Optimization Model for Multi-Stage Train Classification Problem at Tactical Planning Level

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Abstract

Multi-stage train classification is a complex marshalling procedure that could be applied for simultaneous multi-group train formation. Simultaneous train formation is capable of processing a large volume classification insensitive on the number of outbound trains. Through multi-stage classification, wagons are moved several times to achieve desired outbound train sequences. The main optimization issue refers to finding a balance between the number of sorting steps and the total number of wagon movements. The optimization of the classification schedule could be addressed at different levels of the yard planning hierarchy. In this paper we develop mathematical formulation and two different heuristic algorithms to support tactical decisions for the multi-stage train classification problem. The main optimization issue refers to the allocation of tracks for performing multi-stage train classification minimizing annual operating costs. In order to validate the mathematical formulation and evaluate the efficiency of the proposed optimization model, we conduct computational tests and case study experimentations based on infrastructural and operational conditions applied in Belgrade marshalling yard in Serbia.

Keywords

Marshalling yards, Multi-stage train classification, Optimization model, Heuristic algorithms

1 Introduction

Marshalling yards, as consolidation nodes for rail wagonload transportation, play important role in railway freight networks (see e.g. Boysen et al (2012), Belošević et. al (2013) or Gestrelius et al. (2013)). Wagonload transportation, also called Single Wagon Load Service (SWL), consolidate loads composed of single wagons and wagon groups. These wagon loads are collected at different customer sidings and assembled in marshalling yards to full trains on the same routes. Multi-group trains have potential to take a substantial segment in wagonload service. Multi-group trains gather wagons into groups of wagons and serve two or more destinations. The order of the groups in such trains corresponds to the geographical disposition of destinations. The application of multi-group train service leads to the reduction of total layover time of wagons and to the concentration of shunting work on smaller number of main marshalling yards.

Multi-group trains are formed either applying single-stage or multi-stage classification procedures. Multi-stage train classification is a complex marshalling procedure and in some recent research (Dahlhaus et al. (2010), Jocob et al. (2011), Bohlin et al. (2015) or Bohlin et al. (2018)) it is proven that belongs to the class of NP hard problems (nondeterministic in polynomial time). In the paper Jocob et al. (2011), a concise encoding of classification schedules is suggested for multi-stage classification procedure. This way of encoding is used in papers Marton et al. (2009) and Maue et al. (2009) for formulation of linear programming models applicable at the operational planning level. Generally, the models find out an optimal classification schedule with regard to the number of sorting steps as a primary objective and the number of movements as a secondary objective. The same encoding is also applied by Belošević and Ivić (2018). In contrast to above mentioned papers, Belošević and Ivić (2018) provide overall optimization simultaneously minimizing the number of sorting steps and the total number of movements. The formulated model is applicable at strategic planning level. The model integrates the creation of the classification schedule and design of sidings layout. As the multi-stage classification problem is computationally consuming problem, current literature also propose several usages of heuristics for efficient solving large scale instances (see Hauser et al. (2010), Belošević et al. (2013) or Belošević et al. (2018)).

Extending the existing research on this topic, this paper provides the optimization model applicable at tactical planning level. The main optimization issue refers to the allocation of classification tracks for performing multi-stage train classification over forthcoming period of the rail timetable validity. The model provides optimal track allocation minimizing annual operating costs. Based on the heuristic optimization approach presented in Belošević and Ivić (2018), this paper proposes two different Variable Neighborhood search (VNS) algorithms for solving large scale classification schedules. Developed algorithms perform a systematic search of variable neighborhoods either in deterministic or stochastic form. As a part of experiment evaluations, we randomly generate a set of various sized instances and analyze the performances of developed deterministic and stochastic VNS algorithms. The performance comparison of VNS algorithms is based on the objective value and running time of obtained solutions. Finally, case study experimentations are conducted to examine the efficiency of developed heuristic algorithms. The case study depicts infrastructural and operational conditions applied in Belgrade marshalling yard in Serbia.

2 Problem Statement

Multi-group trains are formed using sorting by train or simultaneous strategies. Sorting by train strategies result in a successive train formation procedure. Using a sorting by train strategy, wagons are initially sorted according to their outbound trains. After accumulating all wagons of a common outbound train, the wagons are resorted according to destinations. The duration of sorting by train formation procedures directly depend on the number of outbound trains (see e.g. Ivić et al. (2013)). On other hand, simultaneous strategies can greatly improve classification process, as they enable in parallel formation of several trains. Using a simultaneous train formation strategy, the sorting procedure is altered. Initially, wagons are sorted according to sorting blocks that encompass all groups with the same disposition in all outbound trains. Afterward, the wagons are resorted according to their target trains. This alternation makes the sorting procedure insensitive to the number of outbound trains and therefore capable of processing a large volume classification (see e.g. Belošević et al. (2012)).

Simultaneous formation strategies are based on the multi-stage classification procedure. The multi-stage classification includes iterative repetition of operations:

wagons pulling out from the track (pulling out operation) and wagons rolling in other tracks (disassembling operation). The sorting process starts with wagons pulling out from the first track in sidings for wagons accumulation and follows with their disassembling to other tracks. This process is repeated at all other tracks. At the sidings for wagons accumulation, sorting is performed according to the sorting blocks, while sorting according to outgoing trains is performed at the sidings for final train formation (Figure 1).

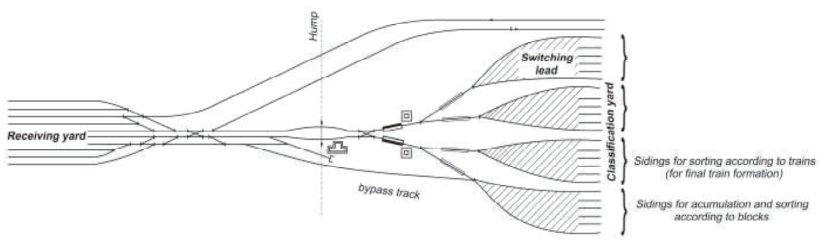

Figure 1: Sorting sidings layout

Through multi-stage classification, wagons are moved several times to achieve a desired order of wagons in an outbound train. The main optimization issue refers to finding a balance between the number of sorting steps and the total number of wagon movements indicating the length and complexity of the classification schedule. The optimization of the classification schedule should respect operational constraints and practical restrictions on the number and capacity of tracks. This optimization could be implemented at different levels of planning hierarchy (operational, tactical or strategic).

In this paper, we consider tactical planning level with the main optimization issue to allocate tracks that will be used for daily multi-stage train classification performed over forthcoming period of the rail timetable validity. Commonly once a year, marshalling yard managers make a decision referring to the distribution of classification tracks in the yard among different marshalling procedures (e.g. single-stage classification, multi-stage classification, empty wagons accumulation etc). In that sense, classification tracks have to be allocated in the way to minimize annual operating costs. As tactical decisions affect organizational processes, they have to be in accordance with the projected volume of daily work.

3 Optimization model

In this section we propose a mathematical programming formulation and a heuristic approach for solving the problem based on Variable Neiboruhood Search (VNS) strategy.

3.1 Mathematical formulation

The formulated mathematical model is based on the binary interpretation of multi-stage classification schedules proposed in Jacob at al. (2011). Let us assume that an arbitrary arranged ingoing sequence of N wagons have to be sorted into a properly sorted sequence $W = \{w_1, \dots, w_N\}$ according to sorting blocks gs ($s = 1, \dots, g_{max}$) where g_{max} denotes the maximum number of groups among all outbound trains. Sorting blocks gather all groups with the same disposition in all outbound trains.

The model minimizes annual operational costs TC_{oper} and could be formulated as follows:

$$\min \; TC_{oper} \tag{1}$$

subject to:

$$\sum_{j=1}^{K} 2^{j-1} x_i^j - \sum_{j=1}^{K} 2^{j-1} x_{i-1}^j \geq 1, \qquad \forall\, i \in F \tag{2}$$

$$\sum_{j=1}^{K} 2^{j-1} x_i^j - \sum_{j=1}^{K} 2^{j-1} x_{i-1}^j \geq 0, \qquad \forall\, i \in W \setminus F \tag{3}$$

$$h_j - x_i^j \geq 0, \qquad \forall\, j \in \{1, \dots, K\}, i \in \{1, \dots, N\} \tag{4}$$

$$h_j - h_{j+1} \geq 0, \qquad \forall\, j \in \{1, \dots, K-1\} \tag{5}$$

$$\sum_{i=1}^{N} x_i^j \leq C, \qquad \forall\, j \in \{1, \dots, K\} \tag{6}$$

where i and j are indices of wagons and sorting steps; F is a subset of the set W whose elements are starting a new group of wagons with the same sorting block index; K is upper bound on the number of tracks that technically could be allocated for multi-stage classification; and C indicates the track capacity limit expressed in the number of wagons.

Decision variables of the model are binary variables x_{ji} and h_j such that: $x_{ij} = 1$ if the wagon w_i participates in the sorting step h_j, or 0 otherwise; $h_j = 1$ if the sorting step h_j is realized, or 0 otherwise.

The objective function in the proposed model minimizes annual operating costs for performing multi-stage classification. The annual operating costs TC_{oper} refer to the costs of wagons' layover and fuel consumption. These costs are calculated on the daily base as a function of classification processing time. Specifically, operating costs could be estimated as a function of the number of sorting steps and total number of movements based on the classification processing time parameters t_h and t_x. The wagon layover cost is weighted with a freight rate e_w, while the fuel consumption is weighted with a fuel price e_c, fuel consumption rate c and the power of the engaged locomotive E_l. Finally, annual operating costs TC_{oper} could be expressed as:

$$TC_{\text{oper}} = 365 \left(\frac{e_w}{60} N + \frac{e_c\, c}{60} E_l\right) t_h \sum_{j=1}^{K} h_j + 365 \left(\frac{e_w}{60} N + \frac{e_c\, c}{60} E_l\right) t_x \sum_{j=1}^{K} \sum_{i=1}^{N} x_i^j \tag{7}$$

Constraints (2) and (3) define the schedule of wagon sorting in compliance with the basic principal of binary encoding. Specifically, wagons with a higher block index have higher code value (2), otherwise wagons with the same index may be assigned with the same code value (3). Constraints (4) and (5) define the relations between decision variables. Constraint (6) specifies the track capacity restriction.

3.2 Heuristic approach

VNS meta-heuristic is developed by Mladenović and Hansen (1997) and is based on the idea of systematic changes of neighborhood structures during a local search. The VNS strategy starts with an initial solution and performs local search procedure within $N_k(1 \leq k \leq k_{max})$ sequential neighbourhoods. If local search results with an improvement, the solution is updated and the procedure is repeated to the new incumbent solution. The final solution presents a local minimum with respect to all k_{max} neighborhoods.

The initial classification schedule X is constructed based on triangular sorting (see Daganzo et al. (1983)). Transformations used to generate candidate schedules within the exploration of neighborhood are presented and explained in details in Belošević and Ivić (2018). The local search procedure performs an iterative evaluation of neighbors and strictly a better solution is returned as the incumbent for the succeeding search. The best improvement is used as a selection mechanism that returns into an incumbent a solution which results in the maximal improvement among all neighbors. We implement Variable Neighborhood Descent (VND) and Reduced Variable Neighborhood Search (RVNS) as two heuristics that differ in the applied strategy of exploring neighborhood structures.

VND is a deterministic heuristic that intensifies a search aiming to improve the solution greedily. Unfortunately, a persistent search of large neighborhoods is mostly time consuming. An attractive approach for improving the performance of VND is to keep the use of large neighborhoods but to reduce the exploration. RVNS is a stochastic heuristic which randomly selects points in a neighborhood and then updates an incumbent solution in the case of an improvement. RVNS diversifies a search aiming to disable improvement stagnation in broad neighborhoods. Steps of the VNS meta-heuristic strategy applied for multi-stage classification problem are presented in Figure 2.

Initialization
Select the set of neighborhood structures that will be used in local search
Find an initial classification schedule X
Repeat the following sequence until no improvement is obtained:
(1) Set $k \leftarrow 1$;
(2) *Repeat* the following steps until $k = k_{max}$:
(a) *Exploration of neighborhood*
Find the best neighbor X' of X ($X' \in N_k$)
(b) *Move or not*
If the obtained solution X' is better than X: set $X \leftarrow X'$ and $k \leftarrow 1$;
Otherwise: set $k \leftarrow k + 1$;

Figure 2: Steps of the VNS meta-heuristic

4 Numerical experiments

In order to evaluate the efficiency of developed VNS algorithms, we conduct numerical experiments. All numerical experiments are done on a working station featuring *Intel Core 2.30 GHz* Dual Core processor with 8GB of RAM memory. The proposed algorithms are coded in *Python 2.7.11*. The exact solving is performed using *CPLEX 12.6* with the running time restriction and memory restriction for the tree structure sets to 3,600 seconds and 8 GB, respectively.

The input data used in experiments are summarized in Table 1 citing costs and other relevant parameters reported in Belošević and Ivić (2018).

Table 1: Input data

Parameters	Value
Freight rate per wagon	1.3 [$/h]
Fuel price	0.3 [$/kg]
Specific fuel consumption	50 [g/kWh]
Locomotive power	650 [kW]
Classification processing time	$19.1h + 0.7x$ [min]
Track capacity	40 [wagons]
Upper bound on the number of tracks in sorting sidings	20

4.1 Computational tests

At first, we test the performance of developed VNS algorithms and show how they behave for a varied set of classification task examples. We vary the total number of wagons (100, 150 and 250 wagons) and the number of sorting blocks (6, 8 and 10 groups in outgoing trains). Combining different number of wagons and number of blocks, we obtain examples with the wide range of complexity. Three random instances are created per each example generating 27 instances in total.

All instances are first computed using the CPLEX solver and then evaluated by VND and RVNS heuristics. Due to the stochastic nature of RVNS, the evaluation procedure is repeated 50 times. The stopping condition for the evaluation is set as a function of N and g_{max}, specifically $t_{max} = Nlog_2g_{max}$.

Results from computational tests are presented in Appendix. Appendix presents the objective value and running time of obtained solutions. Optimal solutions obtained by CPLEX are marked with bold objective function values. Non-bold values refer to the solutions returned in the moment when one of prescribed restrictions is reached, either regarding running time or memory tree. RVNS outputs are indicated by the best and average objective values, the standard deviation of objective values and the average running time. Also, we report a gap obtained by each algorithm on each instance with respect to the CPLEX value.

For all instances with 100 and 150 wagons in inbound flow, CPLEX proves optimality of returned solutions. For these sets of instances, CPLEX running times vary in range from 7 to 2200 seconds. For all instances with 250 wagons, CPLEX fails to prove the optimality of the returned solution. On other hand, VND and RVNS algorithms reach optimal or close optimal solution (with gap lower than 1%) for all instances with 100 and 150 wagons. In several instances, RVNS returns solutions with slight standard deviations due to the stochastic exploration of neighborhood structures. Due to the small neighborhood structures, running times of VND and RVNS algorithms are small and close to each other in most instances with 100 and 150 wagons. Computational tests demonstrate the quality of solutions returned by developed heuristic algorithms for the set of instances with 250 wagons, too. For the most of instances, developed heuristic algorithms return solutions that reach the best objective value. Analyzing the objective values, we can conclude that VND and RVNS algorithms perform almost similar. The minimal values for RVNS solutions are for the most instances equal to the objective values for VND solutions. Furthermore, RVNS solutions have a narrow spread around average values for all instances, so the highest gap for VND is 1.4%, while for RVNS is 1.8%. Finally, RVNS drastically outperforms VND considering running times. Running times for the VND algorithm vary in wide range reaching in some instances more than 30 minutes. On other hand, running times for the RVNS algorithm are mostly below one minute.

4.2 Case study experimentations

In order to construct meaningful experimentations we create a case study that depicts infrastructural and operational conditions applied in Belgrade marshalling yard. Belgrade marshalling yard is main marshalling yard on Serbian railway network. The layout of the yard is shown in the Figure 3. Belgrade marshalling yard features a hump with two parallel tracks. Receiving yard consists of 14 tracks with length in range from 680 to 841 meters. Classification yard consists of 48 tracks arranged in 6 groups with 8 tracks. The length of classification tracks chosen to conduct experimentations ranges from 850 to 1137 meters. Regarding the operation, Belgrade marshalling yard performs primarily single-stage classification. Multi-group trains are formed one by one within secondary sorting, grouping wagons for the same destination on a separate track. In this case study we analyze possibilities for applying the simultaneous train formation strategy. The projected volume of daily work in the forthcoming one year period of timetable validity is presented in Table 2 depicting the classification work on forming a set of outbound trains with maximum 10 groups per train. Each sorting block is assigned with the gamma distributed number of wagons with a rate set to 1.

We analyze the results obtained by developed VNS heuristic algorithms once again. Due to difficulties to use CPLEX for large scale examples, we compare obtained results with the results obtained by elementary simultaneous strategy, sorting by block. This strategy sorts wagons with same block number at a separate track and is close to the applied sorting strategy in the observed yard. The procedure is iterated 250 times. In addition to standard computational outputs regarding the objective function and running time, Table 3 shows key performance indicators in order to analyze the quality of returned classification schedules. Returned classification schedules are indicated with the number of sorting steps and total number of movements.

Table 2: The projected volume of daily work

	Sorting blocks									
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Number of wagons	*35*	*34*	*32*	*29*	*28*	*25*	*21*	*17*	*15*	*12*

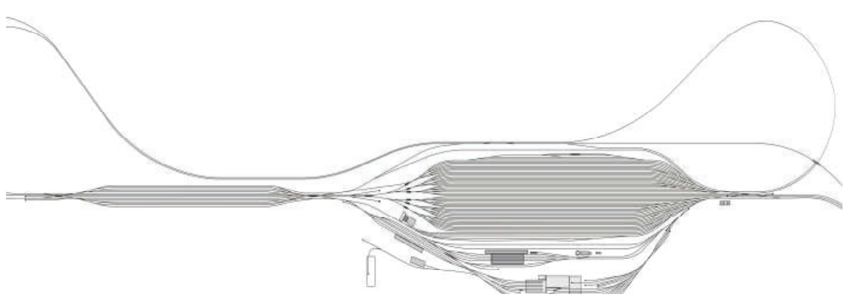

Figure 3: Layout of Belgrade marshalling yard

Table 3: Statistic summary of illustrative experimentations

	VND algorithm		RVNS algorithm		Sorting by block	
	Avg.	Range	Avg.	Range	Avg.	Range
Number of wagons	253	[208, 286]	*Same*		*Same*	
Obj. value [10^3 €]	744.8	[541.3, 974.1]	749.0	[542.7, 977.3]	776.4	[573.0, 995.6]
Gap [%]	0.0	[0.0, 0.0]	0.6	[0.0, 3.8]	4.4	[0.0, 12.8]
CPU [s]	551.7	[99.1, 2181.4]	40.9	[19.5, 216.7]	0.1	[0.0, 0.1]
Sorting steps	9	[8, 11]	9	[8, 11]	11	[10, 12]
Wagon movements	275	[234, 334]	276	[227, 337]	253	[208, 286]

Obtained results by VNS heuristics dominate over sorting by block strategy (see Figure 4). With respect to VND solutions, sorting by block strategy returns solutions with average gap 4.4 %, while the highest gap amounts up to 13 %. In contrast, developed VNS heuristic algorithms return close classification schedules with marginal differences in objective values. The average gap of RVNS solutions (with respect to VND solutions) amounts 0.6%, while the highest gap for RVNS is below 4%. Figure 5 shows the variations on average gap for solutions returned by RVNS and sorting by block. Experimentations confirm that performing complete local search of neighborhood structures could be high consuming. In that sense, VND algorithm requires averagely 551 seconds (in some instances more than 2000 seconds) for performing iterated descend. On other hand, RVNS does not have the problem with this computational issue and requires averagely about 41 seconds. Figure 6 shows these differences between VND and RVNS algorithms in terms of average running time.

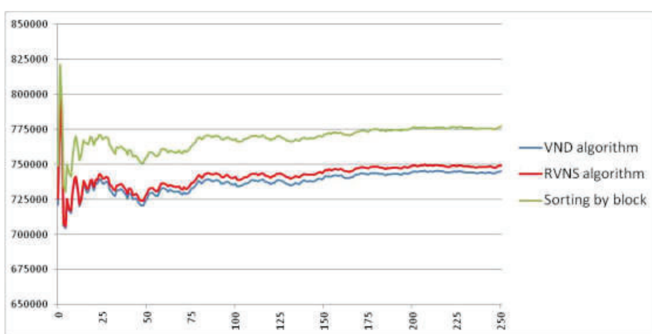

Figure 4: Average objective value

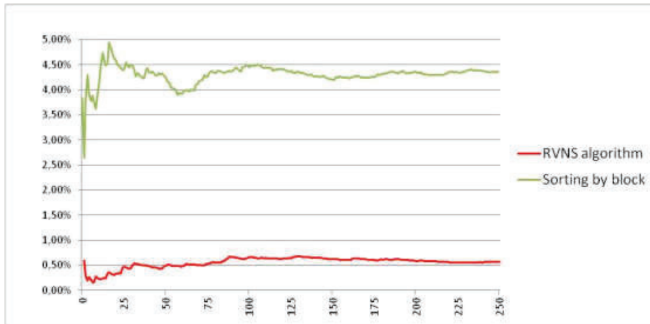

Figure 5: Average gap with respect to VND solutions

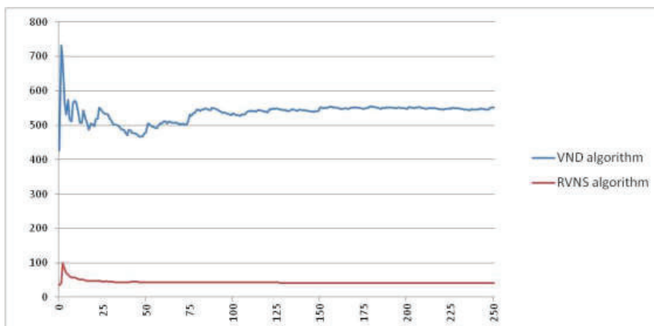

Figure 6: Average running time

Analyzing key performance indicator we can make similar conclusions regarding differences in quality of classification schedules returned by VND and RVNS algorithms. The results obtained by VND and RVNS are almost identical. The average number of sorting steps amounts 9 steps for both algorithms, while VND solutions averagely require 275 movements comparing to 276 movements in RVNS solutions. Comparing to the results from sorting by block, we can say that the heuristic optimization approach makes savings. Although the number of wagon movements is slightly increased, the average number of sorting steps is reduced for 2 steps. In order to define the required number of tracks that should be allocated for performing sorting procedure, we analyze cumulative distribution function (CDF) of the number of sorting steps. For developed heuristics the 0,90th quantile amount 10 tracks, while for sorting by block amounts 11 tracks. The difference is higher considering 0,95th quantile values. For developed heuristics the 0,95th quantile remain 10 tracks, while for sorting by block it increases on 12 tracks.

5 Conclusions

The potential of wagonload transportation is undeniable and confirmed with the increasing interest among shippers in such services. Many environmental and economy of scale benefits are advantages of wagonload transportation, but it needs to improve speed, reliability and cost competitiveness in comparison with other freight alternatives. In this paper, we consider multi-stage classification as one particular marshalling procedure that has the potential to improve the quality of wagonload transport allowing yards to simultaneously compound a set of multi-group trains. Multi-group trains are dominantly used for “last mile” service as local freight trains or industry trains.

In this paper we develop mathematical formulation and two different heuristic algorithms to support tactical decisions for the multi-stage train classification problem. The main optimization issue refers to the allocation of tracks for performing train classification in existing marshalling yards. Optimal track allocation is addressed by minimizing annual operating costs. Focusing on the computational complexity of the multi-stage train classification problem, in this paper we performed comparison of the efficiency of deterministic and stochastic heuristic approaches. The first one is a deterministic VND algorithm where the returned solution is a local optimum with respect to all predefined neighborhood structures. The second one is a stochastic RVNS algorithm which randomly selects points in a neighborhood and then updates an incumbent solution in the case of an improvement. RVNS diversifies a search aiming to disable improvement stagnation in large neighborhoods.

Conducted computational tests shown negligible differences between the solutions returned by developed heuristic algorithms and optimal solutions returned by CPLEX. The highest gap evaluated between the solutions returned by developed heuristics and optimal solutions amounts only 1.4%. Analyzing the objective values returned by developed heuristics, we can conclude that VND and RVNS algorithms perform almost similar. The computational tests show small dispersions of objective values evaluated by RVNS solutions. It is confirmed with low standard deviations obtained in all instances. On other hand, RVNS drastically outperforms VND considering running times. Running times for the VND algorithm reach up to 30 minutes in some instances, while running times for the RVNS algorithm are mostly below one minute. The obtained performances of developed heuristics are also confirmed in the conducted case study experiments. In the case study experiments, VNS heuristic results are compared with the results obtained by sorting by block strategy. Specifically, sorting by block strategy returns solutions with average gap 4.4 %, while the highest gap amounts up to 13 % with respect to VND solutions. These savings are confirmed analyzing key performance indicators, too.

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APPENDIX

Results from computational tests

No. wagons	Groups	Instance	CPLEX Obj. value [$]	CPLEX CPU [s]	VND Obj. value [$]	VND Gap [%]	VND CPU [s]	RVNS Min. obj. value $]	RVNS Aritm. obj. value [$]	RVNS Standard deviation	RVNS Gap [%]	RVNS CPU [s]
100	6	1	**142172,8**	16,8	142172,8	0,0	11,0	142172,8	142232,3	181,4	0,0	8,0
		2	**140983,8**	13,7	140983,8	0,0	2,9	140983,8	140983,8	0,0	0,0	3,3
		3	**143956,4**	28,3	143956,4	0,0	10,7	143956,4	146150,4	2557,4	1,5	6,5
	8	1	**148117,9**	32,2	148117,9	0,0	11,1	148117,9	152958,9	4923,8	3,3	8,4
		2	**152279,5**	8,0	152279,5	0,0	4,5	152279,5	152279,5	0,0	0,0	5,0
		3	**159583,5**	21,5	159583,5	0,0	12,2	159583,5	160059,1	571,5	0,3	7,2
	10	1	**165528,6**	84,0	166123,1	0,4	8,3	166123,1	166202,3	258,1	0,4	7,4
		2	**170284,7**	22,7	170284,7	0,0	8,7	170284,7	170284,7	0,0	0,0	6,0
		3	**165528,6**	54,9	166123,1	0,4	8,7	166123,1	166281,6	346,8	0,5	7,3
150	6	1	**273248,3**	116,5	273248,3	0,0	49,6	273248,3	274293,5	772,3	0,4	13,2
		2	**273248,3**	98,9	273248,3	0,0	50,4	273248,3	274612,9	1706,9	0,5	13,0
		3	**272999,4**	17,6	273248,3	0,1	51,2	273248,3	275222,6	1885,3	0,8	13,3
	8	1	**299378,6**	1110,7	299378,6	0,0	51,2	299378,6	300046,4	747,6	0,2	20,4
		2	**295894,5**	130,7	295894,5	0,0	47,6	295894,5	296446,2	535,6	0,2	18,3
		3	**297636,6**	164,1	297636,6	0,0	42,6	297636,6	298043,0	676,0	0,1	19,5
	10	1	**317669,8**	836,3	317669,8	0,0	22,0	317669,8	317669,8	0,0	0,0	11,6
		2	**308959,7**	290,9	308959,7	0,0	52,4	308959,7	309743,6	979,8	0,3	21,6
		3	**317669,8**	2103,8	317669,8	0,0	107,3	317669,8	326151,8	3407,5	2,7	25,2
250	6	1	692475,7	3600,8	702443,8	1,4	401,0	702443,8	702491,3	260,0	1,4	28,7
		2	668267,6	3605,6	668267,6	0,0	426,3	668267,6	668789,7	698,0	0,1	29,5
		3	671115,6	3600,4	671115,6	0,0	375,2	671115,6	671447,8	612,6	0,0	26,0
	8	1	705698,7	3637,9	666843,5	-5,5	463,6	666843,5	668077,7	1661,2	-5,3	62,0
		2	666843,5	3600,7	666843,5	0,0	393,5	666843,5	666891,0	260,0	0,0	49,8
		3	669691,6	2240,2	669691,6	0,0	370,6	669691,6	671400,4	1467,5	0,3	37,9
	10	1	744553,8	3600,1	744553,8	0,0	1947,6	744553,8	750819,4	5541,7	0,8	71,7
		2	738450,9	3603,1	744553,8	0,8	1473,7	744553,8	746025,3	1650,7	1,0	65,6
		3	737026,9	3600,8	744553,8	1,0	1394,2	744553,8	750439,7	4340,8	1,8	53,0