and uncoupling operations is also sought.

The aim of the paper is to provide a formal model of the G-TUSP. Specifically, the contribution consists in formulating an integrated problem as a mixed-integer linear program (MILP) formulation. The formulation is based on a microscopic representation of the infrastructure and on consideration of dummy train units in order to manage coupling and uncoupling. The rest of the paper is organized as follows. Section 2 reports a summary of the literature on shunting operations planning problems. Section 3 proposes the MILP formulation of the G-TUSP. Section 4 describes the experiments carried out as proof of concept of the applicability of the formulation. Section 5 concludes the paper.

2 Related works

Several contributions introduce problems dealing with various aspects of shunting for passenger transportation.

A part of the literature focuses on the TAP without train matching. A first variant tackled concerns TAP for maintenance. In this problem, it is considered that a train unit may be parked successively on different tracks to use various equipments necessary for its maintenance. The objective is to do so as efficiently as possible. Tomii and Zhou (2000) tackle the SMP and the TAP. Here, the operations scheduling is performed through a PERT network and resource assignments are chosen thanks to a genetic algorithm. Other papers consider TAP for maintenance with a fixed maintenance schedule. Arrival and departure time on shunting tracks can be data of the problem (Li et al. (2017)) or decision variables thanks to a discrete time model (Jacobsen and Pisinger (2011)). A second variant is based on pure TAP. The combinatorial difficulty comes from the fact that several trains can be parked on the same track. When a train leaves a shunting track, it must not be blocked by another train parked in front of it. A constraint based on this requirement is called a *crossing constraint*. Also, the length of trains parked on a shunting track does not exceed the track length. A constraint based on this requirement is called a *length constraint*. Di Stefano and Koči (2004) provide significant theoretical results for TAP without length constraints. Gilg et al. (2018) propose an integer linear programming (ILP) formulation for the TAP with a robust extension and a stochastic version tested on real instances.

A second part of the literature deals with combining TAP and TMP. This combination corresponds to the Train Unit Shunting Problem (TUSP). Winter and Zimmerman (2000) study several algorithms to solve the corresponding problem in tram depots. For what concerns railway, this problem is first introduced by Freling et al. (2005) and solved with a two phases approach. MP is tackled with linear programming solver and then a column generation is used for TAP. Haijema et al. (2006) also consider a two phase approach. It is implemented with a dynamic programming based heuristic. Kroon et al. (2008) give an integrated ILP formulation which gathers TMP and TAP. Haahr et al. (2017) solve the same problem with column generation. This approach is compared with greedy algorithms and a constraint programming method. Lentink et al. (2006) propose an additional step in which they solve SRP thanks to an A* algorithm. Ramond and Marcos (2014) describe a TUSP extension to SMP for ROADEF/EURO challenge. Conflicts between shunting movements are tackled with a macroscopic representation of the infrastructure.

In this paper, we propose an integrated formulation for G-TUSP, while the literature always tackles separately one or few sub-problems.

3 Formulation

3.1 Modeling principles

In our formulation of the G-TUSP, we consider that train units can be coupled or uncoupled to form trains. Three formal sets of trains are introduced to model this: arriving, intermediate and departing trains. Arriving trains are moved from a platform track to the shunting yard. Once there, they are uncoupled if needed, and they become intermediate trains, which are moved in the yard and submitted to maintenance. Finally, intermediate trains are coupled if necessary and become departing trains to be moved to the suitable platform track. Trains move on an infrastructure modeled microscopically through a *track-circuit* scale representation. A track-circuit is a portion of track on which the presence of a train unit is automatically detected. Thanks to this infrastructure model, detailed characteristics of *interlocking systems* are taken into account and train safety is ensured through suitable separation..

Figure 1 represents a simple example in which an orange, a green and a blue path are shown with their respective track-circuits named z followed by a number. Both orange and blue paths use track-circuit z_{15}, therefore they cannot utilize it at the same time. The train with the orange path is an intermediate train whose path starts at shunting track 21. This train results from the arriving train using the green path and has to be cleaned. It is parked at the shunting track 29 for cleaning. The train with the blue path is a departing train which uses platform A.

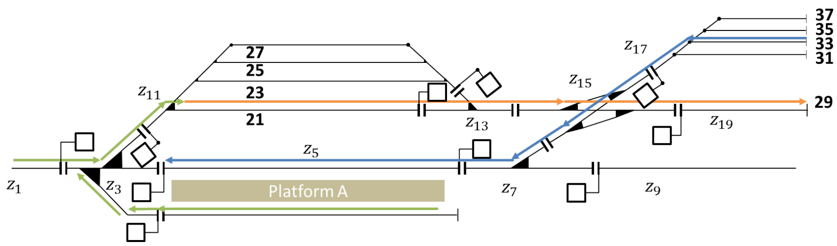

Figure 1: Simple example. Station layout with signals represented by squares. The green arriving train whose path is represented with a green line becomes the orange train at the shunting track 21. The orange intermediate train's path is represented in orange. The blue departing train leaves the shunting track and is moved to platform A. This train uses the blue path.

Trains

We denote T_T the set of arriving trains. Each arriving train can be splitted into several intermediate trains. For an arriving train t', $T_I(t')$ is the set of its intermediate trains. The set of departing trains is denoted T_S. For a departing train we denote $T_I(t)$ the set of intermediate trains which are compatible with t. Those are intermediate trains which can be coupled to obtain t. In this definition intermediate trains in $T_I(t)$ must arrive before t's departure.

Every train is composed of one or several train units. Train units are divided into types so that same type train units get interchangeable. Every arriving train entering the shunting disappear and one or more intermediate trains appear. All intermediate trains do not disappear to become departing trains. Some intermediate trains may remain in the shunting yard at the end of the planning period. For trains that are stored in the station before the planning period, a trivial train is introduced. This arriving train enters the station at the beginning of the planning period on the associated siding.

Besides, by definition, the sets $T_I(t)$ are disjoints. For readability, we introduce $T_I = \cup_{t \in T_T} T_I(t)$ that is the set of intermediate trains. We can remark that a departing train t and an arriving train t' use the same set of train unit if and only if $T_I(t) = T_I(t')$. In Figure 2, three types of train units are considered: hashed ones, full colored ones and white ones. For each arriving train, the set of its intermediate trains is represented by a thick lined dashed box. For each departing train, the set of compatible intermediate trains is represented with a tight lined dashed box. Arrows represent a possible combination of coupling and uncoupling to use the train units available to compose the two departing trains. Here, The arriving train

t_1 is uncoupled in order to obtain train t_A and two intermediate trains are coupled to obtain train t_B.

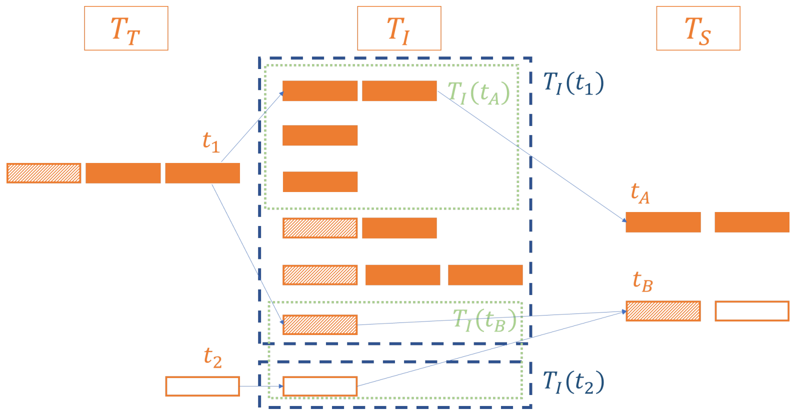

Figure 2: Train matching. Arriving trains T_T on the left are used for the departing trains T_S on the right thanks to intermediate trains T_I. A possible matching is represented with arrows.

We also consider trains which stop at the station without being shunted. Those are *passing trains*. The set of passing trains is denoted T_P.

Infrastructure

A *track-circuit scale* model is used in order to get a rigorous capacity occupation. In the station area, a train follows a path which is a track-circuits succession. As trains can turn around, a path may go twice through a track-circuit. Therefore, we introduce formal track-circuits to precise passing direction. For every real track-circuit, we consider a set of corresponding formal track-circuits. These sets contain up to two formal track-circuits, since there is a formal track-circuit per direction.

We distinguish the notion of path from that of route. Routes are individually handled and defined by signalling control. A path is the concatenation of routes and may include turnarounds. In the turnarounds, a first route is defined up to the turnaround place where a second route starts.

Capacity occupation is based on track-circuit reservation. When a train t needs to go through a track-circuit tc, the signal which allows t to move into the *block section* where tc is located must have a green aspect. A block section is a sequence of track-circuits which can be utilized by at most one train at a time. Thanks to the interlocking system, the green aspect can be obtained once the path r that leads t to tc is formed. This is why we introduce formation times, which depends on block sections characteristics. However r can only be formed if all conflicting routes are released. A block section locked by a train is released shortly after this train clears the last track-circuit it is using in the block section itself.

A path can imply parking on a shunting track. Paths are set such that shunting tracks are at the beginning or the end of the path. For a path r, we define Ps^r the set of shunting tracks where r starts and Pe^r the set of shunting tracks where r ends. Ps^r and Pe^r can contain one shunting track or be empty. Every train has a set of usable paths. Arriving trains paths terminate at a shunting track and go through a platform, while departing trains paths begin at a shunting track and go through a platform. Exactly one path is assigned to arriving, departing and passing trains. Intermediate trains paths begin and terminate on a shunting track. When an intermediate train needs to be parked at several tracks, several paths are assigned to it. In order to define a sequence of paths, two fictive paths r_0 and r_∞ are assigned to intermediate trains. r_0 is at beginning of the sequence while r_∞ terminates it.

The set of exit points of a shunting track p is denoted $Ex(p)$. This set contains at most two elements which indicate a geographical location. We use the locations left and right, respectively denoted L and R. A train enters in (or exits from) a shunting track p with the path r by the exit point $Es(r,p) \in Ex(p)$ (or $Ee(r,p) \in Ex(p)$). A path r_2 can only follow a path r_1 if r_1 ends at the shunting track where r_2 begins: $Ps^{r_2} \cap Pe^{r_1} \neq \emptyset$. In the example of Figure 1, the green path is denoted r_1 and the orange one is denoted r_2. r_2 follows r_1 at the siding track 21. Indeed $Ps^{r_2} = Pe^{r_1} = \{21\}$.

Maintenance operations

Cleaning or maintenance operations may be included in the rolling-stock plan. They are considered to be made on intermediate trains. The operations carried out on an intermediate train $t \in T_I$ form set O_t. An operation $o \in O_t$ can only be performed on shunting tracks with specific facilities. The sequence of operations is given. We introduce P^o set of shunting tracks where o can be carried out. In addition, an operation requires the use of specific human resources. We consider that an operation o requires a crew among the set HR^o of crews which can be assigned to o. Each crew is available from its shift start time to its shift end time.

We also note that when an operation is in progress, the shunting track where it is carried out must be protected to ensure staff safety. Thus, during this period, no other train can enter this shunting track or leave it.

3.2 MILP formulation

In the MILP, we use the following notations:

T_T,T_I,T_S,T_P	set of arriving trains, intermediate trains, departing trains, passing trains
$T = T_T \cup T_I \cup T_S \cup T_P$	set of trains
$T^* = T_T \cup T_I \cup T_S$	set of shunted trains
$T_I(t)$	set of intermediate trains compatible with the arriving or departing train $t \in T_T \cup T_S$
TU,$m_{t,tu}$	set of train unit types, number of train units of type $tu \in TU$ in the train $t \in T^*$
index t	index of train $t \in T$
ty_t,l_t, a_t,d_t	type of train $t \in T$, length of train $t \in T$, arrival time of train $t \in T_T \cup T_P$, departure time of train $t \in T_S \cup T_P$
B_t,Q_t	cancellation cost of train $t \in T_S$, cost associated to the delay of train $t \in T_S \cup T_P$
A_t,Q_R	cost of one time unit duration of a shunting movement performed on the intermediate train $t \in T_I$ and cost of the assignment of a route to the intermediate train $t \in T_I$

$\omega_{t,t'}$	weight associated to the assignment of intermediate train $t' \in T_I(t)$ to departing train $t \in T_S$
Q_C,Q_H	coupling cost, uncoupling cost
b_{t_1,t_2}	indicator function: 1 if $t_1 \in T_I(t)$ (with $t \in T_T$) is placed to the left of $t_2 \in T_I(t)$ with index $t_1 <$ index t_2, 0 otherwise
$i(t,t')$	indicator function: 1 if train $t \in T^*$ is reused for train $t' \in T^*$, $i(t,t') = 1 \iff (t \in T_T, t' \in T_I(t)) \vee (t = t' \in T_I) \vee (t' \in T_S, t \in T_I(t'))$
mp	minimum parking time
R_t,TC_t,Z_t	set of paths, formal track-circuits and real track-circuits which can be used by a train $t \in T$
$TC(z)$	set of formal track-circuits corresponding real track-circuit $z \in \cup_{t\in T} Z_t$
Z^r, TC^r	set of real and formal track-circuits the path $r \in \cup_{t\in T} R_t$
M_R	maximum number of paths which can be assigned to an intermediate train
$OTC_{ty,r,tc}$	set of consecutive formal track-circuits preceding $tc \in TC^r$ which are occupied by a train of type ty traveling along path $r \in \cup_{t\in T} R_t$ if its head is on tc, depending on train and track-circuit length
$pc_{r,t}, sc_{r,t}$	formal track-circuits preceding and following $tc \in TC^r$ along path $r \in R_t$
$rt_{ty,r,tc}$, $ct_{ty,r,tc}$	running and clearing time of $tc \in TC^r$ along $r \in \cup_{t\in T} R_t$ for a train of type ty
$ref_{r,tc}$	reference formal track-circuit for reservation of $tc \in TC^r$ along $r \in \cup_{t\in T} R_t$
$bs_{r,t}$	block section including formal track-circuit $tc \in TC^r$ along $r \in \cup_{t\in T} R_t$
for_{bs},rel_{bs}	formation and release time for block section bs
Ps^r,Pe^r, P^r	set of shunting tracks where $r \in \cup_{t\in T} R_t$ begins, set of shunting tracks where $r \in \cup_{t\in T} R_t$ ends, set of tracks in r $P^r = Ps^r \cup Pe^r$
$Z(p)$,$Ex(p)$	set of real track-circuits and set of exit points composing a shunting track p
L_p	length of shunting track p
$tcp_{r,p}, tce_{r,p}$	reference formal track-circuit for parking at shunting track $p \in P^r$ along $r \in \cup_{t\in T} R_t$, first formal track after shunting track $p \in Ps^r$ along $r \in \cup_{t\in T} R_t$
$Es(r,p)$, $Ee(r,p)$	entrance and exit point of $r \in \cup_{t\in T} R_t$ at shunting track $p \in Pe^r$ and $p \in Ps^r$
O_t	set of operations to carry on $t \in T_I$
pR^o,ω_o	duration and cancellation cost of operation $o \in \bigcup_{t\in T_I} O_t$
HR^o,P^o	set of crews and shunting tracks which can be assigned to operation $o \in \bigcup_{t\in T_I} O_t$
E_t	set of successive operations on $t \in T_I$. $(o,o') \in E_t$ if and only if the operation o' follows the operation o

sR_{hr},eR_{hr}	shift start time and shift end time of crew hr
M,τ_M	large constant compared to event times, end of planning period

In the formulation, we introduce non-negative continuous variables:

- $oc_{t,r,tc}$, $\phi_{t,r,tc}$, $sU_{t,r,tc}$, $eU_{t,r,tc}$, with $t \in T, r \in R_t, tc \in TC^r$: time at which t starts the occupation of tc along r, additional running time of t on tc along r, time at which tc starts being utilized by t along r, time at wich tc ends being utilized by t along r
- $sO_{o,r,r',p,hr}$, with $t \in T_I, o \in O_t, r, r' \in R_t, p \in P^r \cap P^o, hr \in HR^o$: time at which o starts at shunting track p between paths r and r' with crew hr
- D_t, with $t \in T_S \cup T_P$: delay suffered by train t when exiting the control area

Moreover, we introduce binary variables:

- xT_t, with $t \in T_I$, is equal to 1 if t is created and 0 otherwise
- $xS_{t,t'}$, with $t \in T_S, t' \in T_I(t)$, is equal to 1 if t' is assigned to t and 0 otherwise
- $xR_{t,r}$, with $t \in T$, $r \in R_t$, is equal to 1 if t uses r and 0 otherwise
- $xO_{o,r,r',p,hr}$, with $t \in T_I, o \in O_t, r, r' \in R_t, p \in Pe^r \cap Ps^{r'} \cap P^o, hr \in HR^o$, is equal to 1 if o is carried out at shunting track p between paths r and r' with crew hr and 0 otherwise
- qS_t, with $t \in T_S$, is equal to 1 if t is cancelled and 0 otherwise
- $yR_{t,t',r,r',tc,tc'}$ with $t, t' \in T$, $r \in R_t$, $r' \in R_{t'}$, $z \in Z^r \cap Z^{r'}$, $tc, tc' \in TC(z)$, $tc, tc' \in TC^r \cap TC^{r'}$, index $t <$ index t', is equal to 1 if t uses tc along r before t' uses tc' along r' and 0 otherwise
- $k_{t,r,r'}$, with $t \in T_I, r, r' \in R_t, (Ps^{r'} \cap Pe^r \neq \emptyset) \vee (r = r_0) \vee (r' = r_\infty)$ (i.e. r' can follow r), is equal to 1 if t uses r followed by r' and 0 otherwise
- $y_{o,o',hr}$ with $t, t' \in T$, $o \in O_t$, $o' \in O_{t'}$, $hr \in HR^o \cap HR^{o'}$, index $t <$ index t', is equal to 1 if hr performs o before o' and 0 otherwise
- $ysO_{o,t,r_1,r_2,r,p}$, with $t \in T_I$, $t' \in T_I$, $t \neq t'$, $o \in O_{t'}$, $r_1, r_2 \in R_{t'}$, $r \in R_t$, $p \in P^o \cap Pe^{r_1} \cap Ps^{r_2} \cap Pe^{r'}$, is equal to 1 if operation o is carried out at shunting track p between path r_1 and r_2 before t enters shunting track p through r and 0 otherwise
- $yeO_{o,t,r_1,r_2,r,p}$, with $t \in T_I$, $t' \in T_I$, $t \neq t'$, $o \in O_{t'}$, $r_1, r_2 \in R_{t'}$, $r \in R_t$, $p \in P^o \cap Pe^{r_1} \cap Ps^{r_2} \cap Ps^{r'}$, is equal to 1 if operation o is carried out at shunting track p between path r_1 and r_2 before t leaves shunting track p through r and 0 otherwise

We also introduce the following integer variables:

- u_t, with $t \in T_T$ gives the number of uncoupling operations on t

- v_t, with $t \in T_S$ gives the number of coupling operations on t

The objective function to minimize integrates several penalties (1). First, it takes into account the cost of departure cancellations and delays. The function includes uncoupling and coupling operations cost. Then, penalties for intermediate trains assignment to departing trains are added. Moreover, we minimize the number of shunting movements for an intermediate train and the duration of these movements. Finally maintenance operations cancellation costs are introduced. We note that we can have a penalty only if the intermediate train concerned by the operation is actually created.

$$\begin{aligned}\min \sum_{t \in T_S} B_t \cdot qS_t + \sum_{t \in T_S \cup T_P} Q_t D_t + \sum_{t \in T_T} Q_C \cdot u_t + \sum_{t \in T_S} Q_H \cdot v_t + \\ \sum_{t \in T_S} \sum_{t' \in T_I(t)} \omega S_{t,t'} xS_{t,t'} + \sum_{t \in T_I, o \in O_t} \left(xT_t - \sum_{\substack{p \in P^O \cap Pe^r \cap Ps^{r'} \\ r,r' \in R_t, hr \in HR^O}} xO_{o,r,r',p,hr} \right) + \\ \sum_{t \in T_I} \sum_{r \in R_t, p \in Ps^r} Q_R xR_{t,r} + A_t (oc_{t,r,tc_\infty} - oc_{t,r,tce_{r,p}})\end{aligned} \tag{1}$$

Matching constraints
The MILP formulation must consider TMP constraints. First, we need to check train compositions. We introduce constraints for the number of train units of a specific type in trains. For each type, each arriving train must have the same number of train units as intermediate trains created after uncoupling (2). Also, each departing train must have the same number of train units as the intermediate trains assigned to it for coupling (3). As intermediate trains can not be splitted, each of them can be assigned at most to one departing train. If the intermediate train is not created, it can not be assigned to a departing train (4). A departure train is cancelled if no intermediate train is assigned to it (5). Then, the number of uncoupling operations on an arriving train or coupling operations on a departing train is equal to the number of intermediates trains assigned minus one (6), (7).

$$m_{t,tu} = \sum_{t' \in T_I(t)} m_{t',tu} xT_{t'} \;\; \forall t \in T_T, tu \in TU \tag{2}$$

$$m_{tu,t} = \sum_{t' \in T_I(t)} m_{t',tu} xS_{t,t'} \;\; \forall t \in T_S, tu \in TU \tag{3}$$

$$\sum_{t' \in T_S : t \in T_I(t')} xS_{t',t} \leq xT_t \;\; \forall t \in T_I \tag{4}$$

$$1 - qS_t \leq \sum_{t' \in T_I(t)} xS_{t,t'} \;\; \forall t \in T_S \tag{5}$$

$$u_t \geq \sum_{t' \in T_I(t)} xT_{t'} - 1 \;\; \forall t \in T_T \tag{6}$$

$$v_t \geq \sum_{t' \in T_I(t)} xS_{t,t'} - 1 \;\; \forall t \in T_S \tag{7}$$

Routing constraints

An arriving or a passing train cannot be operated before its arrival time (8). The start time of track-circuit occupation by a train along a path is zero if the path itself is not used (9). A train starts occupying a track-circuit along a path after spending in the preceding track-circuit its running time and an additional running time, if the path is used (10). An arriving or a passing train uses exactly one path (11). These sets of constraints are inspired by the RECIFE-MILP model of Pellegrini et al. (2015). A departing train uses exactly one path if it is created and zero otherwise (12). An intermediate uses at most M_R paths if it is created and zero otherwise (13). If an intermediate is created, it uses the dummy paths r_0 (14) and r_∞ (15).

$$oc_{t,r,tc} \geq a_t \cdot xR_{t,r} \ \ \forall t \in T_T \cup T_P, r \in R_t, tc \in TC^r \tag{8}$$

$$oc_{t,r,tc} \leq M \cdot xR_{t,r} \ \ \forall t \in T, r \in R_t, tc \in TC^r \tag{9}$$

$$oc_{t,r,tc} = oc_{t,r,pc_{r,tc}} + \phi_{t,r,pc_{r,tc}} + rt_{t,r,pc_{r,tc}} \cdot xR_{t,r} \ \ \forall t \in T, r \in R_t, tc \in TC^r \tag{10}$$

$$\sum_{r \in R_t} xR_{t,r} = 1 \ \ \forall t \in T_T \tag{11}$$

$$\sum_{r \in R_t} xR_{t,r} = 1 - qS_t \ \ \forall t \in T_S \tag{12}$$

$$\sum_{r \in R_t} xR_{t,r} \leq M_R \cdot xT_t \ \ \forall t \in T_I \tag{13}$$

$$xR_{t,r_0} = xT_t \ \ \forall t \in T_I \tag{14}$$

$$xR_{t,r_\infty} = xT_t \ \ \forall t \in T_I \tag{15}$$

Two constraints model the sequence of path used by an intermediate train. If a path is used by an intermediate train:

- exactly one path follows it (16),
- exactly one path precedes it (17).

$$\sum_{r' \in R_t : (Pe^r \cap Ps^{r'} \neq \emptyset) \vee r' = r_\infty} k_{t,r,r'} = xR_{t,r} \ \ \forall t \in T_I, r \in R_t \setminus \{r_\infty\} \tag{16}$$

$$\sum_{r' \in R_t : (Pe^{r'} \cap Ps^{r} \neq \emptyset) \vee r' = r_0} k_{t,r',r} = xR_{t,r} \ \ \forall t \in T_I, r \in R_t \setminus \{r_0\} \tag{17}$$

A delay is at least equal to the difference between the actual exit time from the infrastructure and the scheduled departure time (18).

$$D_t \geq \sum_{r \in R_t} oc_{t,r,tc_\infty} - d_t \ \ \forall t \in T_S \cup T_P \tag{18}$$

The formulation includes constraints that take into account train matching decisions and the sequence of paths used by an intermediate train. These constraints consider two trains t and t' which use the same rolling-stock. A minimum parking time must be ensured between t's arrival (at the end of t's path) and t''s departure on the shunting track. It happens when an arriving train t becomes an intermediate train t' (19), when an intermediate train uses two path in a row (20) and when an intermediate train becomes an departing train (21).

$$\begin{aligned} oc_{t',r',tce_{r',p}} \geq & \sum_{r \in R_t : p \in Pe^r} [oc_{t,r,pc_{r,tc_\infty}} + (rt_{t,r,pc_{r,tc_\infty}} + mp) \cdot xR_{t,r}] \\ & -M(1 - k_{t,r_0,r'}) \ \ \forall t \in T_T, t \in T_I(t), r' \in R_{t'}, p \in Ps^{r'} \end{aligned} \tag{19}$$

$$\begin{aligned} oc_{t,r',tce_{r',p}} \geq oc_{t,r,pc_{r,tc_\infty}} + rt_{t,r,pc_{r,tc_\infty}} + mp - M(1 - k_{t,r,r'}) \\ \forall t \in T_I, r, r' \in R_{t'} : p \in Pe^r \cap Ps^{r'} \end{aligned} \tag{20}$$

$$\begin{aligned} \sum_{r \in R_{t'} : p \in Ps^{r'}} oc_{t,r,tce_{r',p}} \geq oc_{t,r,pc_{r,tc_\infty}} + (rt_{t,r,pc_{r,tc_\infty}} + mp) \cdot xR_{t,r} \\ -M(1 - xS_{t',t}) \ \ \forall t' \in T_S, t \in T_I(t'), r \in R_t, p \in Pe^r \end{aligned} \tag{21}$$

Moreover, we need to ensure spatial coherence. It means that when an arriving train t becomes an intermediate train t', t uses a path which ends at the same shunting track as the path used by t' (22), (23). The same happens when an intermediate train t' becomes a departing train t (24), (25).

$$\begin{aligned} \sum_{r \in R_t : p \in Pe^r} xR_{t,r} \leq \sum_{r \in R_{t'} : p \in Ps^r} k_{t',r_0,r} + M_R(1 - xT_{t'}) \\ \forall t \in T_T, t' \in T_I(t), p \in \bigcup_{r \in R_t, r' \in R_{t'}} (Pe^r \cup Ps^{r'}) \end{aligned} \tag{22}$$

$$\begin{aligned} \sum_{r \in R_{t'} : p \in Ps^r} k_{t',r_0,r} \leq \sum_{r \in R_t : p \in Pe^r} xR_{t,r} + M_R(1 - xT_{t'}) \\ \forall t \in T_T, t' \in T_I(t), p \in \bigcup_{r \in R_t, r' \in R_{t'}} (Pe^r \cup Ps^{r'}) \end{aligned} \tag{23}$$

$$\begin{aligned} \sum_{r \in R_{t'} : p \in Pe^r} k_{t',r,r_\infty} \leq \sum_{r \in R_t : p \in Ps^r} xR_{t,r} + M_R(1 - xS_{t,t'}) \\ \forall t \in T_S, t' \in T_I(t), p \in \bigcup_{r \in R_t, r' \in R_{t'}} (Ps^r \cup Pe^{r'}) \end{aligned} \tag{24}$$

$$\sum_{r\in R_t:p\in Ps^r} xR_{t,r} \leq \sum_{r\in R_{t'}:p\in Pe^r} k_{t',r,r_\infty} + M_R(1-xS_{t,t'})$$
$$\forall t \in T_S, t' \in T_I(t), p \in \bigcup_{r\in R_t, r'\in R_{t'}} (Ps^r \cup Pe^{r'}) \tag{25}$$

Otherwise, track-circuits of shunting tracks must remain in use when a train is parked. Thus, when an arriving train t becomes an intermediate train t', t' starts using the first track-circuit of its path before t finishes using the last track-circuit of its path (26). The same happens when an intermediate train uses two paths in a row (27) and when an intermediate train becomes a departing train (28).

$$sU_{t',r',sc_{r',tc_0}} \leq eU_{t,r,pc_{r,tc_\infty}} - M(2-k_{t',r_0,r'} - xR_{t,r})$$
$$\forall t \in T_T, t' \in T_I(t), r \in R_t, r' \in R_{t'}, Pe^r \cap Ps^{r'} \neq \emptyset \tag{26}$$

$$sU_{t,r',sc_{r',tc_0}} \leq eU_{t,r,pc_{r,tc_\infty}} - M(1-k_{t,r,r'})$$
$$\forall t \in T_T, r, r' \in R_t, Pe^r \cap Ps^{r'} \neq \emptyset \tag{27}$$

$$sU_{t,r,sc_{r',tc_0}} \leq eU_{t,r',pc_{r,tc_\infty}} - M(2-k_{t',r',r_\infty} - xR_{t,r})$$
$$\forall t \in T_S, t' \in T_I(t), r \in R_t, r' \in R_{t'}, Ps^r \cap Pe^{r'} \neq \emptyset \tag{28}$$

An additional set of constraints deals with formal track-circuit tc reservation. A train's utilization of a track-circuit along a route starts as soon as the train starts occupying the reference formal track-circuit $ref_{r,tc}$ for the reservation of tc minus the formation time (29). A train's utilization of a track-circuit along a route ends when the track-circuit has been physically cleared plus the release time (30). Thus, the equality considers running time, additional running time and clearing time on the track-circuit tc along the path r. Finally, it incorporates possible additional running time on following track-circuits if the train t is long enough to occupy more than one track-circuit at a time. Then, there exists tc' so that tc is physically occupied by t while the head of t reaches the end of track-circuit tc', i.e. $tc \in OTC(t,r,tc')$. There are also disjunctive constraints (31)(32) so that that two trains can not utilize a track-circuit at the same time. These constraint does not affect track-circuits of common shunting tracks.

$$sU_{t,r,tc} = oc_{t,r,ref_{r,tc}} - for_{bs_{r,tc}} xR_{t,r} \quad \forall t \in T, r \in R_t, tc \in TC^r \tag{29}$$

$$eU_{t,r,tc} = oc_{t,r,tc} + ((rt_{t,r,tc} + ct_{t,r,tc} + rel_{bs_{r,tc}})xR_{t,r} + \phi_{t,r,tc})$$
$$+ \sum_{tc'\in TC: tc\in OTC(t,r,tc')} \phi_{t,r,tc'} \quad \forall t \in T, r \in R_t, tc \in TC^r \tag{30}$$

$$eU_{t,r,tc} - M(1 - yR_{t,t',r,r',tc,tc'}) \leq sU_{t',r',tc'}$$
$$\forall t, t' \in T, \text{index } t < \text{index } t', r \in R_t, r' \in R_{t'}, z \in Z^r \cap Z^{r'} \setminus \bigcup_{p\in P^r \cap P^{r'}} Z(p),$$
$$tc \in TC(z) \cap TC^r, tc' \in TC(z) \cap TC^{r'} \tag{31}$$

$$eU_{t',r',tc'} - M \cdot yR_{t,t',r,r',tc,tc'} \leq sU_{t,r,tc}$$
$$\forall t,t' \in T, \text{index } t < \text{index } t', r \in R_t, r' \in R_{t'}, z \in Z^r \cap Z^{r'} \setminus \bigcup_{p \in P^r \cap P^{r'}} Z(p), \tag{32}$$
$$tc \in TC(z) \cap TC^r, tc' \in TC(z) \cap TC^{r'}$$

Maintenance scheduling constraints

For maintenance operations, we specify the inequalities that must be verified at the beginning of the tasks. This must take into account the availability of crew and shunting tracks.

If an intermediate train t is obtained, any operation carried on on t uses only one crew and one shunting track along a given path (33). An operation performed by crew hr must start after the shift start time of hr (34) and before its shift end time (35). An operation carried on on train t at shunting track p between paths r and r' needs to start after t's arrival on p through r. t's arrival time on p through r is given by the expression $sP_{t,r,p}$ (39). If $r \neq \{r_0\}$, $sP_{t,r,p}$ is the moment when t starts using the reference track-circuit for parking at p (37). Else, $r = r_0$ and we need to consider the arriving train which uses the same rolling-stock. Then an intermediate train arrives at its first shunting track when its corresponding arriving train arrives (38). Besides, an operation carried on on train t at shunting track p between paths r and r' needs to finish before t's departure from p through r'. t's departure time from p through r' is given by the expression $eP_{t,r',p}$ (36). If $r \neq \{r_\infty\}$, $sP_{t,r,p}$ is the moment when t starts using the reference track-circuit for parking at p (40). Else, $r = r_\infty$ and we need to consider the departing train which uses the same rolling-stock. Then an intermediate train leaves its first shunting track when its corresponding departing train leaves (41),(42). If no departing train is assigned to t, then t stays at its last shunting track until the end of the planning period (43). Otherwise, if an operation o' follows an operation o, then o' starts after the end of o (44).

$$\sum_{hr \in HR^o, r,r' \in R_t, p \in Pe^r \cap Ps^{r'} \cap P^o} xO_{o,r,r',p,hr} \leq xT_t \quad \forall t \in T_I, o \in O_t \tag{33}$$

$$sO_{o,r,r',p,hr} \geq sR_{hr} \cdot xO_{o,r,r',p,hr}$$
$$\forall t \in T_I, o \in O_t, r, r', \in R_t, p \in Pe^r \cap Ps^{r'} \cap P^o, hr \in HR^o \tag{34}$$

$$sO_{o,r,r',p,hr} + pR^o \leq eR_{hr} \cdot xO_{o,r,r',p,hr}$$
$$\forall t \in T_I, o \in O_t, r, r', \in R_t, p \in Pe^r \cap Ps^{r'} \cap P^o, hr \in HR^o \tag{35}$$

$$sO_{o,r,r',p,hr} \geq sP_{t,r',p} - M \sum_{p \in P^o} (1 - xO_{o,r,r',p,hr})$$
$$\forall t \in T_I, o \in O_t, r, r' \in R_t, p \in Pe^r \cap Ps^{r'} \cap P^o, hr \in HR^o \tag{36}$$

$$sP_{t,r,p} = sU_{t,r,tcp_{t,p}} \quad \forall t \in T_I, r \in R_t \setminus \{r_0, r_\infty\}, p \in Pe^r \tag{37}$$

$$sP_{t,r_0,p} = \sum_{r' \in R_{t'} : p \in Pe^{r'}} sU_{t',r',tcp_{r',p}} \quad \forall t' \in T_T, t \in T_I(t'), p \in \bigcup_{r \in R_t} Ps^r \tag{38}$$

$$sO_{o,p,r,r',hr} + pR^o \leq eP_{t,r',p} + M \sum_{p \in P^o} (1 - xO_{o,p,r,hr})$$
$$\forall t \in T_I, o \in O_t, r, r' \in R_t, p \in Pe^r \cap Ps^{r'} \cap P^o, hr \in HR^o \tag{39}$$

$$eP_{t,r,p} = eU_{t,r,tcp_{r,p}} \quad \forall t \in T_I, r \in R_t \setminus \{r_0, r_\infty\}, p \in Ps^r \tag{40}$$

$$eP_{t,r_\infty,p} \geq \sum_{r' \in R_{t'} : p \in Pe^{r'}} eU_{t,r,tcp_{r',p}} - M(1 - xS_{t',t})$$
$$\forall t' \in T_S, t \in T_I(t'), p \in \bigcup_{r \in R_t} Ps^r \tag{41}$$

$$eP_{t,r_\infty,p} \leq \sum_{r' \in R_{t'} : p \in Pe^{r'}} eU_{t,r,tcp_{r',p}} + M(1 - xS_{t',t})$$
$$\forall t' \in T_S, t \in T_I(t'), p \in \bigcup_{r \in R_t} Ps^r \tag{42}$$

$$eP_{t,r_\infty,p} \geq \tau_M - M \left(1 - \sum_{t' \in T_S : t \in T_I(t')} xS_{t',t}\right)$$
$$\forall t \in T_I, p \in \bigcup_{r \in R_t} Ps^r \tag{43}$$

$$sO_{o',r'_1,r'_2,p',hr'} \geq sO_{o,r_1,r_2,p,hr} + pR^o$$
$$\forall t \in T_I, \forall (o,o') \in E_t, r_1, r'_1, r_2, r'_2 \in R_t, p \in P^o \cap Pe^{r_1} \cap Ps^{r_2},$$
$$p' \in P^{o'} \cap Pe^{r'_1} \cap Ps^{r'_2}, hr \in HR^o, hr' \in HR^{o'} \tag{44}$$

As two operations can not use a crew at the same time, there are disjunctive constraints (45), (46).

$$sO_{o',r'_1,r'_2,p',hr} \geq sO_{o,r_1,r_2,p,hr} + pR^o - M(1 - y_{o,o',hr})$$
$$\forall t, t' \in T, o \in O_t, o' \in O_{t'}, hr \in HR^o \cap HR^{o'}, r_1, r_2 \in R_t, r'_1, r'_2 \in R_{t'},$$
$$p \in P^o \cap Pe^{r_1} \cap Ps^{r_2}, p' \in P^{o'} \cap Pe^{r'_1} \cap Ps^{r'_2}, \text{index } t < \text{index } t' \tag{45}$$

$$sO_{o,r_1,r_2,p,hr} \geq sO_{o',r'_1,r'_2,p',hr} + pR^o - My_{o,o',t,t',hr}$$
$$\forall t, t' \in T, o \in O_t, o' \in O_{t'}, hr \in HR^o \cap HR^{o'}, r_1, r_2 \in R_t, r'_1, r'_2 \in R_{t'},$$
$$p \in P^o \cap Pe^{r_1} \cap Ps^{r_2}, p' \in P^{o'} \cap Pe^{r'_1} \cap Ps^{r'_2}, \text{index } t < \text{index } t' \tag{46}$$

Finally, there is the protection of the garage tracks during an operation. A disjunction sets that trains must enter a shunting track before the beginning (47) or after the end (48) of an operation. An other disjunction sets that trains must leave a shunting track before the beginning (49) or after the end (50) of an operation.

$$\begin{aligned} sP_{t,r,p} \geq sO_{o,r_1,r_2,p,hr} + pR^o + M(1 - ysO_{o,t,r_1,r_2,r',p}) \\ \forall t \in T_I, t' \in T_I, t \neq t', o \in O_{t'}, r_1, r_2 \in R_{t'}, r \in R_t, \\ p \in P^o \cap Pe^{r_1} \cap Ps^{r_2} \cap Pe^{r'} \end{aligned} \tag{47}$$

$$\begin{aligned} sO_{o,r_1,r_2,p,hr} \geq sP_{t,r,p} + MysO_{o,t,r_1,r_2,r',p} \\ \forall t \in T_I, t' \in T_I, t \neq t', o \in O_{t'}, r_1, r_2 \in R_{t'}, r \in R_t, \\ p \in P^o \cap Pe^{r_1} \cap Ps^{r_2} \cap Pe^{r'} \end{aligned} \tag{48}$$

$$\begin{aligned} eP_{t,r,p} \geq sO_{o,r_1,r_2,p,hr} + pR^o + M(1 - yeO_{o,t,r_1,r_2,r',p}) \\ \forall t \in T_I, t' \in T_I, t \neq t', o \in O_{t'}, r_1, r_2 \in R_{t'}, r \in R_t, \\ p \in P^o \cap Pe^{r_1} \cap Ps^{r_2} \cap Ps^{r'} \end{aligned} \tag{49}$$

$$\begin{aligned} sO_{o,r_1,r_2,p,hr} \geq eP_{t,r,p} + MyeO_{o,t,r_1,r_2,r',p} \\ \forall t \in T_I, t' \in T_I, t \neq t', o \in O_{t'}, r_1, r_2 \in R_{t'}, r \in R_t, \\ p \in P^o \cap Pe^{r_1} \cap Ps^{r_2} \cap Ps^{r'} \end{aligned} \tag{50}$$

Parking constraints

Parking constraints are based on constraints which involve precedence between events. In a second step, these precedence variables are used to express the parking constraints.

A first set of variables indicates if two trains use a shunting track at the same time. Thanks to these variables length constraints are set.

For crossing constraints, we introduce two set of binary variables. The first one indicates the relative position of two trains when they enter a shunting track and the second one indicates the relative position of two trains when they leave the track. Two trains must have the same relative position on a shunting track when they enter and when they leave it. These positioning variables are deduced with a disjunction. This disjuntion is based on two assertions:

- if train t enters shunting track p through route r after t' through route r', t is placed on $Es(r,p)$ side of t'
- if train t leaves shunting track p through route r before t' through route r', t is placed on $Ee(r,p)$ side of t'

Table 1 presents a disjunction for entrance relative position variable. This variable is defined with intermediate trains $t, t' \in T_I$, routes $r \in R_t, r' \in R_{t'}$ and shunting track $p \in Pe^r \cap Pe^{r'}$.

$Es(r,p)$	$Es(r',p)$	t enters before t'	t' enters before t
L	L	0	1
L	R	1	1
R	L	0	0
R	R	1	0

Table 1: Values of the entrance relative position variable, with $t \in T_I$, index $t <$ index t', $r \in R_t, r' \in R_{t'}, p \in Pe^r \cap Pe^{r'}$. Variable equal to 1 if t is placed on left side of t' and 0 if t is placed on right side of t'

4 Experiments

In this section, we report on experiments that test the model on a panel of instances. The model is coded in Java and solved exactly using the commercial solver CPLEX. As in principle we shall deploy our solution method for G-TUSP in dispatching centers, it must be able to run on a computer of standard configuration. Therefore, it is executed on a 32 bit operation system equipped with a 2.1 GHz Intel®Core™i3-51010U processor and 4GB RAM. We study Metz-Ville station infrastructure. It is a major hub for Eastern France railway traffic. We tackle real scenarios which include disturbances such as arrival delay or track closure.

4.1 Case study

We consider traffic in Metz-Ville infrastructure and its passengers shunting yards represented in Figure 3. It is a major junction where the Nancy-Luxembourg and Metz-Strasbourg lines intersect. The station mainly hosts regional trains. Many of these trains start or end their service in Metz-Ville. The area is 3.8 km long and has 10 platforms including a dead-end one. The yards F1 and F2 are controlled from the signal box, while switches are directly handled by a ground-agent in yards F3 and F4. The infrastructure is composed of 138 track-circuits, 68 signals, 421 block sections and 405 routes.

The set of path R_t that can be used by a train is computed thanks to breadth-first search (BFS). In preprocessing, this BFS is based on a graph, whose vertices are signals or signs. Its edges are routes between signs and signals or represent turnarounds.

We consider a regular week day and two disturbed week days in 2018. One disturbed day includes several delays form Luxembourg between 16:30 and 19:40. During the other disturbed day, one of the two north side shunting necks is closed. This shunted neck circled in the red (Figure 3) and the available one is circled in green. Here trains perform turnarounds when necessary. A first set of scenarios studies trains between evening peak hour (18:30) and next morning peak hour (07:30). These are scenarios where trains have to be shunted for the night. Trains enters in yards in the evening to leave in the morning. A second set of scenarios considers trains between morning peak hour (07:00) and evening peak hour (19:00). In those scenarios trains are stored during the day. As we need to focus on those trains, we do not have to consider passing trains in the whole time horizon. Indeed, conflicts between shunting movements and passing trains occur during rush hours only. During off-peak time, Metz-Ville dispatchers can trivially find conflict-free shunting routes. Therefore, we only consider passing trains during peak hours (6:30 - 9:00 and 17:00

- 19:30).

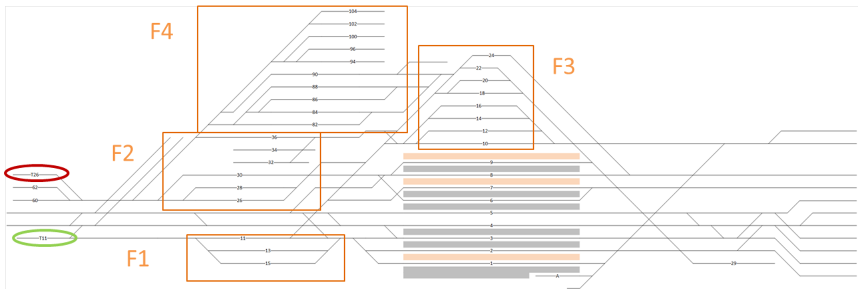

Figure 3: Layout of Metz-Ville station

Name	Day/ Night	Disturbance	$\|T_P\|$	$\|T^*\|$	# of continuous variables	# of binary variables	# of constraints
D1	Day	None	27	6	87 768	891 946	2 402 227
D2	Day	track closure	25	7	91 207	1 345 509	3 134 528
D3	Day	arrival delays	25	6	74 437	804 814	2 182 065
N1	Night	None	22	9	131 084	2 423 404	3 785 881
N2	Night	track closure	24	8	119 640	1 786 528	2 834 102
N3	Night	arrival delays	22	10	153 742	2 659 013	4 257 630

Table 2: Details on the instances tackled in the experimental analysis ($|T_P|$: number of passing trains, $|T^*|$:number of shunted trains)

Table 2 reports the details on the six instances tackled. In each of them there are 7 types of trains on which 4 different operations can be performed: arrival check, internal cleaning, WC cleaning and external cleaning. The track closure scenario reduces the set of possible shunting paths and imply the occurrence of conflicts. Indeed, if a train has to be moved from yard F2 to yard F4, it has to cross main tracks. In instance N3, as trains arrive late in the evening peak hour, their operation can not start on time. In this scenario, in reality as cleaning crews shift ended too early, some cleaning operations were actually postponed to the morning or cancelled. In Table 2, we report the number of passing trains $|T_P|$ and shunted trains $|T^*|$ as well as the number of continuous and binary variables created in the model. Despite the limited set of trains, we get large number of variables. This is essentially because of precedence variables yR which indicate the using order of a track-circuit.

4.2 Results

CPLEX running time is deliberately limited to 3600 seconds. Beyond that duration there is no practical interest for operational planning. Table 3 reports results obtained on the 6 instances described in Section 4.1. It shows the number of coupling and uncoupling required on shunted trains as well as the number of modifications to the planned train matching. It also reports the average number of routes allocated to an intermediate train by our solution and the average number of routes actually allocated by dispatchers. Moreover, we indicate delays taken by departures performed by trains coming from shunting yards. However passing trains departures can also be delayed in addition to shunted trains delays, then the total delay reported in Table 3 comes from these two contributions. The table also shows the actual total delay recorded on traffic database. We remark that the solver does not reach an optimal solution or a proof of optimality in the allotted time. In particular, the gaps exceed 20 % in arrival delay scenarios.

Instance	D1	D2	D3	N1	N2	N3
running time (sec)	3600	3600	3600	3600	3600	3600
# cancelled operations	0	0	0	0	0	1
act. # cancel. op.	**0**	**0**	**0**	**0**	**0**	**2**
# coupling	1	2	0	1	2	2
# uncoupling	2	1	1	2	0	3
# modif. match.	0	0	0	0	2	3
av. # shunt. paths	2.5	3.09	2.67	2.89	3,38	3.10
act. av. # shunt. paths	**2.17**	**2.43**	**2.33**	**2.56**	**2.75**	**2.40**
# shunt. dep. del.	0	1	0	0	0	1
act. # shunt. dep. del.	**0**	**1**	**0**	**0**	**1**	**1**
tot. shunt. mov. time (min)	166.82	287.29	130.04	357.02	434.60	397.55
total delay (min)	0	11.87	54.51	0	3.43	26.32
act. total delay (min)	**0**	**12.5**	**68.5**	**0**	**8.0**	**25.0**
integer solution value	1584.11	1645.80	972.80	1978.45	986.71	2257.32
gap (%)	16.12	7.77	20.56	9.32	13.64	24.30

Table 3: Experiments results (act. # cancel. op.: number of cancelled operations by rolling stock managers, modif. match.: modifications to the planned train matching, av. # shunt. paths: average number of shunting paths allocated to intermediate trains by our solution, act. av. # shunt. paths: average number of shunting paths allocated to intermediate train by dispatchers, shunt. dep. del.: shunted departure trains delayed, act. # shunt. dep. del.: number of shunted departure trains delayed by dispatchers solution, tot. shunt. mov. time: total shunting movement time, act. total delay: total delay in dispatchers solution)

There is no total delay on D1 and N1 instances. However, more shunting movements are performed in our solution than in the one implemented by dispatchers. The solution of D2 brings a departure delayed as in the actual traffic data. It is in both cases the same train, nevertheless it suffers from a 8.34 minute delay in our solution while it was 10 minutes in reality. In instance D3, despite a significant gap, the solution obtained reduces the total delay. The solution of N2 switches two trains in order to reduce the delay. For N3, the result is notably different from the actual decisions. The solution switches three trains in order to cancel fewer operations. However, total delay gets higher. In summary, the implementation of our MILP formulation call the attention on relevant alternatives to the choices of dispatchers in the tested instances. In particular, they highlight the significant effect of changes in the train matching for G-TUSP.

However, we remark that Metz-Ville has a large number of sidings compared to the number of shunted trains. Indeed, it is not necessary to park several trains on the same track except for coupling or uncoupling. Then, in our solutions, trains are always parked on different tracks. It makes a part of TAP constraints useless for these instances.

5 Conclusion

In this paper, we provided a formal description for the G-TUSP, which is the integrated problem of managing shunting operations planning in passenger trains. We tackled a large decision problem that includes many specific operational constraints. We presented a MILP formulation for allocations and continuous time scheduling.

The model copes with both rolling-stock management and capacity management. We extended some literature approaches which combine TAP with TMP. Moreover, we introduced microscopic-scale routing features based on a MILP formulation for real-time traffic management and maintenance scheduling aspects. Maintenance aspects led us to consider that the trains can be successively parked on several tracks which is typically not considered in TUSP literature. The proof of concept carried out on the Metz-Ville instance validates the model relevance. Indeed, it confirms the interest of implementing an integrated approach for improving the operating performance of a station. Even if we can not prove the optimality of the solutions, they are very satisfying compared to the decisions made by dispatchers.

Our study highlights practical issues we will like to tackle in future research. We first need to reduce calculation time. A heuristic phase may provide a first integer solution to the MILP solver, which typically has a major impact on performance. Improvements of the MILP formulation based on valid inequalities may be proposed. In principle, We may also reduce the number of variables, especially precedence ones, by reducing the number of routes to consider. The choice of the remaining routes is in this case critical, and a suitable approach must be found. Other solution techniques such as decomposition can be applied in future works. Moreover, to increase the practical relevance of the formulation, the weights used in the objective function needs to be set in a very accurate way. They are currently quite arbitrary, and they may not properly mimic the need of compromises in real-life situations.

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