A Graph Application for Design and Capacity Analysis of Railway Junctions

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Abstract
In this paper, we developed an analytical model for strategic decision making, for selection of the best solution of the junction layout according to the maximum theoretical infrastructure capacity, completely independent of the timetable. Model achieves triple effects as it enables the selection of the most favorable route sequence, as well as the theoretical capacity calculation. The model uses well known combinatorial problems on graphs, Weighted Vertex Coloring Problem (WVCP) and Traveling Salesman Problem (TSP) to determine the minimum time of the infrastructure occupancy. The model is tested on three different junction layouts.

Keywords
Railway Junction, Capacity, Weighted Vertex Coloring Problem, Traveling Salesman Problem,

1 Introduction

In the recent years, the capacity utilization on the main railway lines and corridors has been increasing. Modern trends in strategic policy such as the opening of a railway market and the appearances of new railway operators led to increase in the number of trains and the capacity of the railway infrastructure has become a bottleneck for the entire railway system. Consequently, there is a decline in the quality of transport service due to the occurrence of train delays.

Railway infrastructure is the most expensive subsystem of the entire railway system. However, the maximum utilization of railway infrastructure capacity should not be the ultimate aim. A high value of the infrastructure capacity utilization coefficient leads to train delays, as well as an exponential increase in these delays (Yuan and Hansen (2004), Landex (2008)). Furthermore, train delays cause a drastic reduction in the quality of transport services. As a result, there is a demand for the construction of new railway lines, as well as for the reconstruction and modification of existing ones.

The term ”railway infrastructure capacity”, in academic and especially in professional publications, mainly refers to the capacity of railway lines. Existing methods, such as UIC 406 (Union International des Chemins de Fer - UIC (2013)), focus on the calculation of railway track capacity, while capacity issues addressing railway nodes are considered as specific cases. However, junctions and stations as nodes in railway networks are essential to the entire railway line capacity evaluations. The capacity of junctions is a complex param-

eter and its calculation is a difficult task primarily due to various train movements that are allowed to be set through a switching area. In such situations, some train routes are compatible and can be executed simultaneously, whereas other train movements are not compatible and have to be separated by a time interval. The minimum time intervals between two successive but incompatible train movements differ depending on the sequences of train route realizations.

Permanent development in computer science and technologies put forwards simulation methods as a reliable approach for evaluating railway capacity. Simulation methods enable the representation of dynamic behavior of a rail traffic system duplicating its real-world operations. Basically simulation models are categorized as macroscopic (e.g. Kecman et al. (2013)) or microscopic (e.g. Nash and Huerlimann (2004) or Radtke and Hauptmann (2004)) models. However, simulation methods have to be adapted to each specific application environment requiring a large amount of preprocessing input data. It could be extremely difficult to collect all required input data, especially for conception solutions characterized with imprecisely defined infrastructure (either regarding track layout or interlocking components) or timetable data. In contrast, analytical methods present a convenient approach aimed to preliminary evaluate capacity of different conception solutions and to identify bottlenecks. Analytical methods utilize mathematical expressions to obtain theoretical upper bound on capacities. Main advantages of analytical methods are fast and simple calculations that provide sufficiently accurate results.

Analytical methods that address capacity evaluations of railway nodes are presented in Malavasi et al. (2014) referring to the mathematical expressions given by Potthoff (1980), Corazza and Musso (1991) and guidelines provided by German railways from 1979. In addition to these simple analytical approaches, Huisman et al. (2002) proposed an analytical approach for the analysis of railway nodes based on the queuing theory. Yuan and Hansen (2007) proposed a stochastic model for train delay propagation that could be used to estimate capacity utilization. Lindner (2011) presented the application of UIC 406 method for station capacity evaluations. The UIC approach was adopted by Landex and Jensen (2013) to analyze capacity at stations with simple track layouts. Also, authors proposed additional measures to analyze and describe track complexity and robustness of train operations. The similar topic on understanding the relationships between capacity utilization and performances of railway stations and junctions is analyzed by Armstrong and Preston (2017). Finally, Jensen et al. (2017) expanded the UIC approach to calculate infrastructure utilization in networks, considering different sequences of a train route realization and their dependence on the infrastructure occupation. As authors stated, the approach is ideal for strategic planning providing the evaluation of different infrastructure solutions.

In this paper, we developed an analytical model applicable for design and capacity analysis of railway junctions. The proposed method determines the sequence of train routes that guarantees the lowest capacity utilization. Based on the proposed approach, it is possible to compare different junction layouts determining the capacity utilization coefficient for each of them. The model is developed as a reverse approach to the graphic Potthoff model. Its main advantages are simplicity and the fact that the model does not require train schedules (timetables). For input data, the model requires only conceptual solutions with defined sets of feasible train routes characterized with the average duration of train routes and mean time intervals between each of them.

2 Problem description and model formulation

The term capacity of the railway infrastructure includes the number of train movements that can be realized in the considered time. The calculation process of the line capacity between two stations involves determining the exact line occupation time by all trains. The time obtained in this manner is used to calculate the utilization coefficient of the railway infrastructure. However, during the calculation process of the capacity of junctions, this procedure becomes significantly complicated, primarily because some train routes can be realized simultaneously with some other routes.

The model proposed in this paper requires the construction of a route compatibility matrix in the first step, as in most of the previously described models. In addition, the model uses a graphical interpretation similar to the Potthoff model. After the construction of the route compatibility matrix, the graph should be constructed such that every possible train movement should be presented as a vertex. An example of junctions used for a detailed description of the model is taken from (Pachl (2004)) as shown in Figures 1 and 2. In these figures, the letters represent the start and end points of the considered routes.

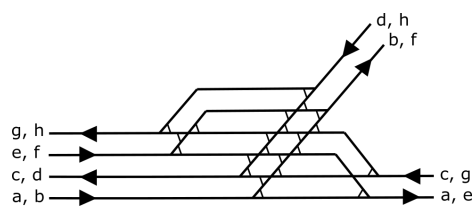

Figure 1: ”Inferior” design of the example junction

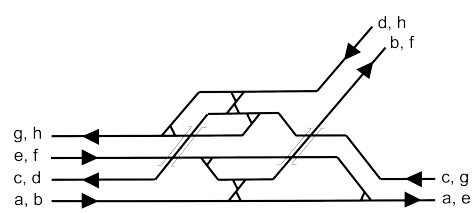

Figure 2: ”Improved” design of the example junction

Based on the provided example junction, in the first step, the matrices of compatible train routes should be constructed. The compatibility matrix is formed by assigning a ”+” sign to the element of matrix $c_{i,j}$ if routes *i* and *j* are compatible with each other. Conversely, the ”-” sign is assigned to the element of matrix $c_{i,j}$ if routes *i* and *j* are incompatible with each other. At the same time, the matrix of minimum time intervals should be created, in such a way that for each element in the compatibility matrix with sign ”-”, for each pair of routes, one calculate and enter the value of the minimum time interval since previous route releases the last joint infrastructure element, until the moment when a consecutive route can start.

Now, the model is developed on the basis of a simple variation in graphical interpretation of the Potthoff method: each possible route is represented as a vertex of the graph, and the edges link the vertices that represent *mutually incompatible* routes, i.e., those train movements that cannot be executed simultaneously. Thus, the graph $G = (V, E)$ is constructed, where V represents a set of vertices, and with E a set of edges are marked. The graph defined in this manner is complementary to that defined by the original Potthoff method (Pachl (2004)). For the junctions presented in 1 and 2, the constructed graphs are shown in Figures 3 and 4 for the "inferior" and "improved" layouts, respectively.

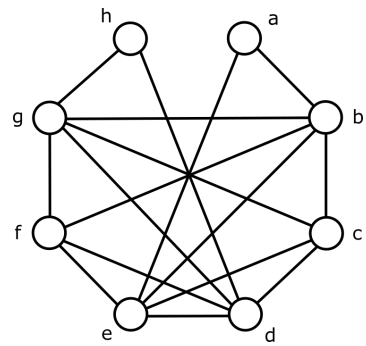

Figure 3: Graph of incompatible train routes for "inferior" layout of the example junction

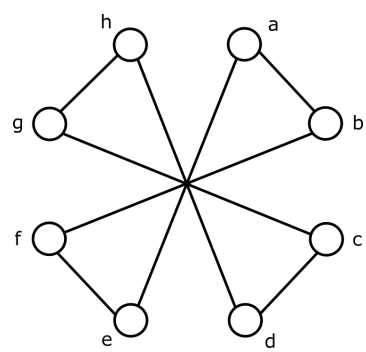

Figure 4: Graph of incompatible train routes for "improved" layout of the example junction

Keeping in mind the rule that in one moment in time, one infrastructure segment can be allocated to only one train movement, the next question can be asked: how to execute all intended routes in such a way that each train movement must be performed at least once and that there is no collision between any two train routes?

Let S denote the set of all infrastructure segments in the switching area and V the set of all possible train routes through the considered switching area. For any train route x, S_x is a set of infrastructure segments that will be occupied during the realization of route x, at least in one moment. If y denotes another route, then we will call x and y incompatible routes if

they cannot be executed simultaneously, i.e., if they must be separated in time, if and only if it is valid

$$S_x \cap S_y \neq \emptyset \tag{1}$$

Nodes of graph G, which are linked by an edge, represent train routes that require at least one "common" element of the infrastructure.

2.1 Weighted Vertex coloring-based approach to junction design analysis

In graph theory, the coloring of a graph is a simple marking of the graph's elements. Similar to the coloring of edges, researches have dealt with the problem of vertex coloring, the problem that we use in our model. The vertex coloring problem (VCP) assumes that each vertex (node) is attributed by a certain marking (color), such that two neighboring vertices, i.e., vertices connected by an edge, cannot have the same marking (color). Formally, if we denote *K=(1,...,m)* as a set of markings (colors), the problem of the vertex coloring for graph G, with m colors, is mapping $C : V \rightarrow K$. The graph is correctly colored for

$$c(i) \neq c(j), \forall \{i, j\} \in E. \tag{2}$$

The smallest number of colors that is sufficient for a graph to be correctly colored is defined as a chromatic number of graph G and is marked as $\chi(G)$. Graph G is $k-colored$ if it is not $(k-1)-colored$. The graph coloring is optimal if all vertices are colored and if k is a minimal number of colors that can be used to color the graph. Although, complexity of the chromatic number computation is known to be NP-hard, for every $k > 3$, a $k-coloring$ of a graph exists by the so called "four color theorem", and it is possible to find such a coloring in polynomial time.

VCP can be modeled by integer linear programming. First, we define two sets of binary variables:

- x_{ij} - a variable that defines whether the marking (color) j is assigned to vertex i; the variable has value 1 if and only if color j is assigned to vertex i,
- y_j - a variable that defines whether the marking (color) j is used in the process of mapping; the variable has a value 1 only if color j is assigned to at least one of the vertices.

The goal is coloring all vertices of the graph using the minimal number of colors; that is, to establish a chromatic number of the graph, the objective function is defined as

$$min \sum_j y_j \tag{3}$$

with a set of constraints

$$\sum_j x_{ij} = 1, \ i \in V \tag{4}$$

$$x_{ij} + x_{kj} \leq 1, \ \forall (i, k) \in E, \ j = 1, ..., n \tag{5}$$

$$x_{ij} \in \{0, 1\}, \ i \in V, \ j = 1, ..., n \tag{6}$$

$$y_j \in \{0,1\},\ j = 1, ..., n. \tag{7}$$

If we apply a VCP on previously described graphs of incompatible routes, the chromatic number of a graph, i.e., the number of used colors for an optimal coloring of incompatible routes graph, will represent a minimal number of the groups of routes that should be formed so that each route is performed exactly once. All vertices that are marked with the same color belong to a set of routes that are mutually compatible and can be executed simultaneously. Colored graphs of "inferior" and "improved" designs of the switching area are shown in Figures 5 and 6.

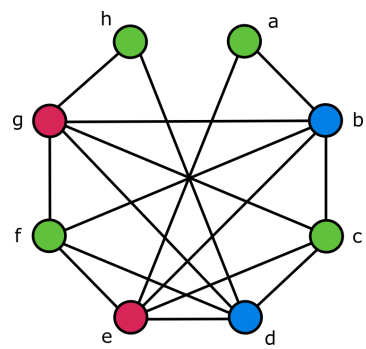

Figure 5: Colored graph of incompatible routes for "inferior" design of switching area

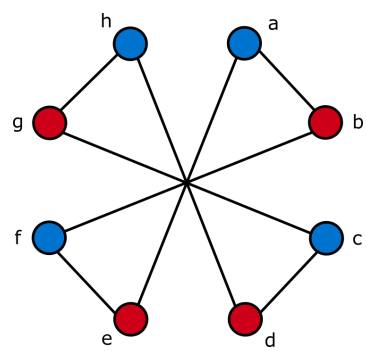

Figure 6: Colored graph of incompatible routes for "improved" design of switching area

For the realization of each set of mutually compatible routes, one after another, in several iterations, each of the defined routes will be completed. Now, it can be confirmed that through the analysis of "inferior" and "improved" designs of the switching area, all routes for the "improved" design can be executed in two iterations, while for the "inferior" design, for completing all routes, we need to form at least three sets of mutually compatible routes.

Based on such a simplified approach for presenting a problem, a model will allow a creative analysis for the layout of the switching area, according to a possible number of required sets for exactly one execution of each route.

In the previously described model, graph coloring does not consider time for route execution, but only their mutual compatibility. A consequence of such model application leads to the generation of the so-called ”unproductive” times. ”Unproductive” time represents a time elapsed from the end of one route within one set of mutually compatible routes (within vertices in one color) until the end of the longest route of the same set. In situations where it is possible to color a graph in more than one way, the time difference between the moments of finished routes and that when the route that needs maximum time to finish is over and belongs to the same set of compatible routes, it is considered as an unproductive time. Even with a previously introduced constraint which imposes that all routes from the next set start their execution simultaneously, after the competition of all defined routes, there is a ”lost” time. To fully understand unproductive and lost time, let us assume that we observe some junction and it is possible to define five routes and that these routes can be grouped in several ways – in Figure 7, there is a diagram of the time distribution.

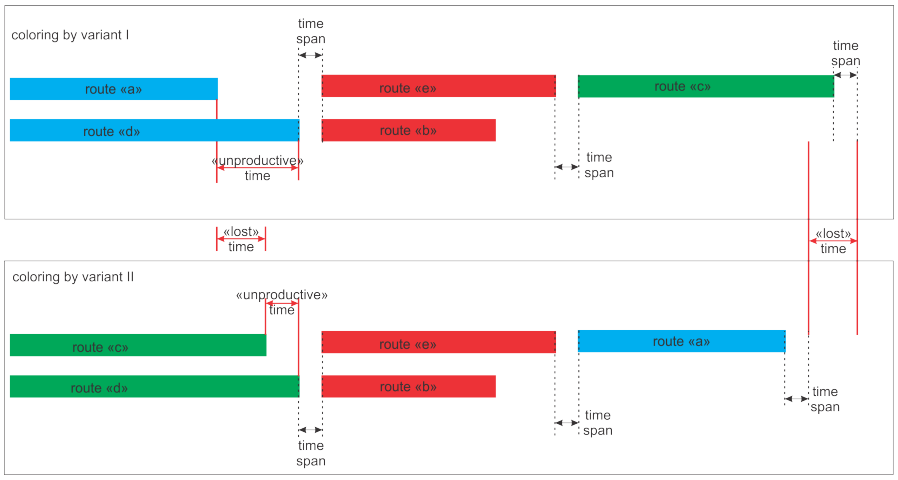

Figure 7: Two alternatives of the Gantt diagram of train routes when graph coloring for incompatible routes is possible in many ways

As presented in Figure 7, ”lost” time is the difference between ”unproductive” times within different sets. Due to the constraint imposed by the simultaneous start of the routes within the next set, ”unproductive” time cannot be eliminated and ”lost” time is generated as an extension of total time of the switching area occupied by all routes.

To reduce the produced negative effects, in the process of coloring the incompatibility graph, it is necessary to group the routes where the time difference between the longest route and a previous route is the smallest within the same set. This can be achieved by assigning each vertex j of graph G a nonnegative value w_j^v. The value of w_j^v is a weight of vertex j, and in the model, it represents the execution time of a route j.

The weighted vertex coloring problem (WVCP) is an extension of the basic graph VCP, where the basic principles of graph coloring are the same. Connected vertices of the graph should be assigned different colors, by defining a minimization of the sum of the cost for the used colors as an objective function. The cost of the used colors is the maximum value

of the vertex weight coefficients that were assigned the same color (Malaguti et al. (2009); Furini and Malaguti (2012)). WVCP is known to be NP-hard.

The model is based on the assumption that the graph vertex weight coefficients w_j^v, $\forall$ $j \in V$, are nonnegative integer values. However, without lack of generalization, we can consider them as real values, ordered by descending values. The model is then shaped as mixed integer programming, as we define the following two sets of variables (Malaguti et al. (2009); Malaguti (2009)):

- x_{ij} - a binary variable with a value of 1 if and only if the color j is assigned to vertex i,
- z_j - a real variable that has a value of the cost for color j.

Now, we can define a basic model with the objective function

$$min \sum_j z_j \tag{8}$$

and constraints

$$z_j \geq w_j^v \cdot x_{ij},\ i \in V,\ j = 1, ..., n \tag{9}$$

$$\sum_j x_{ij} = 1,\ i \in V \tag{10}$$

$$x_{ij} + x_{kj} \leq 1,\ (i, j) \in V,\ j = 1, ..., n \tag{11}$$

$$x_{ij} \in \{0, 1\},\ i \in V,\ j = 1, ..., n. \tag{12}$$

In the defined model, relation (8) is an objective function, constraint (9) defines a cost for each color, and (10) formulates a demand that all vertices must be assigned a color. Constraint (11) represents a basic limitation of the graph VCP, i.e., the neighboring vertices cannot be assigned the same color, while (12) defines a binary variable x (Malaguti (2009); Malaguti et al. (2009)).

As opposed to the basic graph VCP, the solution for WVCP does not have to provide an optimal graph coloring, according to a chromatic number of the graph, $\chi(G)$. Hence, it is possible to group mutually compatible routes in a larger number of groups than it would be minimum necessary, with an assumption that vertex weights are defined as a time to perform certain routes represented by vertices. The model objective function gives the shortest occupation time for the junction *only by time for the completion of a routes*. By each increase in the number of different sets of compatible routes, the total occupation time of the junction is increased by a necessary time interval between each newly added set and its predecessor set of compatible routes. Therefore, through the application of the WVCP model, improvement is evident only if the solution is optimal by the defined objective function (8) as well as by the objective function (3). For this reason, the final number of groups is adopted from the results of VCP. After that, in the case of a different manner of combining routes obtained by VCP and WVCP, in order to improve the results we accept the WVCP solution.

An improvement that is imposed by the application of the WVCP model is a consequence of the comparison of grouped compatible routes with the longest route within the same set while ignoring the "short" routes within a set. However, besides in extreme situations, this will not affect the result.

2.2 Weighted Vertex coloring-based approach for capacity determination

To determine the capacity of a junction, it is necessary to define the time needed for the realization of all routes assuming that each route is realized at least once. Furthermore, we assume that the realization of all routes within a single group is simultaneous and that it starts once all infrastructural and rail operational conditions are met. The assumption that all routes within the same group of mutually compatible routes begin its realization simultaneously allows the formation of a simplified graph, $D(V', E')$. In this simplified graph, vertices are groups of mutually compatible routes, defined by the solution of the WVCP model (relations (8)-(12)). In such a graph, "compatible groups" cannot exist because they would be returned as a joined group by the WVCP model. Thus, the graph created is a complete graph with edges between all pairs of vertices. Now the weight coefficient of the edge is introduced as the maximum value of the required interval between the longest route in group i and all routes within group j of mutually compatible routes, $\tau_{i,j}$:

$$w_{ij}^{e} = max\ \tau_{ij}, \forall (i,j) \in V',\ i \neq j. \tag{13}$$

However, as the minimum necessary time interval between incompatible routes does not have to be equal and most often is not, there are two possibilities. First, a higher value is chosen for the weight coefficient of the edge:

$$w_{ij}^{e} = max\ (w_{ij}^{e}, w_{ji}^{e}). \tag{14}$$

The second possibility, which is used in this paper, imposes the formation of independent edges for each of these two intervals. In this way, the model defines a graph of "incompatible groups of routes" creating a complete digraph, i.e., a directed graph with a pair of edges between all pairs of vertices.

Besides the weight coefficients of the edges, those of the vertices can be assigned to graph D as the maximum realization time of the routes that are grouped together. Bearing in mind the assumption that all routes within one group start simultaneously, the duration of the realization of all routes within one group of mutually compatible routes will be equal to that of the longest route within that group. If we assume that t_{rj} is the duration of a route j in group r, the realization time of all routes from that group will be the same:

$$w_{j}^{r} = \max_{j}\ t_{rj}. \tag{15}$$

To determine the most favorable sequence in which the routes will be executed, it is necessary to first determine the order of the groups of mutually compatible routes. In addition, to determine the capacity of the entire switching area, it is necessary to determine the total time of occupation of the switching area through the realization of all routes when each of them is realized exactly once. Given the characteristics of the defined graph D, both problems can be solved by finding the shortest Hamilton cycle in graph D. The problem of finding the shortest Hamilton cycle, if there is one, is known as the traveling salesman problem (TSP), the famous combinatorial problem, from the NP-complete class. In order to allow periodic repetition of the most favorable sequence throughout observation period, we need to determine Hamiltonian cycle, i.e. Hamiltonian path would not be sufficient for total occupation time determination.

The most favorable sequence in which the routes will be executed is gained by determining the order of realization of groups of mutually compatible routes, as a solution to the

shortest allowed Hamilton cycle, while the total time of occupying the switching area, T_g^s, will be equal to the sum of the solution of TSP problem and the sum of realization times of the longest routes within each group. According to relation (8), the sum of the realization times of the longest routes within each group equals

$$\sum_c w_c^v = T_{WVCP} = min \sum_j z_j. \tag{16}$$

Thus, the total occupation time of the switching area T_g^s by all routes and all necessary time intervals between them equals

$$T_g^s = T_{WVCP} + T_{TSP}. \tag{17}$$

The coefficient of utilization is defined as the ratio of the total occupation time T_g^s and observation time U

$$\eta = \frac{T_g^s}{U}. \tag{18}$$

On the other hand, the total theoretical number of routes N_r that can be executed during a certain period U is defined as

$$N_r = \frac{U}{T_g^s} \cdot \nu \tag{19}$$

where ν signifies the total number of defined routes in the switching area, i.e., the sum of all routes from all groups.

In this way, the model can be used not only for the design analysis of switching areas but also for determining the most favorable sequence of route realization and for approximate capacity determination. The approximate capacity of the switching area, i.e., the maximum number of routes in the observed switching area, can be determined exclusively with the assumption that the traffic pattern, i.e., the specified order of route realization, is unchangeable.

The formed direct graphs, after applying WVCP on the aforementioned examples for ”inferior” and ”improved” track layout designs, are shown in Figures 8 and 9, respectively. The determination of vertex weight coefficients as the maximum duration of route realization within each group is shown in red text, while the procedure of determining the weights of the edges is shown next to each edge.

Considering the developed model, it is easy to compare the two junction layouts, both in terms of the number of simultaneous routes and from the aspect of determining the most favorable sequence of route realization and determining the total capacity.

2.3 Model expansion to achieve demanded route sequences and to deal with heterogeneity

In the case of a timetable with an unequal number of routes from and for different directions, i.e., when some of the routes should be executed more often than other train movements, these routes must be presented as distinct vertices in the graph. Moreover, they have to be connected by edges with all vertices that their base routes are connected with, including the additional edge to the base route. All such ”additional” routes entered into the graph

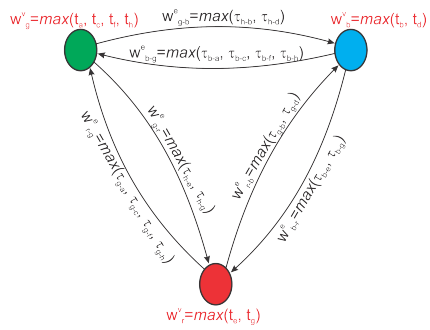

Figure 8: Reduced direct graph coloring of incompatible rides for ”inferior” design of the switching area

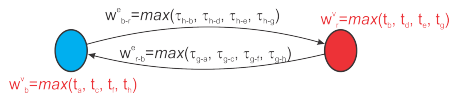

Figure 9: Reduced direct graph coloring of incompatible rides for ”improved” design of the switching area

as distinct vertices and have all characteristics of their base routes. Moreover, they have to respect the same compatibility rules with other routes, with which they are also in conflict. A graph for a case with an unequal number of routes for/from different directions (a and c represent base routes that should be realized twice as often as the rest) and for the ”inferior” design of the switching area is shown in Figure 10. Since the execution of all routes, including additional routes a' and c' represents a cycle, the order of routes in the cycle can be changed, i.e., in the vertex coloring process, additional routes are equal to their base routes, so it is possible to change the execution sequence, as shown in Figure 11.

On the other hand, a case may arise where, with the change in the frequency of certain route realizations, certain limitations concerning the order of their execution are required. Namely, when a certain base route has a higher realization frequency than others, e.g., route a in Figures 10 and 11, there is no logic to allowing successive realization of two, or even more, same routes, especially in case of passenger trains. Actually, it is necessary to introduce additional restrictions in TSP, preventing the procurement of an optimal solution with the adjacent vertices of the same route. At the lowest level, this can be achieved by the removal of edges from digraph $D(V', E')$ that connect ”critical” groups of routes.

Besides the abovementioned case, the requirements for the successive execution of individual routes may occur, especially in the case of passenger trains, in order to obtain connections for the transfer of passengers from one train to another. As in the previous case, simply by modifying the digraph $D(V', E')$, it is possible to impose the successive realization of the two groups of routes, but this time, by forcing the path, from one vertex into another, i.e., through the existence of obligation of a particular edge in the TSP problem solution.

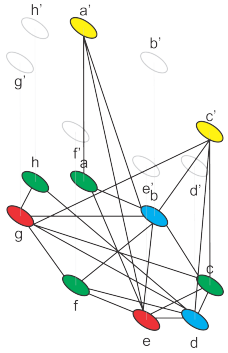

Figure 10: Incompatibility graph for additional routes and different frequency – alt. I

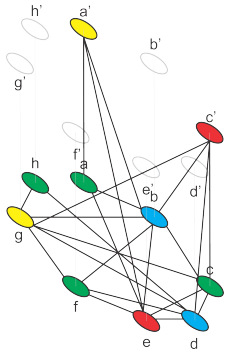

Figure 11: Incompatibility graph for additional routes and different frequency – alt. II

The process of determining the total occupancy time of the junction for a cycle period remains completely unchanged - if there is a change in the number of groups of simultaneous routes, they are equal with other groups, so the algorithm should be applied entirely. Ideally, routes with a higher frequency can be realized simultaneously with the routes of another group, so the graph will accordingly be colored.

In cases where it is predicted that an identical route is carried out by trains whose paths in the timetable are different, i.e., in the case of heterogeneous traffic, as well as in the case of different route frequencies, the vertices of routes using identical parts of the infrastructure but different technical parameters (running speed, train length, etc.) are added to the graph of mutually incompatible routes, while the mutual relations with remaining routes in the incompatibility matrix do not change.

3 Case study and result analysis

For complete application and testing of the defined model, we created three different track layout alternatives for flaying (or grade separated) railway junction. The examined railway

junction has a track configuration in which two main double track railway lines cross each other by a bridge to avoid conflicts of their 4 main routes (a, b, c, d). Furthermore, all three alternatives have track connections that enable additional 8 routes for crossing trains over both railway lines in both directions (e, f, g, h, i, j, k, l). However, the alternatives differ in the complexity of their track layouts expressed either in the number of installed switches, diamond crossings or bridges. The applied track layout directly influences the compatibility of train routes.

Alternative 1 - a basic layout that provides single track connections required to enable trains to cross over railway lines. The track layout consists of two main double track lines, 4 single track connections with installed 24 switches. The layout provides 52 compatibilities among the observed 12 routes. This junction layout is shown in Figure 12

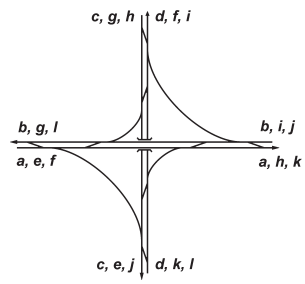

Figure 12: Alternative I of the conceptual solution of test junction

Alternative 2 - a layout that provides double track connections between main railway lines (Figure 13). Double track connections enable two heading trains to cross between main lines in parallel. In addition to two main double track lines, the layout consists of 4 double track connections with installed 16 switches and 8 fixed diamond crossings. The layout provides 60 compatibilities among the observed 12 routes.

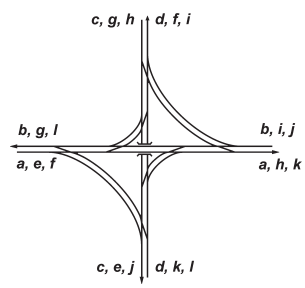

Figure 13: Alternative II of the conceptual solution of test junction

Alternative 3 - a layout that additionally reduce route conflicts providing grade separated track connections instead of fixed diamond crossings. In addition to main double track lines, the layout consists of 4 double track connections with installed 16 switches and 8 bridges. The layout provides 84 compatibilities among the observed 12 routes. This layout is shown

in Figure 14.

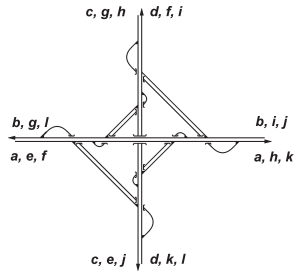

Figure 14: Alternative III of the conceptual solution of test junction

In addition to the base traffic pattern with exactly one train run per route, we analyze two variants where we increased number of trains on some routes. All routes, together with the estimated duration time for each route, are shown in Table 1.

Table 1: Assumed routes and their duration in minutes

Route symbol	Route duration [min.]
a	1.72
b	1.78
c	1.69
d	1.71
e	2.13
f	2.35
g	2.07
h	2.22
i	2.14
j	2.23
k	2.20
l	2.27

To demonstrate how the developed model responds to traffic heterogeneity, we analyze two more variants where we increased number of trains on some routes. The number of trains on each route in observed traffic pattern variants is shown in Table 2.

Table 2: The number of trains on each route in one cycle

Traffic pattern	a	b	c	d	e	f	g	h	i	j	k	l
variant I	1	1	1	1	2	4	2	4	2	4	2	4
variant II	1	1	1	1	2	4	4	2	2	4	4	2

Following a defined method, for every variant, an incompatibility graph was formed and then we applied VCP and WVCP on them. With finding the optimal solutions of WVCP for each defined variant, we obtained the minimum junction occupation times only by route realization, for each alternative separately. The obtained results are shown in Table 3.

After the minimum occupation times by route realization were established, graph reduction was executed. The reduced digraphs were used as an input to TSP and the solutions were obtained using OPL models. The results are shown in Tables 3 and 4. In this manner, we obtained junction occupation time only by minimal necessary time intervals between the groups of mutually incompatible routes, as well as the best feasible sequences of the groups, for each alternative and each variant separately.

Table 3: Acquired results, by variant

	N_{route}	N_{inc}	N_c	T_{WVCP}	T_{TSP}
alternative I	12	92	4	8.60	2.60
alternative II	12	84	3	6.40	2.05
alternative III	12	60	3	6.33	2.00
alt. I - variant I	28	564	11	24.66	6.80
alt. I - variant II	28	500	9	24.58	6.76
alt. II - variant I	28	508	7	20.26	5.86
alt. II - variant II	28	420	11	15.72	4.67
alt. III - variant I	28	308	7	15.58	4.69
alt. III -variant II	28	308	7	15.58	4.78

In the Table 3, column names represent:

- N_{route} - Number of routes,
- N_{inc} - Number of incompatibilities between the routes,
- N_c - Number of colors,
- T_{WVCP} - Total running time [min.] (solution of WVCP) and
- T_{TSP} - Total time intervals [min.] (solution of TSP).

Table 4: Junction capacity, by alternative and by variant

	U	N^h_{route}	η	N_r
alternative I	11.20	64	18.70[%]	1542
alternative II	8.45	85	14.10[%]	2044
alternative III	8.33	86	13.90[%]	2074
alt. I - variant I	31.46	53	52.40[%]	1281
alt. I - variant II	31.34	53	52.20[%]	1286
alt. II - variant I	26.12	64	43.50[%]	1543
alt. II - variant II	20.39	82	34.00[%]	1977
alt. III - variant I	20.27	82	33.80[%]	1989
alt. III -variant II	20.36	82	33.90[%]	1980

Column names in the Table 4 represent:

- U - Total utilization time [min.],
- N^h_{route} - Theoretical maximum number of routes, per hour,

- η - Utilization coefficient for one hour [%] and
- N_r - Theoretical maximum number of routes, per day

By analyzing the obtained results, we can conclude that the best design solution is alternative III, according to the maximum theoretical capacity. As the second-best solution, alternative II was selected.

Obtained results clearly indicate that in a defined model segment of determination of minimum occupation time by route realization, obtaining WVCP solution, is equally important as a segment of determination of minimum occupation time by necessary time intervals between the routes and the best feasible sequence of the routes.

4 Conclusions

Although, thus far, considerable software has been developed for a precise determination of infrastructure capacity, the existence of simple, analytical methods has always had its advantages, especially when quick solutions with satisfactory accuracy are required. A simulation model, although very fast, often requires long-term preparation for precise data acquisition and storing them in a database.

The developed model provides the possibility of a relatively simple junction capacity determination when there are no details regarding train sequence and no timetable. It's extremely useful when it is necessary to quickly obtain solutions for the comparison of several different junction designs, particularly conceptual solutions, considering that all elements are not yet determined. In addition, the model provides the possibility of precise determination of capacity utilization in the time period and determination of the best sequence of train routes.

Although all combinatorial problems used in the paper belong to the NP class (VCP in the scope of decision problem is NP-complete, WVCP is NP-hard, while TSP is also NP-complete), the application of the developed model in practice will be possible, since it is almost impossible to find a junction with so many possible routes, which would make the model too extensive for the application.

In the case study, our developed method was strictly applied on theoretical junction designs, which could be classified as of medium-heavy complexity, or, at the very least, not of easy one. Quality results were obtained, especially since the effects of different conceptual designs were immediately noticeable, even in the case of very small changes in layout. In addition, it was determined that by adopting a better design of the future junction, the utilization coefficient could be reduced by almost 5%, comparing the most favorable and most unfavorable alternatives and equal number of routes. With different train frequencies, this improvement is even more noticeable.

The model has no implemented buffer times, in order to maintain timetable robustness and stability. The implementation of these times should represent the next step in the proposed model development.

Finally, it must be noted that the construction or modernization of a junction is an investment project with various criteria, and hence, the proposed model should be incorporated into a comprehensive decision support system, where infrastructure capacity would be only one criterion.

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