A train timetabling and stop planning optimization model with passenger demand

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Abstract

Train timetabling plays an important part in train management, not only for passengers, but also for train operators. In a highly dynamic transportation market, train timetabling is an essential bridge connecting the service supplier with transportation demand. However, in present operations, train scheduling without considering passenger demand can reduce competitive advantages of railway in the multimodal transportation market and will further lead to passenger dissatisfaction. Therefore, it's important to schedule trains responding to passenger demand in the train planning process. In this paper, we focus on the problem of train timetabling with passenger demand, specifically deciding train stop plan based on different origin-destination passenger demand pairs. Taking the stop indicators as important decision variables, a mixed integer linear programming model is proposed to address this train timetabling and stopping plan integration issue, with minimizing total train travel time and maximizing the number of transported passengers. The weighted-sum method is used to find the Pareto optimal solutions for the proposed bi-objective mathematical model. A set of numerical tests is presented based on Beijing-Jinan high-speed railway line (part of Beijing-Shanghai high-speed railway line) by Cplex optimization solver to validate the model.

Keywords

Train timetabling, Stop planning, Passenger demand, Mixed integer programming, Pareto optimization

1 Introduction

In the rapidly changing multimodal transportation market with intense competition, various transportation modes make efforts to enlarge their own service scope. Providing punctual and flexible service considering passenger demand is especially essential for railway transportation to improve its competitiveness and increase market share in such a situation. An effective train operation plan can provide better service for passengers who choose railway transportation to complete their trips. Due to the growing passenger demand of railway, train operators incline to plan train schedule considering the nature of passengers instead of assuming that passengers will adjust their behaviours to the provided train service. Hence, the scheduling process for railway system has been more and more significant for ensuring punctuality of train operation and for guaranteeing passenger satisfaction.

To provide passenger oriented train service, the key of train scheduling is to meet passenger demand while reducing the cost of operation and management. This complex task requires a comprehensive consideration of passenger demand patterns and train unit resources. For scheduling with passenger demand, urban rail operation under passenger demand concentrates on minimizing passengers' waiting time at metro stations instead of highlighting the origin and destination of passengers, since metro train always stops at each station. While railway pays closer attention to whether there are enough trains to take these passengers at the station as many as possible and how to schedule these trains in an economic way, such as determining stopping plan and frequency. Therefore, from this point, the train timetable and train stops are both determined according to the passenger demand.

Train timetabling and stop planning are regarded as two critical parts in train scheduling. In tradition, these two parts are separated because there is a sequential planning process that is divided in several steps when schedule trains, as Fig.1 shows. Generally, each previous step is taken as an input of the latter one. After a demand analysis, line planning determines train service frequency and different stopping plans of each train to meet passenger demand, also constrained by infrastructure. Then, based on line plan, the train timetable is given to determine the departure time and arrival time of each train at each station, and provides a foundation of rolling stock schedules and crew schedules. At the same time, the latter two process may need to adapt the departure/arrival times of the obtained train schedule. However, in the real world, the adjusted stopping plan and train timetable might not be the best solution for train operators as well as might not meet passenger demand.

In this work, we focus on the integrated optimization problem of train timetabling and train stop planning (ITTSP), which embeds the train stopping planning constraints, based on potential passenger flow for different origin-destination pairs, into the train timetabling stage. To solve this ITTSP problem, a bi-objective mixed integer linear programming model is formulated, in which passenger demand with different origin and destination stations, train stop planning, train routing and train timetabling are included in the model formulation. A weighted-sum method solution approach is then used to solve the resulting integrated optimization problem, where both the objective functions are directly optimized proportionally to the assigned weights.

The reminder of this paper is organized as follows. Section 2 provides a literature review on demand oriented train timetabling and on the integration of train stop planning and timetabling. Then, a detailed problem statement and model assumptions are given first, followed by a bi-objective model that formulates the ITTSP problem based on passenger demand in Section 3. Next, a weighted-sum method is introduced to solve the resulting ITTSP problem. To evaluate the effectiveness of bi-objective model, a case study based on Beijing-Jinan high-speed railway line (BJ railway line) is tested in Section 4. Finally, conclusions and future research are presented in Section 5.

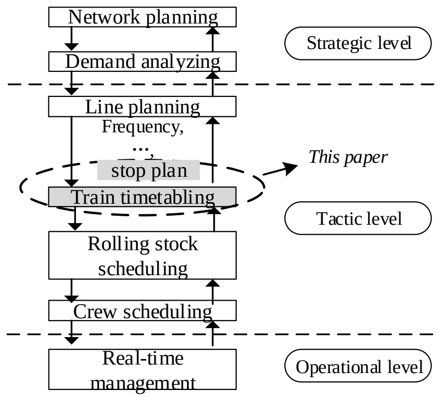

Fig.1: The sequential railway planning process

2 Literature review

In this section, we review the state of the art in two directions: 1) train timetabling considering passenger demand; 2) integrated optimization of train timetabling and stop planning.

Train timetabling plays an important role in train scheduling operation. Several researchers (Szpigel, 1973; Higgins, Kozan and Ferreira, 1996; Caprara et al., 2002; Caprara et al. 2006; Zhou and Zhong, 2007) made some related investigations on this problem. Based on the job-shop scheduling problem, Szpigel (1973) firstly modeled the single-track train scheduling problem to determine the location of crossing and overtaking. Higgins, Kozan and Ferreira (1996) scheduled trains optimally on a single line track and presented a lower bound to reduce the search space in the branch and bound tree. Caprara et al. (2002) and Caprara et al. (2006) gave a graph description of the train timetabling problem with fixing train running time and headway time, while it is not practical due to train acceleration and deceleration. Zhou and Zhong (2007) used branch-and-bound solution algorithms to solve a single-track train timetabling problem and generalized station headway capacities-constrained scheduling formulation.

In recent years, studies on demand-sensitive train timetabling have attracted more and more attention, in order to provide higher level of the train service for transportation demand, especially for the passenger demand. Sun et al. (2014) provided a demand-driven timetable for metro services, which adjusted service frequency dynamically instead of setting it for peak/off-peak time respectively, aiming at minimizing the total passenger waiting time. Canca et al. (2014) put variable demand in a long period into their timetabling model, where vehicle capacity was considered to generate effective solution quality. Barrena et al. (2014a) proposed three exact linear formulations and the branch-and-cut algorithm to design train timetables consistent with dynamic demand. In order to solve large-scale instances, Barrena et al. (2014b) presented an adaptive large neighborhood search (ALNS) meta-heuristic that was able to solve larger and more realistic instances. Niu, Zhou and Gao (2015) proposed quadratic and quasi-quadratic objective functions to formulate total passenger waiting time based on time-varying origin-to-destination demand. Wang et al. (2018) concentrated on time-varying passenger demand of the segment between two adjacent stations to integrate train scheduling and rolling stock circulation planning on an urban rail transit line. Robenek et al. (2018) formulated a passenger centric train timetabling problem under elastic passenger demand and used a logit model to reflect the unknown demand elasticities. Researchers who studied the problem of passenger demand oriented train scheduling were mostly concerned with adjusting train timetable, but line planning is another essential part reflected by passenger demand. Optimizing both line planning and train timetable can better adapt to passenger demand in practice. In the stage of line planning, train stop planning is of particular importance.

In the literature, most existing researches focused on planning train stop plan. Lan (2002) explained that different stopping programs should be included when designing operation plans for Beijing-Shanghai high-speed railway. Besides, Cheng and Peng (2014) developed a 0-1 bi-level mathematical programming model for urban rail transit special stop schedule scheme, considering elastic passenger demand. Yue et al. (2016) optimized train stopping patterns and schedules for high-speed passenger trail corridors and developed an innovative methodology using a column-generation-based heuristic algorithm to simultaneously consider passenger demand and train scheduling. Yang et al. (2016) proposed a new collaborative optimization method for train scheduling and stop planning problem and handled it through linear weighted method, where the model considered the satisfaction of macro demands on each station. Qi, Cacchiani and Yang (2018) emphasized uncertain passenger demand and aimed to determine both train timetable as well as stop plan.

Different from metro rail with all-stop operations, in railway operation plan, passenger demand has a straightforward influence on train stop patterns. Although the all-stop operation is obviously the simplest way for satisfying passenger demand, it may take long-distance passengers' travel time as an extra cost. Therefore, the integration of passenger demand oriented train timetabling and stop planning is a hot researching direction. Nevertheless, the integrated optimization of train timetabling and stop planning with passenger demand could put stress on the computation time and model difficulty.

In this paper, we take passenger demand into train timetabling and highlight the relationship between stop plan and passenger demand pairs with different origin-destination stations, in order to design a train timetable consistent with demand. This paper proposes the following contributions:

- This work embeds passenger demand into the timetabling phase by choosing the train stop on its travel route. A bi-objective linear programming model is proposed, rigorously considering passenger demand constraints.
- We combine train scheduling and stop planning with passenger demand to generate a train timetable and stop plan simultaneously. The objectives of the model we proposed are to minimize total train travel time, in order to reduce the management costs for rail operators, and to maximize the number of transported passengers, in order to better satisfy passenger demand.

3 Problem statement and model

3.1 Problem statement

Before the mixed-integer linear programming model is described, the problem statement and model assumptions are given sequentially. First, inputs of this problem are explained as below:

(1) A railway network

A railway network is given with a number of stations and segments between adjacent stations, in which the segment between two adjacent stations is set as one section, the station is set including specific siding tracks.

(2) Train information

For each train, we know its origin and destination station, the earliest starting time at its origin station, running time between two adjacent stations, minimum and maximum dwell time at intermediate stations, headway time of two consecutive trains, train carrying capacity, characteristics (i.e. train type).

(3) Passenger demand

We consider passenger demand, in this paper, as different sets of passenger pairs who have different origin and destination stations. For each passenger pair, we know its origin station, destination station and volume.

The ITTSP problem has three decision variables:

(1) For each train, its stopping plan needs be determined, that is whether the train chooses to stop and how long it will stop at this station.

(2) For each train scheduled, we need to determine its departure time at the origin station, the arrival, dwell time, and departure time at intermediate stations, as well as the arrival time at the last station.

(3) For each passenger pair, we need to determine which train it is assigned to and how many of passengers in the passenger pair are assigned.

In our model, we make the following assumptions:

(1) In this paper, our purpose is to provide a train schedule to satisfy passenger demand from the view of train operators, so the response of passenger behaviours to the resulting train service is not included.

(2) In our proposed model, the station dwell time occurs only if the train is required to stop due to the passenger demand, and minimum dwell time is fixed without changing with passenger flow variation at a station.

(3) We assume that the station can accommodate enough trains, which means that the station capacity is not considered.

The general subscripts and input parameters of the proposed formulations are introduced in Table 1 and 2, respectively, and the decision variables are given in Table 3.

Table 1: General subscripts

Symbol	**Description**
i, j, k	Physical node index, $i, j, k \in N$, N is the set of nodes in a railway network.
e	Physical cell index, $e \in E$, E is the set of cells in a railway network.
t, t'	Time index, $t, t' \in \{1...T\}$, T is the planning horizon, e.g. 3 hours.
p	Passenger origin-destination (OD) pair index, $p, p' \in P$, P is the set of passenger OD pairs. One passenger OD pair refers to a group of passengers who have the same origin and destination stations.
f	Train index, $f \in F$, F is the set of all trains which need to be scheduled.
m, m'	Station index, $m, m' \in S$, S is the set of all stations in a railway network.

Table 2: Input parameters

Symbol	**Description**
E_f	Set of cells train *f* may use, $E_f \subset E$.
E_c	Set of cells of sections between two adjacent stations, $E_c \subset E$.
E_m	Set of cells of station *m*, $E_m \subset E$.
E_i^{o}	Set of cells starting from node *i*.
E_i^{s}	Set of cells ending at node *i*.
$\delta_{f,i,j}$	Free-flow running time for train *f* to drive through cell (*i*, *j*).
$\vartheta_{f,i,j}^{\min}$	Minimum dwell (waiting) time for train *f* on cell (*i*, *j*).
$\vartheta_{f,i,j}^{\max}$	Maximum dwell (waiting) time for train *f* on cell (*i*, *j*).
$g_{f,i,j}$	Safety time interval between train *f*'s occupancy and arrival on cell (i, j).
$h_{f,i,j}$	Safety time interval between train *f*'s departure and release on cell (i, j).
$c_{i,j,t}$	Flow capacity on cell (i, j) at time *t*.
O_f	Origin node of train *f*.
S_f	Destination (sink) node of train *f*.
O_m	Origin node of station *m*.
S_m	Destination (sink) node of station *m*.
m_p^o	Origin station of passenger pair *p*.
m_p^s	Destination station of passenger pair *p*.
EST_f	Predetermined earliest starting time of train *f* at its origin node.
η_p	Volume of passenger OD pair *p*.
C_f	The capacity of train *f*
α	The maximum passenger carrying coefficient of the scheduled trains.

Table 3: Decision variables

Symbol	Description
$a_{f,i,j,t}$	0-1 binary train arrival variables, =1 if train *f* has already arrived at cell (*i*, *j*) by time *t*; =0 otherwise.
$d_{f,i,j,t}$	0-1 binary train departure variables, =1 if train *f* has already departed from cell (*i*, *j*) by time *t*; =0 otherwise.
$u_{f,i,j,t}$	0-1 binary infrastructure usage variables, =1, if train *f* occupies cell (*i*, *j*) at time *t*; =0 otherwise.
$x_{f,i,j}$	0-1 binary train routing variables, =1, if train *f* selects cell (*i*, *j*) on the network; =0 otherwise.
$y_{f,p}$	Passenger assignment variables, passenger volume of passenger OD pair *p* that is assigned to train *f*.
$z_{f,p,i,j}$	Passenger assignment variables on cell (*i*, *j*), passenger volume of passenger OD pair *p* on cell (*i*, *j*) that is assigned to train *f*.
$r_f^{m,m'}$	0-1 binary train stopping variables, =1, if train *f* stops at both station *m* and *m'*, =0 otherwise.
$k_{p,i,j}$	0-1 binary passenger travel route variables, =1, if passenger pair *p* travel on cell (*i*, *j*), =0 otherwise.
$TT_{f,i,j}$	Travel time for train *f* on cell (*i*, *j*).

3.2 Formulation of the mathematical model

The objective function consists of two parts: one is to maximize the number of transported passengers that are carried by planned trains.

$$Z_{passenger} = \sum_{p\in P}\sum_{f\in F} y_{f,p} \tag{1}$$

Another one is to minimize total train travel time from its origin station to destination station.

$$Z_{time} = \sum_{f\in F}\sum_{t\in T}\left[t\times \sum_{i:(i,s_f)\in E^s_{s_f}\cap E_f} (a_{f,i,s_f,t} - a_{f,i,s_f,t-1}) - t\times \sum_{j:(O_f,j)\in E^o_{O_f}\cap E_f} (d_{f,O_f,j,t} - d_{f,O_f,j,t-1}) \right] \tag{2}$$

The bi-objective function can be presented as:

$$Z = \max Z_{passenger} + \min Z_{time} \tag{3}$$

Subject to:

Group 1: Train running constraints

Train timetabling is actually to determine travel routes of each train on a time-space network, so, based on it, cumulative flow variables (Meng and Zhou, (2014)) $a_{f,i,j,t}$ and $d_{f,i,j,t}$ are introduced to represent both temporal and spatial consumption of trains.

In the network, trains' start is restricted. For trains' start time, constraint (4) and (5) make sure that each train do not depart earlier than predetermined earliest starting time at their origin nodes. Within cell to cell transition, to guarantee the passing time at each cell, the time when train f departs from the forward cell (i, j) and arrives at the later cell (j,k) should be the same.

$$\sum_{j:(o_f,j)\in E_f} a_{f,o_f,j,t} = 0, \forall f \in F, t < EST_f \quad (4)$$

$$\sum_{j:(o_f,j)\in E_f} d_{f,o_f,j,t} = 0, \forall f \in F, t < EST_f \quad (5)$$

$$\sum_{i,j:(i,j)\in E_f} d_{f,i,j,t} = \sum_{j,k:(j,k)\in E_f} a_{f,j,k,t}, \forall f \in F, j \in N \setminus \{o_f, s_f\}, t = 1,...,T \quad (6)$$

In this train scheduling problem, all trains are supposed to meet the flow balance when trains run in the railway network. In this model, we separate nodes in a network into three parts (origin node, intermediate node and destination node) to explain the flow balance problem. At the origin node and destination node, there is only one routing choice for train f to go through. Constraint (7)-(9) ensure flow balance on the network at the origin node, intermediate nodes, and the destination node of train f respectively.

$$\sum_{j:(i,j)\in E^{o}_{o_f} \cap E_f} x_{f,i,j} = 1, \forall f \in F \quad (7)$$

$$\sum_{i:(i,j)\in E^{s}_{j} \cap E_f} x_{f,i,j} = \sum_{k:(j,k)\in E^{o}_{j} \cap E_f} x_{f,j,k}, \forall f \in F, j \in N \setminus \{o_f, s_f\} \quad (8)$$

$$\sum_{i,j:(i,j)\in E^{s}_{s_f} \cap E_f} x_{f,i,j} = 1, \forall f \in F \quad (9)$$

Constraints (10) is imposed to map the variables $a_{f,i,j,t}$ in space-time network to the variables $x_{f,i,j}$ in physical network, so as to describe whether cell (i, j) is selected by train f for traversing the network from its origin to destination.

$$x_{f,i,j} = a_{f,i,j,T}, \forall f \in F, (i, j) \in E_f \quad (10)$$

Here, we use decision variables $a_{f,i,j,t}$ and $d_{f,i,j,t}$ to represent running time $TT_{f,i,j}$, which is the difference of exit time and entrance time for train f on cell (i, j), as constraint (11) shows.

$$TT_{f,i,j} = \sum_t \{t \times [d_{f,i,j,t} - d_{f,i,j,t-1}]\} - \sum_t \{t \times [a_{f,i,j,t} - a_{f,i,j,t-1}]\}, \forall f \in F, (i, j) \in E_f \quad (11)$$

In practice, due to station stops and some unexpected disturbance, such as bad weather, total travel time on cell (i, j) must be equal or larger (smaller) than its free flow travel time plus it minimum (maximum) planned dwell time at the station. Constraint (12) specifies it in an inequality. The minimum planned dwell time is larger than zero, only if there is a train stop at a station in the timetable.

$$\vartheta^{\min}_{f,i,j} + \delta_{f,i,j} \le TT_{f,i,j} \le \vartheta^{\max}_{f,i,j} + \delta_{f,i,j}, \forall f \in F, (i, j) \in E_f, p \in P \quad (12)$$

When train stops at s station, train acceleration and deceleration operations can occur in many real-world cases. In order to formalize them in train timetabling problem, the occupancy for train f on cell (i, j) is used by introducing extra running times. Constraint (13) links $u_{f,i,j,t}$ with $a_{f,i,j,t}$ and $d_{f,i,j,t}$. Hence, train f contributes a value of 1 to the occupancy on cell (i, j) when it has arrived at cell ($a_{f,i,j,t+g} = 1$) but not departure from it by time t ($d_{f,i,j,t-h} = 0$). Furthermore, the number of trains that occupies the same cell (i, j) is limited by the capacity of cell (i, j) to avoid conflicts in railway stations. Usually, the capacity of cell (i, j) in station is set as 1.

$$u_{f,i,j,t} = a_{f,i,j,t+g} - d_{f,i,j,t-h}, \forall f \in F, (i, j) \in E_f, t = 1,...,T \tag{13}$$

$$\sum_{f:(i,j)\in E_f} u_{f,i,j,t} \le c_{i,j,t}, \ \forall (i, j) \in E_m, t = 1,...,T \tag{14}$$

To better describe the train time-dimension routing in a railway network using cumulative flow variables, we give definitional constraint (15) and constraint (16). Specifically, if train f has arrived or departed on cell (i, j) by time t, $a_{f,i,j,t}$ and $d_{f,i,j,t}$ will have a value of 1 for all later time periods.

$$a_{f,i,j,t} \ge a_{f,i,j,t-1}, \forall f \in F, (i, j) \in E_f, t = 1,...,T \tag{15}$$

$$d_{f,i,j,t} \ge d_{f,i,j,t-1}, \forall f \in F, (i, j) \in E_f, t = 1,...,T \tag{16}$$

Group 2: Passenger assignment constraints

For each passenger pair, the total number of passengers carried by planned trains should be no more than the volume of passenger pair. Besides, for each train scheduled, the total number of passengers that can be assigned to a train is limited by its maximum passenger carrying capacity. Constraint (19) is a mapping constraint between $z_{f,p,i,j}$ and $k_{p,i,j}$.

$$\sum_{f\in F} y_{f,p} \le \eta_p, \forall p \in P \tag{17}$$

$$\sum_{p:p\in P} z_{f,p,i,j} \le \alpha \times C_f, \forall f \in F, (i, j) \in E_c \tag{18}$$

$$z_{f,p,i,j} = y_{f,p} \times k_{p,i,j}, \forall f \in F, p \in P, (i, j) \in E_c \tag{19}$$

Group 3: Mapping constraints between passenger assignment and stopping pattern

The following constraint presents that if train f carry passenger pair p, the train stops at both the origin station and destination station of pair p. It is a constraint between passenger demand and stop planning.

$$y_{f,p} \le r_f^{m_p^o, m_p^s}, \forall f \in F, p \in P \tag{20}$$

Further, if train f stops at the origin station and destination station of passenger pair p ($r_f^{m_p^o, m_p^s} = 1$), the departure time and arrival time for train f departing/arriving at each station is not equal, in order to provide waiting time for trains to stop at a station. Constraint (21) and constraint (22) enforced the waiting time for train f at the origin station and destination station of pair p respectively.

$$\sum_{t\in T}\left[t\times\sum_{i:(i,S_{m_p^o})\in E^s_{S_{m_p^o}}\cap E_f}(d_{f,i,S_{m_p^o},t}-d_{f,i,S_{m_p^o},t-1})-t\times\sum_{j:(O_{m_p^o},j)\in E^o_{O_{m_p^o}}\cap E_f}(a_{f,O_{m_p^o},j,t}-a_{f,O_{m_p^o},j,t-1})\right]\succ \tag{21}$$

$$M\times(r_f^{m_p^o,m_p^s}-1),\forall f\in F$$

$$\sum_{t\in T}\left[t\times\sum_{i:(i,S_{m_p^s})\in E^s_{S_{m_p^s}}\cap E_f}(d_{f,i,S_{m_p^s},t}-d_{f,i,S_{m_p^s},t-1})-t\times\sum_{j:(O_{m_p^s},j)\in E^o_{O_{m_p^s}}\cap E_f}(a_{f,O_{m_p^s},j,t}-a_{f,O_{m_p^s},j,t-1})\right]\succ \tag{22}$$

$$M\times(r_f^{m_p^o,m_p^s}-1),\forall f\in F$$

3.3 Solution approach

Regarding the two objectives of our proposed model, one is to maximize the number of transported passengers to get on the trains from the view of passengers, and another one is to minimize total travel time from the view of train operators. When maximizing the passengers, the train has to stop to meet passenger demand and the dwell time that can increase trains travel time will occur. On the other hand, when reducing train travel time as much as possible, there will be some passengers that fail to take the train service. The problem is that these two aims are associated by different stakeholders with different cost functions (i.e. tickets) and economic interests.

The multi-objective optimization problem has been widely used in railway management. Some researches enforced ε-constraint method to solve it. (Ghoseiri et al. (2014); Yang et al. (2017); D'Ariano et al. (2017)). Meanwhile, many studies adopted weighted-sum method to handle the multi-objective model and generate the Pareto solutions. Burdett et al. (2015) used the weighted-sum method to analyse the absolute capacity in railway networks. Yang et al. (2016) optimized train scheduling problem on high-speed railway through linear weighted methods. D'Ariano et al. (2017), based on weighted-sum method, developed a formulation to integrate train scheduling and railway infrastructure maintenance.

Based on the existing literature of solving multi-objective problem, we apply a formulation that the objective functions are optimized by setting different assigned weights. It is achieved by two input parameters $\boldsymbol{\alpha}_1$ and $\boldsymbol{\alpha}_2$ fixed by the decision maker. And the parameters are constrained: $\boldsymbol{\alpha}_1 \geq 0$, $\boldsymbol{\alpha}_2 \geq 0$, $\boldsymbol{\alpha}_{1+}\ \boldsymbol{\alpha}_2=1$. Therefore, the Pareto solutions can be obtained by varying $\boldsymbol{\alpha}_1$ and $\boldsymbol{\alpha}_2$ to satisfy different demands. In this approach, we first get the results of f_1 and f_2, where $f_1= min\ Z_{time}$, and $f_2= max\ Z_{passsenger}$. Then, set $m_1= \boldsymbol{\alpha}_1/ f_1$, $m_2= \boldsymbol{\alpha}_2/ f_2$, Z= $m_1* Z_{time}$- $m_2* Z_{passsenger}$. Finally, replace the objective function with $N=min\ Z$, restricted by constraint (4)-(22).

4 Case study

In this section, we first describe the dataset in Section 4.1 and then we demonstrate experimental results in 4.2.

We adopt the CPLEX solver version 12.6.3 with default settings to solve the MILP models. The following experiments are all performed on a server with two Intel(R) Xeon(R)

CPU E5-2660 v4 @ 2.00GHz 2.00GHz processors and 512GB RAM.

4.1 Description of the test dataset

To evaluate the effectiveness of the model, we performed numerical experiments on a railway corridor (BJ railway line) with 6 stations of Beijing-Shanghai high-speed railway line, as shown in Fig.2. To determine the route in railway station, we illustrate BJ railway network in appendix. In BJ railway network, only the down direction is considered for simplicity. In this experiment, a total of 7 trains will be taken into consideration and a total of 15 passenger pairs among these stations is included. We assume that the start time of the first train is at 8:03 and the minimum time interval between two consecutive trains is set as 9 min. Besides, the minimum and maximum dwell time at its stop is fixed as 2 min and 5 min respectively to ensure the necessary operation time if the train needs to stop. The maximum passenger carrying coefficient of trains scheduled in this work is all set as 1.2. Detailed train information can be seen from Table 4.

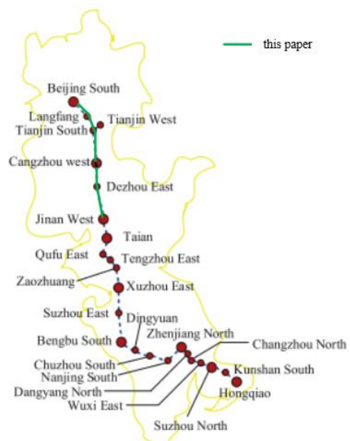

Fig.2: BJ railway line

Table 4: Train origin/destination station and carrying capacity in the test

Number of trains	Train number	Origin station	Destination station	Passenger carrying capacity
7	No.1	BJS	JNW	535
	No.2	BJS	JNW	535
	No.3	BJS	JNW	450
	No.4	TJS	JNW	400
	No.5	BJS	JNW	450
	No.6	TJS	JNW	400
	No.7	BJS	JNW	463

In addition, we give the passenger demand of different origins and destinations on BJ railway line in Table 5 and the total number of passengers that are going to be transported by seven trains is 3619. The passenger data is obtained by the historical passenger flow of one day on BJ railway line.

Table 5: Passenger volume between stations on BJ railway line

Volume	BJS	LF	TJS	CZW	DZE	JNW
BJS	-	801	211	596	241	1118
LF	-	-	141	81	10	68
TJS	-	-	-	92	44	76
CZW	-	-	-	-	13	98
DZE	-	-	-	-	-	29
JNW	-	-	-	-	-	-

4.2 Results of the experiments

In the set of experiments, we vary $\boldsymbol{\alpha}_1$ from 0.1 to 0.9, (by step of 0.1) to observe the set of optimal solutions. Here, we analyse the solution when $\boldsymbol{\alpha}_1 = \boldsymbol{\alpha}_2 = 0.5$ more in detail. We show train timetable, stopping plan and passenger assignment plan of the experiment result in Fig.3, Fig.4 and Table 6 respectively. In Fig.4, the solid dot "●" means that the train has to stop at this station for passengers getting on/off the train or for train preparing for its operation at its origin station.

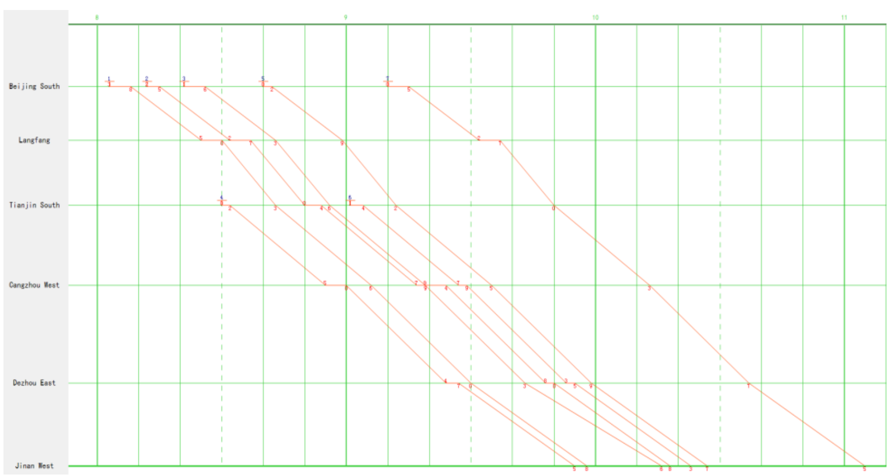

Fig.3: Train timetable for 7 trains on BJ railway line

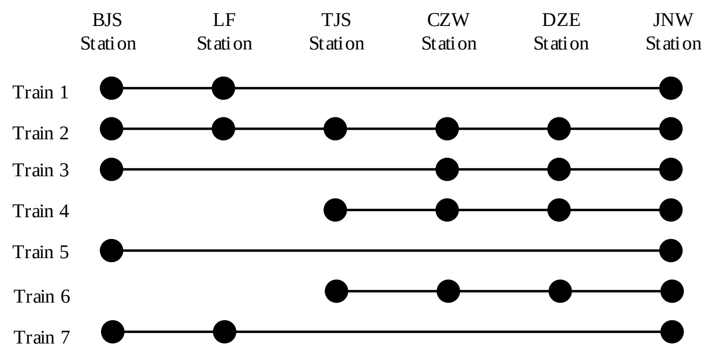

Fig.4: Train stop plan for 7 trains on BJ railway line

Table 6: Passenger assignment plan

Passenger pair	**Number of trains**							**Volume of passenger pair**
	1	2	3	4	5	6	7	
BJS- LF	642	124	0	0	0	0	45	773
BJS- TJS	0	211	0	0	0	0	0	211
BJS- CZW	0	61	289	0	0	0	0	350
BJS- DZE	0	0	241	0	0	0	0	241
BJS- JNW	0	0	0	0	540	0	510	1050
LF- TJS	0	87	0	0	0	0	0	87
LF- CZW	0	81	0	0	0	0	0	81
LF- DZE	0	10	0	0	0	0	0	10
LF- JNW	0	68	0	0	0	0	0	68
TJS- CZW	0	0	0	92	0	0	0	92
TJS- DZE	0	0	0	44	0	0	0	44
TJS- JNW	0	0	0	0	0	76	0	76
CZW-DZE	0	0	0	0	0	13	0	13
CZW-JNW	0	0	0	0	0	98	0	98
DZE - JNW	0	0	0	0	0	29	0	29

Fig.3 illustrates a train timetable of these 7 trains, in which we can obtain the information of train stop and dwell time at each station. Fig.4 details train stop plan for the tested trains on BJ railway line. Then, Table 6 represents the plan that transported passengers are assigned to the seven trains. It shows that when there are 7 trains in the railway network, total 3223 passengers have been delivered already, with 396 passengers not transported yet. To deliver these transported passengers, total travel time is 745 minutes, considering passenger demand.

5 Conclusion and future research

In this paper, we have put passenger demand into consideration when design a train timetable, and tackled the integration problem of train timetabling and train stop planning by using a bi-objective mixed integer linear programming model. Our aim is to compute train timetables (i.e. departure times and arrival times of all train at their stations), stop plan (including the choice of station that train stop and the dwell time at the station) and passenger assignment plan (including the resulting train that passenger get on it and the number of passengers that are carried). In this model, based on the origin station and destination station, we divide them into different passenger pairs in order to link the passenger pair with train stopping plans, and then generate train stop plans and timetable simultaneously. Furthermore, the weighted-sum method is used to find optimal solutions for the proposed bi-objective model. The validity of our model on solving this integration problem is shown by testing it on a part of Beijing-Shanghai high-speed railway line.

For future research, we will focus on the following main extension. Firstly, we will formulate the response of passenger behaviour to existing train service into our mathematical model to maximize the satisfaction of passengers. Train service operation is actually a mutual process. Next, a challenging extension is demand variation, as we know that passenger demand is elastic rather than fixed. Therefore, robust timetable is increasingly needed to adapt to the changing passenger demand (i.e. flexible dwell time in accordance to changing passenger demand). Finally, it is necessary to develop heuristic algorithm and dynamic programming method to improve the solution quality and computational efficiency for the real-time train scheduling problem, as passenger demand enhances the computational complexity.

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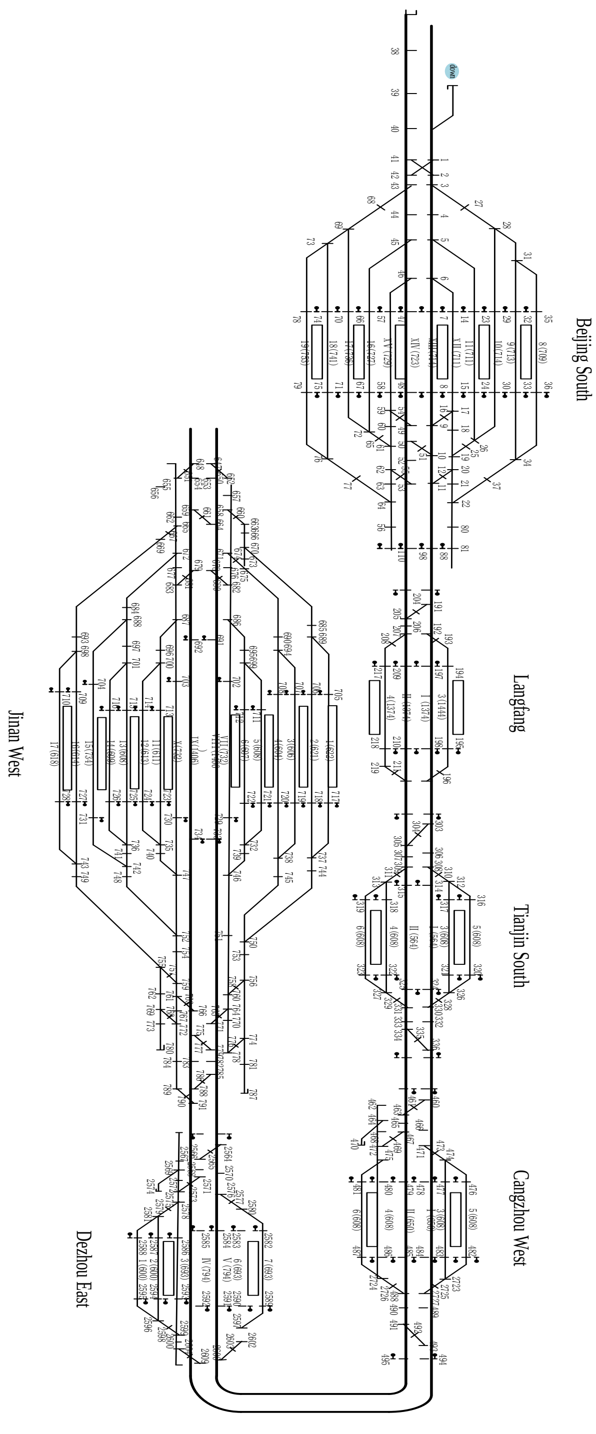
Appendix: BJ railway network in the test


Beijing South
Langfang
Tianjin South
Cangzhou West
Jinan West
Dezhou East

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