Train Rescheduling for an Urban Rail Transit Line under Disruptions

Yihong Chang [a], Ru Niu [a], Yihui Wang [a,1], Xiaojie Luan [b] Marcella Samà [c]
Andrea D'Ariano [c]
[a] State Key Laboratory of Rail Traffic Control and Safety, Beijing Jiaotong University
Beijing 100044, P.R. China
[1] E-mail: yihui.wang@bjtu.edu.cn, Phone: +86 10 5168 8607
[b] Section Transport Engineering and Logistics, Delft University of Technology
2628 CD Delft, the Netherlands
[c] Deparment of Engineering, Roma Tre University
via della vasca navale 79,00146 Rome, Italy

Abstract
Disruptions in urban rail transit systems usually result in serious incidents due to the high density and the less flexibility. In this paper, we propose a novel mathematical model for handling a complete blockage of the double tracks for 5-10 minutes, e.g., lack of power at a station, where no train can pass this area during the disruption. Under this disruption scenario, train services may be delayed or cancelled, some rolling stock may be short-turned at the intermediate stations with either single or double crossovers. To ensure the service quality provided to passengers, the back-up rolling stock inside depots may also be put into operation depending on the consequences of the disruptions. Thus, the number of rolling stock in the depot is considered. We discuss the disruption management problem for urban rail transit systems at a macroscopic level. However, operational constraints for the turnaround operation and for the rolling stock circulation are modelled. A mixed-integer non-linear programming (MINLP) model, which can be transformed into mixed-integer linear programming (MILP) problem, is proposed to minimize the train delays and the number of cancelled train services as well as to ensure a regular service for passengers, while adhering to the departure and arrival constraints, turnaround constraints, service connection constraints, inventory constraints, and other relevant railway constraints. Existing MILP solvers, e.g. CPLEX, are adopted to obtain near-optimal solutions. Numerical experiments are conducted based on real-world data from Beijing subway line 7 to evaluate the effectiveness and efficiency of the proposed model.

Keywords
Urban rail transit, Train rescheduling, Complete blockage, Short-turn, Rolling stock circulation

1 Introduction

Urban rail transit is of crucial importance for transporting commuters and travelers in big cities due to its advantages, such as large capacity, high efficiency, and the ability to provide safe, reliable and fast service. However, with the rapid development of urban rail transit, plenty of new technologies and new equipment have been used, which bring in many uncertain factors that affect the normal operation of urban rail transit systems. Unexpected events, such as infrastructure failures, rolling stock failures and signal malfunctions, happen frequently and have significant impacts on the operation of train services as well as the safety of passengers. When a disruption occurs, it is important that dispatchers quickly present a good solution to reschedule trains so as to recover to the planned schedule as quickly as possible and minimize the inconvenience of passengers. On the one hand, the headway of urban rail transit lines has become smaller and smaller due to the increasing passenger demand, e.g., the headway is 2 minutes during peak hours for most of the metro lines in Beijing. On the other hand, the layout, especially the station layout, of urban rail transit lines is much simpler when compared with mainline. In most of the urban rail transit lines, trains do not overtake or meet each other in general during normal operations due to the limited infrastructure (in terms of tracks and platforms) available. So the disruptions in urban rail transit systems usually cause serious consequences due to the dense traffic and the limited operation flexibility.

The real-time railway traffic management problem has attracted more and more attention in recent years. Advances in scheduling theory have made it possible to handle railway traffic management problem effectively, in which not only the adjustment of running time and dwell time is considered (Ginkel and Schöbel (2007)), but also reordering, rerouting, cancellation of trains and other measures are adopted to change the connection between trains to ensure the quality of service provided to passengers (Corman et al. (2012)). According to Clausen et al. (2010), a disruption is an event or a series of events that render the planned schedules for trains, crews, etc. infeasible. When a disruption occurs, some effective measures which can quickly help the system return to normal operation and reduce the negative impact on passengers should be taken to adjust train schedules in a safe, effective and well-organized way. Jespersen-Groth et al. (2013) split the disruption management process for passenger railway transportation as three main sub-problems: timetable adjustment, rolling stock rescheduling and crew rescheduling. For more information, we direct to the review papers (Cacchiani et al. (2014); Narayanaswami and Rangaraj (2011)).

However, most existing literatures on train rescheduling problems are based on mainline railway systems. Since extra tracks, platforms and multiple routes are available, rescheduling in mainline railway systems usually involves reordering and rerouting strategies. Ghaemi, Cats and Goverde (2017) considered a complete blockage of double tracks for several hours, a MILP model is proposed at the microscopic level to select the optimal short-turning stations and reroute for all the services to continue operating in opposite direction. Louwerse and Huisman (2014) focused on adjusting the timetable of a passenger railway system in case of major disruptions, in which both partial and complete blockage of tracks are formulated. They also investigated the trade-off between delaying and cancelling trains. Zhan et al. (2015) investigated the real-time rescheduling of railway traffic on a high speed railway line in case of a complete blockage of double tracks, in which disrupted trains do not turn around but wait at stations until the disruption ends. Main decisions, including in which stations do trains wait, in which order do they leave after the disruption, and the cancellation of trains, are optimized by a MILP model. Zhan et al. (2016) rescheduled train services on a double-track high speed railway under disruptions, in which one of the double tracks is temporarily unavailable. They assumed that the exact duration of the disruption is not known as a priori but been updated gradually, thus trains are rescheduled according to the latest information of the disruption. Alternative graph models, which combine job shop and alternative graph techniques, are developed in a series of papers (D'Ariano et al. (2008); D'Ariano and Pranzo (2009); D'Ariano, Pranzo and Hansen (2007)) and applied in a real traffic management system ROMA (railway traffic optimization by means of alternative graphs) to resolve conflicts in recent years. In the alternative graph model, the operation of trains is regarded as jobs associated to a prescribed sequence of operations which denote the processing on block sections.

The researches with regard to the rescheduling problems for urban rail transit systems are limited. In comparison to mainline railway systems, the objectives and formulation approaches for urban rail transit systems are slightly different due to their specific characteristics. As an early literature on train rescheduling in urban rail transit systems, Eberlein et al. (1998) tried to improve the headway regulation after a disturbance by using deadheading strategy. A MIP model is constructed to determine which trains should be deadheaded and how many stations should be skipped by certain trains to shorten the average passenger waiting time. Kang et al. (2015) proposed a model to reschedule the last trains in urban rail networks after a disturbance. The objective is to minimize the running time and the dwelling time, and meanwhile to maximize the average transfer redundant time and the network accessibility, as well as to minimize the difference between the planned timetable and the rescheduled one. A genetic algorithm was developed to solve the problem. Gao, Yang and Gao (2017) proposed a mathematical optimization model to calculate real-time automatic rescheduling strategy for an urban rail line by integrating the information of fault handling. However, they just considered small faults and recovered the timetable by modifying dwelling time and running time at a macroscopic level. Xu, Li and Yang (2015) considered an incident on one track of a double-track subway line and formulated an optimization model to calculate the rescheduled timetable with the objective to minimize the total delay time of trains. Crossover tracks are considered to balance the service quality under emergent situations. Taking passengers demand in consider, Gao et al. (2016) proposed an optimization model to reschedule a metro line with an over-crowded and time-dependent passenger flow after a short disruption, in which the pure running time between consecutive stations is fixed and stop-skip strategy is presented in the model to speed up the circulation of trains. An iterative algorithm is used to solve the model.

In this paper, we focus on a complete blockage of the double tracks for 5-10 minutes, e.g., an accident happened and the operator shut down the power supply system at a station, where no train can pass this area during the disruption. Therefore, some rolling stock may be short-turned at the intermediate stations with either single or double crossovers. The rolling stock circulation is also formulated in our disruption management model, where the rolling stock performed a disrupted service can turn around at a turnaround station and take over another service in the opposite direction. To ensure the service quality provided to passengers, the back-up rolling stock inside the depot may also be put into operation depending on the consequences of the disruptions, thus the number of rolling stock in the depot is considered. A mix integer non-linear programming (MINLP) model is proposed to handle the disruption management problem, which can be transformed into mix integer linear programming (MILP) model and then solved by exciting solvers.

The remainder of this paper is organized as follows: Section 2 describes the disruption management problem considered in this paper. The MINLP model for the disruption management problem in urban rail transit systems in term of a complete blockage of the double tracks for 5-10 minutes is proposed in Section 3. In Section 4, the formulated optimization model is transformed into an MILP problem. Experimental results based on the real-world

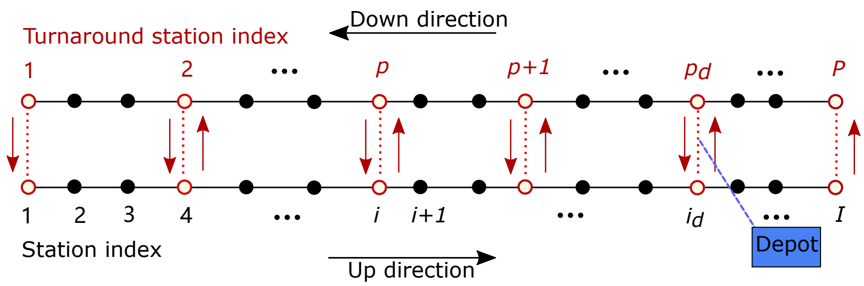

Figure 1: The layout of an urban rail transit line

data from Beijing subway line 7 are given in Section 5. The paper ends with conclusions in Section 6.

2 Problem Description

2.1 Operation of An Urban Rail Transit Line

An urban rail transit line mainly consists of stations, turnaround stations, open tracks, crossovers and depots. Figure 1 shows the layout of an urban rail transit line, which has I stations, P turnaround stations and a depot linked to turnaround station pd. One station is separated into two platforms. Open tracks are separated into two directions and each track is designed for rolling stock to operate in only one direction during normal operation but can be used in opposite direction under emergent situations. The crossovers connecting two parallel open tracks at turnaround stations can be used by rolling stock to turn around and take over another train service in the opposite direction.

This paper considers the disruption management problem for urban rail transit systems at a macroscopic level, however, the sufficient details for the turnaround operation and the rolling stock circulations are involved. In this paper, “train service” is defined as a rolling stock operating in one direction from its origin to destination. In detail, we use “service” to represent a rolling stock’s operation from station 1 to station I in the up direction or from station I to station 1 in the down direction. Once a rolling stock turns around using crossovers at turnaround stations, the corresponding “service” ends, while the rolling stock keeps circulating in the urban rail transit line. Rolling stock is stored in the depots when out of usage and the number of rolling stock in depots is limited.

2.2 Dispatching Measures

This paper considers the rescheduling problem in case of an incident of the railway infrastructure. Due to the disruption, the double tracks in a railway segment are out of order for 5-10 minutes and no train services can pass this area during the time period. The dispatching measures used to ensure the capacity of urban rail transit systems and quickly recover from the disruption include:

- Adjustments of running times and dwell times for train services;

- Rolling stock performed a disrupted service in one direction can turn around at the turnaround stations and take over another service in opposite direction;
- The back-up rolling stock inside depots can be put into operation when necessary, e.g., performing a train service that cannot be executed by the predefined rolling stock;

2.3 Assumptions

In order to formulate the disruption management model for the complete blockage scenario, we make following assumptions according to the special characteristics of urban rail transit systems:

- Rolling stock do not meet or overtake each other during operation due to the limited infrastructure (in terms of tracks and platforms) available;
- Connection between train services will change when rolling stock turning around at intermediate stations, cancelling train services and using the back-up rolling stock inside depots;
- Stopping in an interval is not allowed to avoid panicking passengers;
- Since the potential accumulation of rolling stock on the line due to the disruption, adding of new train services is not avaiable;
- Train services can depart before the departure time specified in the timetable, since the urban rail transit is more focus on the headway between train services and the passengers do not know the exact departure times;

3 Mathematical Formulation

3.1 Parameters and Variables

Parameters and decision variables adopted in the mathematical model are listed in Table 1 and Table 2 for the convenience of formulating the disruption management problem.

3.2 Objective Function

The objective function of the disruption management problem involves three parts:

- Minimize the train delay times at all visited stations;
- Minimize the deviation of the current train operations and the predefined timetable in terms of the number of cancellation services and intermediate turnaround services;
- Minimize the headway deviations between train services to ensure a regular operation and minimize passengers' waiting time;

Table 1: General subscripts, sets, input parameters

Symbol	Description
$\mathbf{I}$	set of stations, I is the last station in the line
$\mathbf{P}$	set of turnaround stations, P is the last turnaround station in the line
$\mathbf{F}$	set of train services in the up direction
$\mathbf{G}$	set of train services in the down direction
i	station index, $i \in \mathbf{I}$, i_d is the station corresponding to turnaround station p_d
p	turnaround station index, $p \in \mathbf{P}$, p_d is the turnaround station connected with depot
f	train service index in the up direction, $f \in \mathbf{F}$
g	train service index in the down direction, $g \in \mathbf{G}$
$\bar{x}^{\mathrm{up}}_{f,p,p+1}$	given binary value, $\bar{x}^{\mathrm{up}}_{f,p,p+1} = 1$ if service f in the up direction operates between turnaround station p and $p+1$ for $p \in \{1, 2, \ldots, P-1\}$ in the timetable
$\bar{y}^{\mathrm{up}}_{f,i,i+1}$	given binary value, $y^{\mathrm{up}}_{f,i,i+1} = 1$ if service f in the up direction operates between station i and $i+1$ for $i \in \{1, 2, \ldots, I-1\}$ in the timetable
$\bar{x}^{\mathrm{dn}}_{g,p,p-1}$	given binary value, $x^{\mathrm{dn}}_{g,p,p-1} = 1$ if service g in the down direction operates between turnaround station p and $p-1$ for $p \in \{2, 3, \ldots, P\}$ in the timetable
$\bar{y}^{\mathrm{dn}}_{g,i,i-1}$	given binary value, $y^{\mathrm{dn}}_{g,i,i-1} = 1$ if service g in the down direction operates between station i and $i-1$ for $i \in \{2, 3, \ldots, I\}$ in the timetable
$\bar{\beta}^{\mathrm{up}}_{f,g,p}$	binary variable, $\bar{\beta}^{\mathrm{up}}_{f,g,p} = 1$ if service f in the up direction is connected with service g in the down direction at turnaround station p in the timetable
$\bar{\beta}^{\mathrm{dn}}_{g,f,p}$	binary variable, $\bar{\beta}^{\mathrm{dn}}_{g,f,p} = 1$ if service g in the down direction is connected with service f in the up direction at turnaround station p in the timetable
$\bar{a}^{\mathrm{up}}_{f,i}/\bar{d}^{\mathrm{up}}_{f,i}$	planned arrival/departure time of service f at station i in the up direction in the timetable
$\bar{a}^{\mathrm{dn}}_{g,i}/\bar{d}^{\mathrm{dn}}_{g,i}$	planned arrival/departure time of service g at station i in the down direction in the timetable
h_{min}	minimum headway between two successive train services in the same direction in the timetable
$w^{\mathrm{up,max}}_{i}/w^{\mathrm{up,min}}_{i}$	maximum/minimum dwell time of train services at station i in the up direction
$w^{\mathrm{dn,max}}_{i}/w^{\mathrm{dn,min}}_{i}$	maximum/minimum dwell time of train services at station i in the down direction
$r^{\mathrm{up,max}}_{i,i+1}/r^{\mathrm{up,min}}_{i,i+1}$	maximum/minimum running time between station i and station $i+1$ in the up direction
$r^{\mathrm{dn,max}}_{i,i-1}/r^{\mathrm{dn,min}}_{i,i-1}$	maximum/minimum running time between station i and station $i-1$ in the down direction
$t^{\mathrm{turn,max}}_{p}/t^{\mathrm{turn,min}}_{p}$	maximum/minimum turnaround time at turnaround station p
w_{cr}	extra waiting time at turnaround stations needed to let all the passengers alight from the train
N_{p_d}	number of rolling stock in the depot before the disruption, $N_{p_d} \geq 1$
t_d	the start time point for disruption

Table 2: Decision variables

Symbol	Description
$x^{\mathrm{up}}_{f,p,p+1}$	binary variable, $x^{\mathrm{up}}_{f,p,p+1} = 1$ if service f in the up direction operates between turnaround station p and $p+1$ for $p \in \{1, 2, \ldots, P-1\}$
$y^{\mathrm{up}}_{f,i,i+1}$	binary variable, $y^{\mathrm{up}}_{f,i,i+1} = 1$ if service f in the up direction operates between station i and $i+1$ for $i \in \{1, 2, \ldots, I-1\}$
$x^{\mathrm{dn}}_{g,p,p-1}$	binary variable, $x^{\mathrm{dn}}_{g,p,p-1} = 1$ if service g in the down direction operates between turnaround station p and $p-1$ for $p \in \{2, 3, \ldots, P\}$
$y^{\mathrm{dn}}_{g,i,i-1}$	binary variable, $y^{\mathrm{dn}}_{g,i,i-1} = 1$ if service g in the down direction operates between station i and $i-1$ for $i \in \{2, 3, \ldots, I\}$
$\beta^{\mathrm{up}}_{f,g,p}$	binary variable, $\beta^{\mathrm{up}}_{f,g,p} = 1$ if service f in the up direction is connected with service g in the down direction at turnaround station p
$\beta^{\mathrm{dn}}_{g,f,p}$	binary variable, $\beta^{\mathrm{dn}}_{g,f,p} = 1$ if service g in the down direction is connected with service f in the up direction at turnaround station p
$a^{\mathrm{up}}_{f,i}/d^{\mathrm{up}}_{f,i}$	arrival/departure time of service f at station i in the up direction
$a^{\mathrm{dn}}_{g,i}/d^{\mathrm{dn}}_{g,i}$	arrival/departure time of service g at station i in the down direction
$w^{\mathrm{up}}_{f,i}$	dwell time of service f at station i in the up direction
$w^{\mathrm{dn}}_{g,i}$	dwell time of service g at station i in the down direction
$r^{\mathrm{up}}_{f,i,i+1}$	running time of service f between station i and station $i+1$ in the up direction
$r^{\mathrm{dn}}_{g,i,i-1}$	running time of service g between station i and station $i-1$ in the down direction
$t^{\mathrm{turn}}_{f,p}/t^{\mathrm{turn}}_{g,p}$	turnaround time of service f/g at turnaround station p
$\alpha^{\mathrm{up}}_{f,p_d}$	binary variable,$\alpha^{\mathrm{up}}_{f,p_d} = 1$ if the rolling stock performing service f in the up direction go back to the depot at turnaround station p_d
$\alpha^{\mathrm{dn}}_{g,p_d}$	binary variable,$\alpha^{\mathrm{down}}_{g,p_d} = 1$ if the rolling stock performing service g in the down direction go back to the depot at turnaround station p_d
$\theta^{\mathrm{up}}_{f,p_d}$	binary variable,$\theta^{\mathrm{up}}_{f,p_d} = 1$ if the rolling stock performing service f in the up direction come out from the depot at turnaround station p_d
$\theta^{\mathrm{dn}}_{g,p_d}$	binary variable,$\theta^{\mathrm{down}}_{g,p_d} = 1$ if the rolling stock performing service g in the down direction come out from the depot at turnaround station p_d
$N^{\mathrm{in}}_{f,p_d}/N^{\mathrm{in}}_{g,p_d}$	total number of rolling stock going back to depot before the departure of train service f/g at turnaround station p_d
$N^{\mathrm{out}}_{f,p_d}/N^{\mathrm{out}}_{g,p_d}$	total number of rolling stock coming out from depot before the departure of train service f/g at turnaround station p_d

Thus, the objective function can be formulated as

$$
\begin{aligned}
Z = \min \Bigg(w_1 * \Bigg(& \sum_{f\in\mathbf{F}} \sum_{i\in\mathbf{I}, i\neq 1} y^{\mathrm{up}}_{f,i-1,i} \Big(\max\big(0, (d^{\mathrm{up}}_{f,i} - \bar{d}^{\mathrm{up}}_{f,i})\big)\Big) \\
& + \sum_{g\in\mathbf{G}} \sum_{i\in\mathbf{I}, i\neq I} y^{\mathrm{dn}}_{g,i+1,i} \Big(\max\big(0, (d^{\mathrm{dn}}_{g,i} - \bar{d}^{\mathrm{dn}}_{g,i})\big)\Big)\Bigg) \\
+ w_2 * \Bigg(& \sum_{f\in\mathbf{F}} \sum_{p\in\mathbf{P}, p\neq P} \big(\bar{x}^{\mathrm{up}}_{f,p,p+1} - x^{\mathrm{up}}_{f,p,p+1}\big) + \sum_{g\in\mathbf{G}} \sum_{p\in\mathbf{P}, p\neq 1} \big(\bar{x}^{\mathrm{dn}}_{g,p,p-1} - x^{\mathrm{dn}}_{g,p,p-1}\big)\Bigg) \\
+ w_3 * \Bigg(& \sum_{f\in\mathbf{F}, f\neq 1, f\neq F} \sum_{i\in\mathbf{I}, i\neq 1} \big(y^{\mathrm{up}}_{f-1,i-1,i} y^{\mathrm{up}}_{f,i-1,i} y^{\mathrm{up}}_{f+1,i-1,i} (d^{\mathrm{up}}_{f+1,i} + d^{\mathrm{up}}_{f-1,i} - 2d^{\mathrm{up}}_{f,i})\big) \\
& + \sum_{g\in\mathbf{G}, g\neq 1, g\neq G} \sum_{i\in\mathbf{I}, i\neq I} \big(y^{\mathrm{dn}}_{g-1,i+1,i} y^{\mathrm{dn}}_{g,i+1,i} y^{\mathrm{dn}}_{g+1,i+1,i} (d^{\mathrm{dn}}_{g+1,i} + d^{\mathrm{dn}}_{g-1,i} - 2d^{\mathrm{dn}}_{g,i})\big)\Bigg)\Bigg)
\end{aligned}
\tag{1}
$$

3.3 Operational Constraints

Departure and Arrival Times

As shown in Figure 2, in the disruption scenario considered in this paper, train service f in up direction can operate continuously to the next station or turn around to connect with train service g in down direction at station i (corresponding to turnaround station p). Thus, the calculation of departure times can be analysed into two cases according to the layout of station i:

- Normal Stations

  In this case, service f can only depart from station i and operate to station $i + 1$, the departure time of service f at station i can be calculated by

$$d^{\mathrm{up}}_{f,i} = y^{\mathrm{up}}_{f,i-1,i}(a^{\mathrm{up}}_{f,i} + w^{\mathrm{up}}_{f,i}), \forall f \in \mathbf{F}, i \in \{2, 3, \ldots, I\}, \tag{2}$$

  where $w^{\mathrm{up}}_{f,i}$ denote the dwell time of service f at station i, which satisfies the following constraint

$$w^{\mathrm{up,min}}_{i} \leq w^{\mathrm{up}}_{f,i} \leq w^{\mathrm{up,max}}_{i}, \forall f \in \mathbf{F}, i \in \mathbf{I}. \tag{3}$$

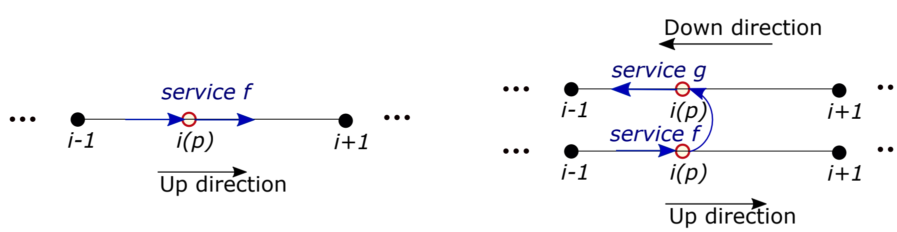

Figure 2: Departure options of train service f at station i

- Turnaround Stations
  If service f in the up direction turns around at station i (corresponding to turnaround station p) and connects with service g in the down direction, i.e., $\beta^{\mathrm{up}}_{f,g,p} = 1$, then we have

$$d^{\mathrm{up}}_{f,i} = y^{\mathrm{up}}_{f,i-1,i}(a^{\mathrm{up}}_{f,i} + w^{\mathrm{up}}_{f,i} + \beta^{\mathrm{up}}_{f,g,p}w_{\mathrm{cr}}), \forall f \in \mathbf{F}, g \in \mathbf{G}, p \in \mathbf{P}, i \in \{2, 3, \ldots, I\}, \tag{4}$$

  where w_{cr} is the extra time needed to let all the passengers alight from the train.

The calculation of arrival times can also be analysed into two cases:

- Normal Stations
  The arrival time of service f at station i from station $i-1$ can be calculated by

$$a^{\mathrm{up}}_{f,i} = y^{\mathrm{up}}_{f,i-1,i}(d^{\mathrm{up}}_{f,i-1} + r^{\mathrm{up}}_{f,i-1,i}), \forall f \in \mathbf{F}, i \in \{2, 3, \ldots, I\}, \tag{5}$$

  where $r^{\mathrm{up}}_{f,i-1,i}$ denotes the running time of service f between station $i-1$ and i, which satisfies the following constraint

$$r^{\mathrm{up,min}}_{i-1,i} \leq r^{\mathrm{up}}_{f,i-1,i} \leq r^{\mathrm{up,max}}_{i-1,i}, \forall f \in \mathbf{F}, i \in \{2, 3, \ldots, I\}. \tag{6}$$

- Turnaround Stations
  If train service f is taken over by the rolling stock performed train service g in the down direction, which turns around at turnaround station i (corresponding to turnaround station p), i.e., $\beta^{\mathrm{dn}}_{g,f,p}$=1,the arrival time of service f at station i in up direction can be calculated by

$$a^{\mathrm{up}}_{f,i} = (1-y^{\mathrm{up}}_{f,i-1,i})y^{\mathrm{dn}}_{g,i+1,i}\beta^{\mathrm{dn}}_{g,f,p}(d^{\mathrm{dn}}_{g,i}+t^{\mathrm{turn}}_{g,p}), \forall f \in \mathbf{F}, g \in \mathbf{G}, p \in \mathbf{P}, i \in \{2, 3, \ldots, I\}, \tag{7}$$

  where $t^{\mathrm{turn}}_{g,p}$ denotes the turnaround time of service g at turnaround station p, which satisfies the following constraint

$$t^{\mathrm{turn,min}}_{p} \leq t^{\mathrm{turn}}_{g,p} \leq t^{\mathrm{turn,max}}_{p}, \forall f \in \mathbf{F}, p \in \mathbf{P}. \tag{8}$$

  When combining equation (5) and equation (7), the arrival time of service f at station i in the up direction can be calculated by

$$a^{\mathrm{up}}_{f,i} = \beta^{\mathrm{dn}}_{g,f,p}(1 - y^{\mathrm{up}}_{f,i-1,i})y^{\mathrm{dn}}_{g,i+1,i}(d^{\mathrm{dn}}_{g,i} + t^{\mathrm{turn}}_{g,p}) + (1 - \beta^{\mathrm{dn}}_{g,f,p})y^{\mathrm{up}}_{f,i-1,i}(d^{\mathrm{up}}_{f,i-1} + r^{\mathrm{up}}_{f,i-1,i}),$$
$$\forall f \in \mathbf{F}, g \in \mathbf{G}, p \in \mathbf{P}, i \in \{2, 3, \ldots, I-1\}. \tag{9}$$

Similarly, the departure time and arrival time for train service g at station i can be calculated in two cases as well.

Headway Constraints
In the disruption scenario, the headway between train services should be larger than the minimum headway determined by the train control systems. Therefore, we have the headway between service $f-1$ and f

$$\begin{aligned} y^{\mathrm{up}}_{f-1,i-1,i}y^{\mathrm{up}}_{f,i-1,i}(d^{\mathrm{up}}_{f,i} - d^{\mathrm{up}}_{f-1,i}) \geq y^{\mathrm{up}}_{f-1,i-1,i}y^{\mathrm{up}}_{f,i-1,i}h_{\mathrm{min}}, \\ \forall f \in \{2, 3, \ldots, F\}, i \in \{2, 3, \ldots, I\}. \end{aligned} \tag{10}$$

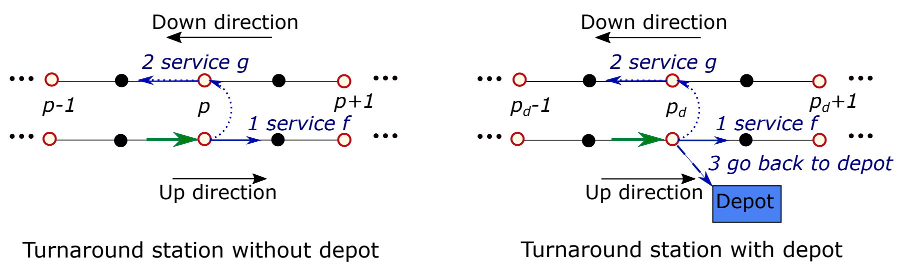

Figure 3: Departure directions of train service at turnaround stations

If $y^{\text{up}}_{f,i-1,i} = 0$ or $y^{\text{up}}_{f-1,i-1,i} = 0$ (one of the two consecutive train services was cancelled or turn around at intermediate stations), the constraint above is satisfied automatically. However, if train service f at station i is canceled, i.e., $y^{\text{up}}_{f,i-1,i} = 0$, then we need to calculate the headway using service $f+1$ and $f-1$ as follow:

$$y^{\text{up}}_{f-1,i-1,i} y^{\text{up}}_{f+1,i-1,i}(1 - y^{\text{up}}_{f,i-1,i})(d^{\text{up}}_{f+1,i} - d^{\text{up}}_{f-1,i}) \geq y^{\text{up}}_{f-1,i-1,i} y^{\text{up}}_{f+1,i-1,i}(1 - y^{\text{up}}_{f,i-1,i}) h_{\min}, \quad \forall f \in \{2, 3, \ldots, F-1\}, i \in \{2, 3, \ldots, I\}. \tag{11}$$

Service Connection Constraints

The rolling stock performed train service f in the up direction can turn around at turnaround stations and take over another service in the opposite direction in the disruption scenario. However, train service f can be connected with at most one train service in the down direction, i.e.,

$$\sum_g \sum_p \beta^{\text{up}}_{f,g,p} \leq 1, \forall f \in \mathbf{F}, g \in \mathbf{G}, p \in \mathbf{P}, \tag{12}$$

where $\beta^{\text{up}}_{f,g,p}$ denotes the connection between service f in the up direction and service g in the down direction.

Similarly, we have

$$\sum_f \sum_p \beta^{\text{dn}}_{g,f,p} \leq 1, \forall f \in \mathbf{F}, g \in \mathbf{G}, p \in \mathbf{P}. \tag{13}$$

to ensure train service g is connected with at most one train service in the up direction.

As shown in Figure 3, train service f in the up direction has more than one departure option at turnaround stations, especially turnaround stations with depot. Therefore, services connection constraints should be discussed separately according to different turnaround stations.

- Turnaround Stations without Depot
  In this case, service f in up direction at turnaround station p has two options: operate continuously to next station in the up direction or turn around at turnaround station p and connect to service g in the down direction. The relationship between $\beta^{\text{up}}_{f,g,p}$ and $x^{\text{up}}_{f,p,p+1}$ can be formulated as follow

$$\beta^{\text{up}}_{f,g,p} + x^{\text{up}}_{f,p,p+1} = x^{\text{up}}_{f,p-1,p}, \forall f \in \mathbf{F}, g \in \mathbf{G}, p \in \{2, 3, \ldots, P-1\}. \tag{14}$$

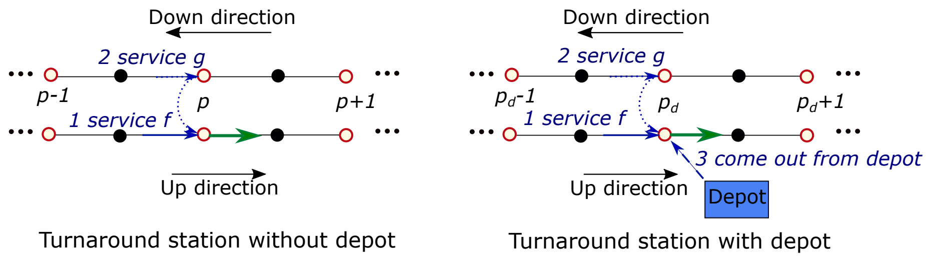

Figure 4: Sources of train service at turnaround stations

- Turnaround Stations with Depot
  Except the two options described above, service f can also go back to depot directly at turnaround station p_d which connects with depot, the equation can be proposed as

$$\beta^{\mathrm{up}}_{f,g,p_d} + x^{\mathrm{up}}_{f,p_d,p_d+1} + \alpha^{\mathrm{up}}_{f,p_d} = x^{\mathrm{up}}_{f,p_d-1,p_d}, \forall f \in \mathbf{F}, g \in \mathbf{G}, \tag{15}$$

  where $\alpha^{\mathrm{up}}_{f,p_d}$ denotes whether service f goes back to depot at turnaround station p_d.

At the same time, train service f departs from turnaround station p in the up direction also has different sources according to the layout of turnaround stations as shown in Figure 4:

- Turnaround Stations without Depot
  In this case, service f departs from turnaround station p has two sources: come from station $p-1$ in the up direction or connect with service g in the down direction, so we have

$$\beta^{\mathrm{dn}}_{g,f,p} + x^{\mathrm{up}}_{f,p-1,p} = x^{\mathrm{up}}_{f,p,p+1}, \forall f \in \mathbf{F}, g \in \mathbf{G}, p \in \{2, 3, \ldots, P-1\}. \tag{16}$$

- Turnaround Stations with Depot
  Except the two sources described above, service f departs from turnaround station p_d in the up direction may also come from depot directly, the equation can be proposed as

$$\beta^{\mathrm{dn}}_{g,f,p_d} + x^{\mathrm{up}}_{f,p_d-1,p_d} + \theta^{\mathrm{up}}_{f,p_d} = x^{\mathrm{up}}_{f,p_d,p_d+1}, \forall f \in \mathbf{F}, g \in \mathbf{G}, \tag{17}$$

  where $\theta^{\mathrm{up}}_{f,p_d}$ denotes whether service f is come out from the depot at turnaround station p_d.

Since the adding of new train services is not included in the this model, we have

$$x^{\mathrm{up}}_{f,p,p+1} \leq \bar{x}^{\mathrm{up}}_{f,p,p+1}, \forall f \in \mathbf{F}, p \in \{1, 2, \ldots, P-1\}. \tag{18}$$

Similarly constraints about service connection of train service g in the down direction can be presented.

Inventory Constraints

For turnaround stations connected with the depot, train services can be performed by rolling stock coming out from the depot directly and the rolling stock performed a service can also go back to the depot. However, the number of back-up rolling stock inside depots for urban rail transit lines is fixed. We need to consider the availability of rolling stock when adjusting the connection between train services at turnaround stations with depot.

When a rolling stock inside the depot is required to perform train service f, i.e., $\theta^{\mathrm{up}}_{f,p_d} = 1$, the number of rolling stock going back to and coming out from the depot before train service f should satisfy inventory constraints

$$\theta^{\mathrm{up}}_{f,p_d}(N^{\mathrm{out}}_{f,p_d} - N^{\mathrm{in}}_{f,p_d}) \leq N_{p_d} - 1, \forall f \in \mathbf{F}, \tag{19}$$

where N_{p_d} is the number of rolling stock in the depot before the disruption, N^{out}_{f,p_d} and N^{in}_{f,p_d} denote the total number of rolling stock coming out from and going back to the depot before the departure of train service f at turnaround station p_d after the disruption happened, which can be calculated by

$$N^{\mathrm{out}}_{f,p_d} = \sum_{f'} \epsilon^{\mathrm{up}}_{f',p_d}\delta^{\mathrm{up}}_{f',f,p_d}\theta^{\mathrm{up}}_{f',p_d} + \sum_{g'} \epsilon^{\mathrm{dn}}_{g',p_d}\delta^{\mathrm{dn}}_{g',f,p_d}\theta^{\mathrm{dn}}_{g',p_d}, \forall f \in \mathbf{F}, f' \in \mathbf{F}, g' \in \mathbf{G}, \tag{20}$$

$$N^{\mathrm{in}}_{f,p_d} = \sum_{f'} \lambda^{\mathrm{up}}_{f',p_d}\eta^{\mathrm{up}}_{f',f,p_d}\alpha^{\mathrm{up}}_{f',p_d} + \sum_{g'} \lambda^{\mathrm{dn}}_{g',p_d}\eta^{\mathrm{dn}}_{g',f,p_d}\alpha^{\mathrm{dn}}_{g',p_d}, \forall f \in \mathbf{F}, f' \in \mathbf{F}, g' \in \mathbf{G}. \tag{21}$$

A set of binary variables is presented to describe the sequence between train services, in which $\delta^{\mathrm{up}}_{f',f,p_d} = 1$, means service f' in the up direction departs from turnaround station p_d (corresponding to station i_d) before the departure of service f, i.e.,

$$d^{\mathrm{up}}_{f,i_d} - d^{\mathrm{up}}_{f',i_d} \geq 0, \forall f \in \mathbf{F}, f' \in \mathbf{F}, \tag{22}$$

$\delta^{\mathrm{dn}}_{g',f,p_d} = 1$, means service g' in the down direction departs from turnaround station p_d before the departure of service f, i.e.,

$$d^{\mathrm{up}}_{f,i_d} - d^{\mathrm{dn}}_{g',i_d} \geq 0, \forall f \in \mathbf{F}, g' \in \mathbf{G}, \tag{23}$$

$\eta^{\mathrm{up}}_{f',f,p_d} = 1$, means service f' in the up direction arrives at turnaround station p_d before the departure of service f, i.e.,

$$d^{\mathrm{up}}_{f,i_d} - a^{\mathrm{up}}_{f',i_d} \geq 0, \forall f \in \mathbf{F}, f' \in \mathbf{F}, \tag{24}$$

$\eta^{\mathrm{dn}}_{g',f,p_d} = 1$, means service g' in the down direction arrives at turnaround station p_d before the departure of service f, i.e.,

$$d^{\mathrm{up}}_{f,i_d} - a^{\mathrm{dn}}_{g',i_d} \geq 0, \forall f \in \mathbf{F}, g' \in \mathbf{G}, \tag{25}$$

Moreover, a set of binary variables is considered to identify if the train service arrives at or depart from turnaround station p_d after the disruption happened, in which $\epsilon^{\mathrm{up}}_{f',p_d} = 1$ means service f' in the up direction departs from turnaround station p_d after the disruption happened, i.e.,

$$d^{\mathrm{up}}_{f',i_d} - t_d \geq 0, \forall f' \in \mathbf{F}, \tag{26}$$

$\epsilon^{\mathrm{dn}}_{g',p_d} = 1$, means service g' in the down direction departs from turnaround station p_d after the disruption happened, i.e.,

$$d^{\mathrm{dn}}_{g',i_d} - t_d \geq 0, \forall g' \in \mathbf{G}, \tag{27}$$

$\lambda^{\mathrm{up}}_{f',p_d}$, means service f' in the up direction arrives at turnaround station p_d after the disruption happened, i.e.,

$$a^{\mathrm{up}}_{f',i_d} - t_d \geq 0, \forall f' \in \mathbf{F}, \tag{28}$$

$\lambda^{\mathrm{dn}}_{g',p_d}$, means service g' in the down direction arrives at turnaround station p_d after the disruption happened, i.e.,

$$a^{\mathrm{dn}}_{g',i_d} - t_d \geq 0, \forall g' \in \mathbf{G}, \tag{29}$$

Similarly, when a rolling stock inside the depot is required to perform train service g, i.e., $\theta^{\mathrm{dn}}_{g,p_d} = 1$, the inventory constraints can also be proposed.

4 MILP Solution

The mixed-integer nonlinear programming (MINLP) model which is formulated in Section 3 can be transformed into a mixed-integer linear programming (MILP) problem according to the transformation properties introduced in (Bemporad et al. (1999)).

- Property I: Consider a real-valued variable $f(x)$ and a logical variable $\theta \in [0, 1]$. if we let $M = f(x)_{max}$, $m = f(x)_{min}$, the product term $\theta f(x)$ can be replaced by an auxiliary real variable $z = \theta f(x)$, where $z = \theta f(x)$ is equivalent to

$$\begin{cases} z \leq M\theta, \\ z \geq m\theta, \\ z \leq f(x) - m(1-\theta), \\ z \geq f(x) - M(1-\theta). \end{cases} \tag{30}$$

- Property II: Consider two logical variables $\theta_1 \in [0, 1]$ and $\theta_2 \in [0, 1]$. the product term $\theta_1\theta_2$ can be replaced by a logical variables $\theta_3 \in [0, 1]$, where $\theta_3 = \theta_1\theta_2$ is equivalent to

$$\begin{cases} -\theta_1 + \theta_3 \leq 0, \\ -\theta_2 + \theta_3 \leq 0, \\ \theta_1 + \theta_2 - \theta_3 \leq 1. \end{cases} \tag{31}$$

- Property III: Consider a real-valued variable $f(x) \leq 0$, and let $M = f(x)_{max}$, $m = f(x)_{min}$. If we introduce a logical variable $\theta \in [0, 1]$, it can be verified that $[f(x) \leq 0] \longleftrightarrow [\theta = 1]$ is true if

$$\begin{cases} f(x) \leq M(1-\theta), \\ f(x) \geq \epsilon + (m-\epsilon)\theta. \end{cases} \tag{32}$$

Through property I the nonlinear constraints (4) and (9) can be transformed by using auxiliary real variables. Constraints (20) and (21) can be transformed by adding another logical variables according to property II. Constraints (9), (10) and (11) can be transformed by combining property I and II. The statements (22) to (29) can be transformed into logical dynamic constraints through property III.

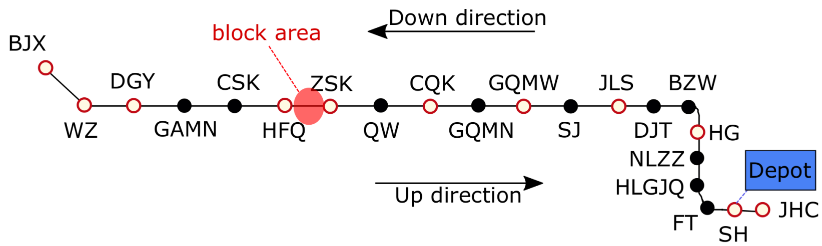

Figure 5: Layout of Beijing subway line 7

Table 3: Detailed status of train services

Number of train services	Direction	Status
f1	up	dwelling at DJT
f2	up	running from GQMN to GQMW
f3	up	running from QW to CQK
f4	up	running from CSK to HFQ
f5	up	running from DGY to GQMN
f6	up	dwelling at BJX
g1	down	running from QW to ZSK
g2	down	running from GQMN to CQK
g3	down	running from JLS to SJ
g4	down	running from HG to BZW
g5	down	running from FT to HLGJQ

5 Case Study

In this section, the experimental results of the proposed model is demonstrated based on the data from Beijing subway line 7 and IBM CPLEX 12.8 is used as the solver for the MILP problem.

The layout of Beijing subway line 7 is shown in Figure 5, which is 23.7 km long with 21 stations and one depot connected with SH station. Stations denoted by red circles are turnaround stations which provide single or double crossovers for rolling stock to turn around and take over another service in opposite direction, while stations denoted by black dots are normal stations where train services can only run directly to next station in the same direction. Train services running from BJX to JHC are in up direction while services running from JHC to BJX are in down direction. In this case study, we consider the time period from 11:00 am to 12:00 am, 10 services in each direction, which departure from its origin during this period are considered. The track blockage between HFQ and ZSK starts at 11:29 am and ends at 11:39 am, during which no trains can pass the block area. At 11:29 am, the time point which the disruption occurs, 6 services in the up direction as well as 5 services in the down direction considered in this case study are operating on the line, the detailed status are given in table 3. The maximum and minimum running times in each section are defined by adding extra 10s or reducing 10s based on the predefined timetable. The minimum dwell times at each station are defined as 20s to let passengers get on or alight from the trains while the maximum dwell times are defined by adding extra 120s in

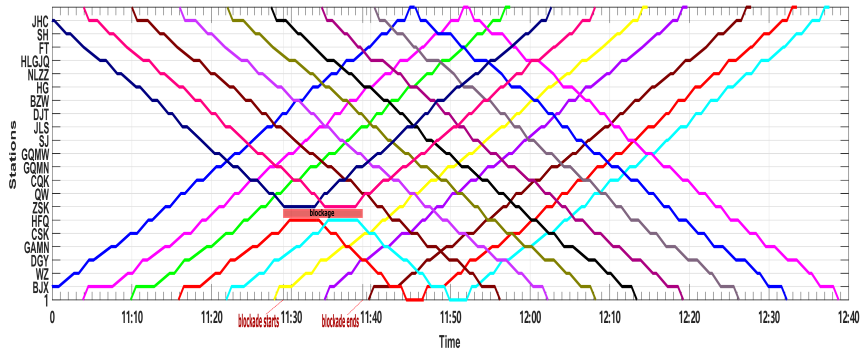

Figure 6: The rescheduled timetable

case of holding the train in station if necessary. Furthermore, the turnaround time should be between 120s and 600s. The headway of two consecutive train services should be more than 240s. The number of rolling stock in the depot is taken as 2 at the beginning of disruption. The extra waiting time at turnaround stations is 60s. The weights in the objective function are set to $w_1 = 2$, $w_2 = 100$ and $w_3 = 1$ based on several experiments.

The rescheduled timetable for train services in this disruption scenario is shown in Fig-

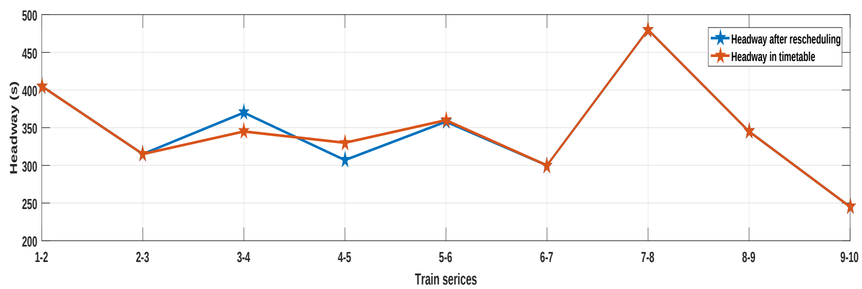

Figure 7: The headway in up direction

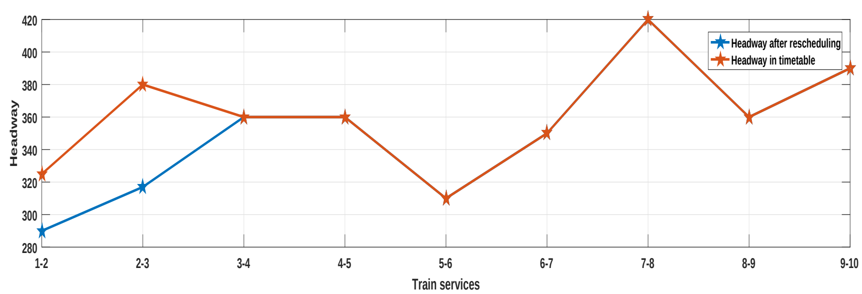

Figure 8: The headway in down direction

ure 6, in which different colors denoted train services performed be different rolling stock and the track blockage is denoted by a red rectangular inserted between HFQ and ZSK, which appears at 11:29 am and disappears at 11:39 am. It can be observed that, two train services ($f3$ and $f4$) in the up direction turn around at turnaround station HFQ and connect to train services ($g1$ and $g2$) in the down direction, accordingly, train services ($g1$ and $g2$) in the down direction turn around at turnaround station ZSK and connect to train services ($f3$ and $f4$) in the up direction without huge impact on other train services. The circulation plan of rolling stock does not change, in which train services $g8$ and $g9$ in the down direction are performed by the rolling stock which performed $f1$ and $f2$ in the up direction and turned around at JHC. The headways between train services at the station close to the block area in up and down direction are illustrated in Figure 7 and Figure 8 respectively, in which the red line denoted the predefined headway in timetable and the blue line denoted the headway after rescheduling. As can be observed in Figure 7, the headway between service $f2$ and $f3$ and the headway between service $f3$ and $f4$ are slightly changed since services $f3$ and $f4$ are disrupted and turn around before the block area, while other headways remain the same in timetable. The result is similar in down direction.

The experimental results demonstrate the effectiveness and efficiency of the proposed disruption management model. A rescheduled timetable and rolling stock circulation plan can be obtained in a few seconds, which can be used to handle disruptions so as to ensure the capacity of urban rail transit and the service quality provided to passengers.

6 Conclusion

In this paper, a disruption management model is proposed to rescheduling train services in term of a complete blockage of the double tracks for 5-10 minutes in urban rail transit systems. The objective of the model is to minimize the train delays and the number of canceled train services as well as to ensure a regular service for passengers, while constrains, such as departure and arrival constraints, turnaround constraints, service connection constraints, inventory constraints are considered. The case study based on the real-world data from Beijing subway line 7 demonstrated that an acceptable rescheduled timetable and rolling stock circulation plan can be obtained within a few seconds, which can be adopted in real-time disruption management.

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