Some aspects of the modeling of tube-and-shell heat-exchangers

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Abstract

Modern control design of large industrial plants relies strongly on modelling, while experimentation on the real system is extremely limited. The usable model should be intuitive and easily reusable to model potential plant adaptations or similar plants.

In this paper some aspects of the modelling a recuperator plant consisting of four heat exchangers are presented. With the object-oriented modelling tool Dymola, using Modelica as a modelling language, the building of clear, intuitive and reusable model of basic, but most important plant's blocks, heat exchangers is enabled. However, the proposed model of heat exchanger proved to be intrinsically stiff and therefor at the moment problematic to be used as a submodel of more complex models.

Keywords: recuperator, heat exchanger; tube-and-shell, cross-flow, object-oriented modelling

1 Introduction

It is well known that practical constraints exists in control design of production lines, where one of the most problematic is the limited experimentation possibilities. The production line should be namely brought in operation as fast as possible but as rarely as possible halted for maintenance during its life-time. So there are not many opportunities to learn about the system through experimentation and to improve the control design. Consequently tere is enough time merly for tuning the controllers. Control design must be somehow supported by modelling.

In this paper a model of recuperator is presented, shown schematically in Fig. 1. The plant comprises of four heat exchangers in series which are used to recover waste heat from exhaust gases. The latter are created by combustion of a mixture of the waste gases coming from the main plant and a natural gas. The pipe they flow through is shown in the middle of the Fig. 1. On the secondary side of the heat exchangers incoming gases have various purposes and destinations in the main plant.

The flow of gases through recuperator can be manipulated by flap valves and ventilators (not shown in Fig. 1).

The recuperator plant is included in different industrial lines and can be modified according to the needs and specifics of the plant. The model of the process must thus be very flexible and user friendly for a control designer. As an appropriate modelling environment Dymola with object-oriented modelling language Modelica [6] was chosen. Object-oriented and acausal modelling approach simplifies modelling process. On contrary to traditional modelling, where model is represented by a set of functional blocks with causal connections (inputs and outputs), object-oriented acausal models are composed as sets of related, interacting objects (submodels) and transformation of the model in a proper form suitable for computation is left to translator. Object-oriented models thus preserve topology of the system being modelled and are as such more intuitive and easily reusable since submodels do not have explicitly defined computational order.

Modelica was found as appropriate also because free Modelica libraries from the domain of thermofluids [1, 3] are available what significantly mitigates the model development procedure.

2 Mathematical model of the heat exchanger

Heat exchanger is a device in which energy in the form of heat is transferred what is usually realized by the confinement of both fluids in some geometry in which they are separated by a conductive material. The properties of heat exchanger are strongly dependent on geometry and material as well as on properties of both fluids. It is known that such devices had usually nonlinear behaviour [5].

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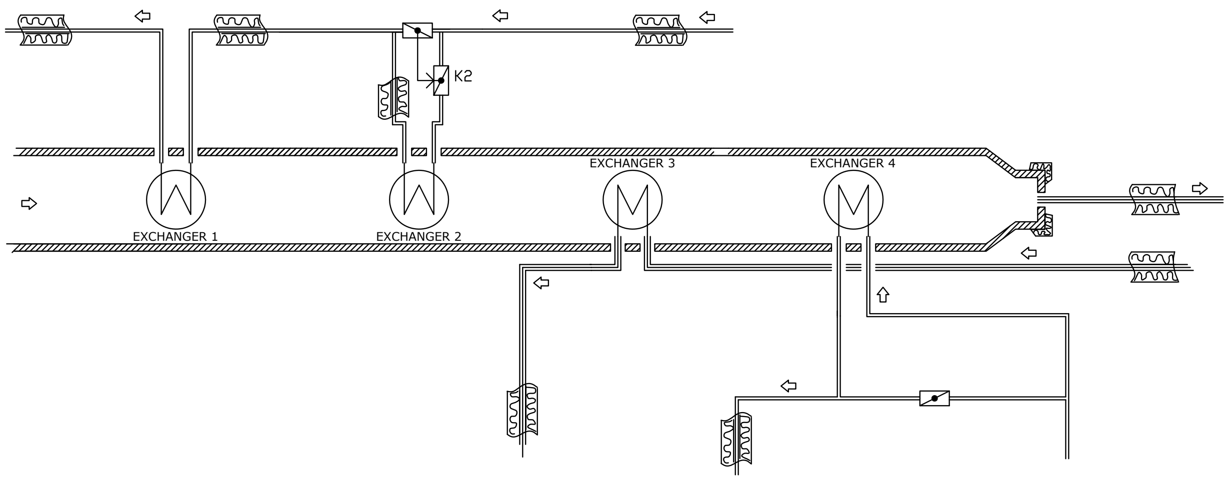

Figure 1: A scheme of the recuperation process: hot exhaust gases flow pass four heat exchangers and heat up gases used in other part of the plant.

2.1 Model of the heat flow rate between fluids

Our aim is to build a reusable model which will enable better understanding of the process. So the model should be described by analytical equations. The observed recuperator process comprises of shell-and-tube cross-flow heat exchangers depicted in Fig. 2. Particular heat exchanger consists of a bundle of tubes with circular cross section which is inserted into a shell so that a flow through the tubes is perpendicular to the flow through shell as illustrated in Fig. 3. Since heat transfer takes place across the tubes surface, the arrangement of flows (geometry of the heat exchanger) has important impact on the efficiency of the device.

However, analytical solution for shell-and-tube heat exchanger exists only when the flows of both fluids are parallel. They can be co-current or counter-current. The energy balance for tube-side fluid and shell-side fluid for a tubular counter-current heat exchanger is given in Eq. (1) and Eq. (2) respectively. Equations

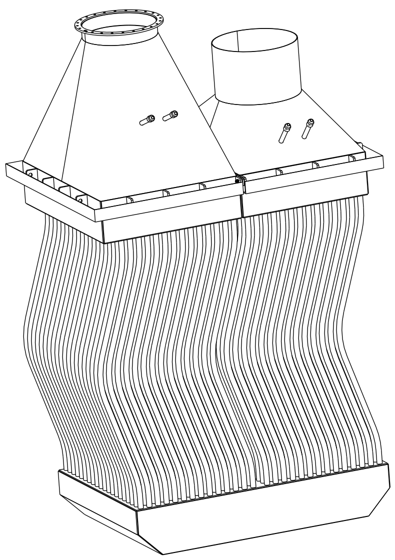

Figure 2: Scheme of the cross-flow heat exchanger.

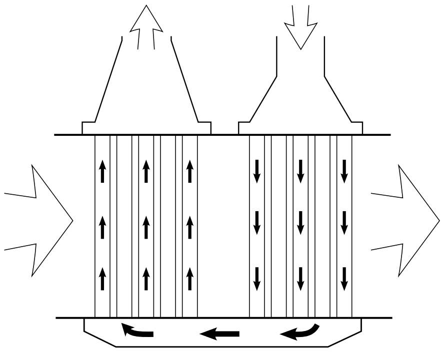

Figure 3: Flow directions in cross flow heat exchanger.

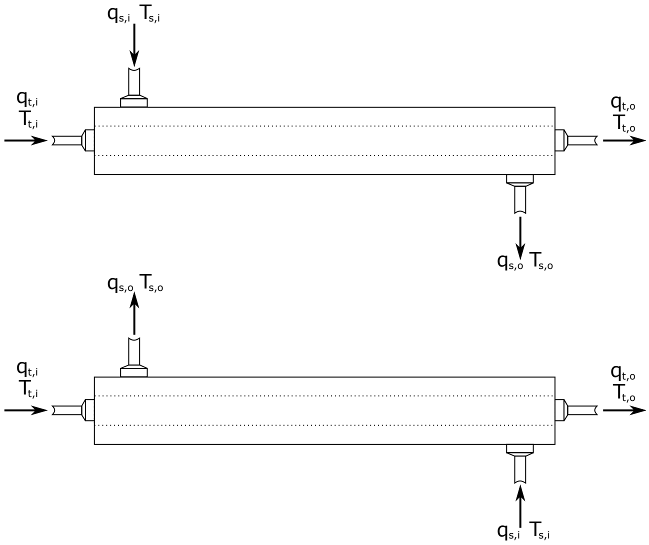

Figure 4: Scheme of the simple co-current (above) and counter-current (below) double-pipe heat exchanger

for co-current heat exchanger are different (due to flow direction) only in the sign of the first term on the right side of Eq. (2).

$$\frac{\partial}{\partial t}(\rho_t \cdot A_{c,t} \cdot \hat{C}_{p,t} \cdot T_t) = -\frac{\partial}{\partial z}(\rho_t \cdot q_t \cdot \hat{C}_{p,t} \cdot T_t) - \frac{U \cdot A}{L} \cdot (T_s - T_t) \tag{1}$$

$$\frac{\partial}{\partial t}(\rho_s \cdot A_{c,s} \cdot \hat{C}_{p,s} \cdot T_s) = \frac{\partial}{\partial z}(\rho_s \cdot q_s \cdot \hat{C}_{p,s} \cdot T_s) - \frac{U \cdot A}{L} \cdot (T_s - T_t) \tag{2}$$

In Eq. (1) and (2) indices t and s denote quantities of tube-side and shell-side fluid respectively; T designates mean temperature of cross section which depends on time and position along the pipe's length (z-axis): $T = T(t,z)$, A_c is area of the pipe's cross section, ρ and $\hat{C}_p$ are density and specific heat capacity of the fluid respectively (also time and position dependent), q is the mass flow, U is the overall transfer coefficient, A is area through which heat is exchanged and L is length of the pipes.

From Eq. (1) and (2), analytical expressions for the temperatures of the fluids at the pipes' outlets and heat flow rate between the fluids can be derived [4]. However, for a cross-flow heat exchanger it is known only that its performance is worse from counter-current and better from co-current heat exchanger [5]. However, no analytical expressions exists. So the following supposition was taken into account: while the shell's length is relatively small in comparison to its cross section and the flow through it is highly turbulent (Reynolds number is of the order 10^5), temperature differences across the shell are negligible and shell can be modelled sufficiently accurate as a lumped model. The energy balance equations are then:

$$\frac{\partial}{\partial t}(\rho_t \cdot A_{c,t} \cdot \hat{C}_{p,t} \cdot T_t) = -\frac{\partial}{\partial z}(\rho_t \cdot q_t \cdot \hat{C}_{p,t} \cdot T_t) - \frac{U \cdot A}{L} \cdot (T_s - T_t) \tag{3}$$

$$\frac{d}{dt}(\rho_s \cdot V_s \cdot \hat{C}_{p,s} \cdot T_s) = \rho_s \cdot q_s \cdot \hat{C}_{p,s} \cdot (T_{s,i} - T_{s,o}) - U \cdot A \cdot (T_s - \bar{T}_t) \tag{4}$$

In Eq. (4) V_s denotes the volume of the shell, $T_{s,i}$ and $T_{s,o}$ temperatures of the shell-side fluid at inlet and outlet respectively and $\bar{T}_t$ is a mean temperature of fluid in tube bundle at a given time.

As Modelica does not support solving partial differential equations implicitly, Eq. (3) was discretized by a finite volume method.

2.2 Model of the wall's heat capacity

In the proposed heat exchangers fluids in tube- and shell-side are hot gases that passes through the exchanger at relatively high speed (about 15 m/s in the shell), and their mass (and thus heat capacity) is much smaller then the mass of the tube-bundle's wall which weights 3.4 tons. The influence of the wall heat capacity is thus considerable and must be included in the model.

On the other hand, the thermal conductivity of the wall is a few orders greater then conductivity of the gas and was thus neglected.

In order to consider dynamics due to wall's heat capacity, additional equation has to be added to the energy-balance equations Eq. (3) and Eq. (4):

$$\frac{\partial}{\partial t}(\rho_t \cdot A_{c,t} \cdot \hat{C}_{p,t} \cdot T_t) = -\frac{\partial}{\partial z}(\rho_t \cdot q_t \cdot \hat{C}_{p,t} \cdot T_t) - \frac{\alpha_t \cdot A}{L} \cdot (T_w - T_t) \tag{5}$$

$$\rho_w \cdot A \cdot d \cdot \hat{C}_{p,w} \cdot \frac{dT_w}{dt} = -\alpha_t \cdot A(T_w - \bar{T}_t) - \alpha_s \cdot A \cdot (T_w - T_s) \tag{6}$$

$$\frac{d}{dt}(\rho_s \cdot V_s \cdot \hat{C}_{p,s} \cdot T_s) = \rho_s \cdot q_s \cdot \hat{C}_{p,s} \cdot (T_{s,i} - T_{s,o}) - \alpha_s \cdot A \cdot (T_s - T_w) \tag{7}$$

In the new equations, index w designates quantities of the wall, α_t and α_s are the convective heat transfer factors of the tube- and shell-side gas respectively and d is thickness of the wall.

2.3 Model of the fluid dynamics

The purpose of the model is temperature dynamics description of the recuperator. The thermal dynamics of its component, heat exchanger, is described by Eqs. (5), (6) and (7). However, some parameters, for example, ρ_t, $C_{p,t}$, are not constant (or nearly constant) in operating-temperature range, so additional relations must be introduced. It means flow-equations derived from the laws of mass and momentum balances (to calculate mass flow and pressure respectively) and algebraic equations (Eqs. (8)) which determine properties of the media. They describe relation among pressure p, temperature T, internal energy u, enthalpy h and density ρ.

$$\begin{aligned} p &= p(\rho, T) \\ u &= u(\rho, T) \\ h &= u + \frac{p}{\rho} \end{aligned} \tag{8}$$

Eqs. (8) introduce a non-negligible nonlinearites in the model and also increase differential algebraic equations index of the system [7].

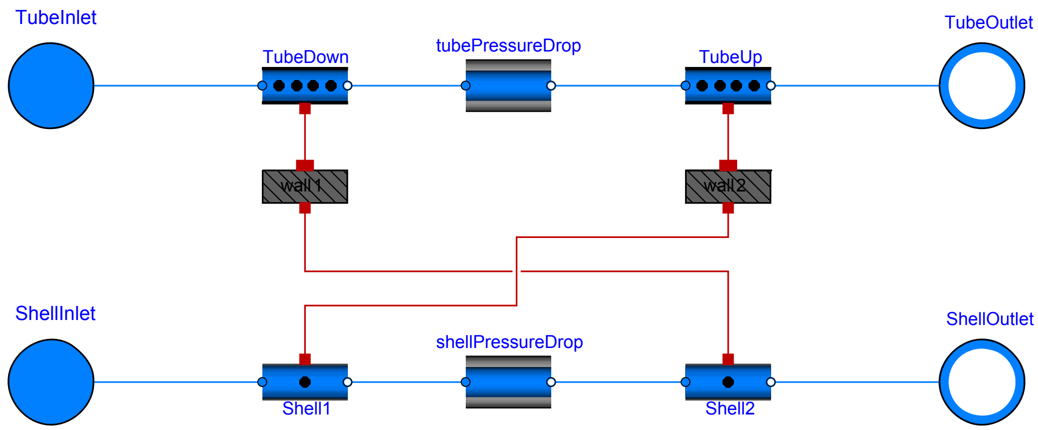

Figure 5: Component scheme of the heat exchanger submodel in Dymola

2.4 Model of the pressure drop

Determination of the pressure drop plays a major role in the design of heat exchanger along with heat-transfer coefficient [5]. However, for the control design of the observed recuperator is not very important. Partly it is the consequence of the lack of measurements of the pressure-drop dynamics. Only nominal pressure drop at nominal volume flow rate was namely available. So, a pressure drop Δp of the whole pipe (shell and tube) of the heat exchanger is modelled:

$$\Delta p = kv^2 \tag{9}$$

Coefficient k in Eq. (9) was calculated by inserting nominal pressure drop and nominal velocity (derived from nominal volume flow) into the equation.

2.5 Convective heat-transfer coefficient

The convective heat-transfer coefficient of the gas is problematic as it is influenced by geometry of the heat exchanger and chemical properties of the gases. Usually it is provided as an empirical expression including Reynolds and Prandtl numbers. To keep the model simple, the expressions in Eq. (10) and (11) (for tube- and shell-side respectively) were found to be sufficient:

$$\frac{\alpha_t \cdot d_{h,t}}{\lambda_t} = 0.040 \cdot (Re_t \cdot Pr_t)^{0.75} \tag{10}$$

$$\frac{\alpha_s \cdot d_{h,s}}{\lambda_s} = 0.113 \cdot (Re_s \cdot Pr_s)^{0.75} \tag{11}$$

In Eq. (10) and (11) indices t and s indicate tube- and shell-side respectively, d_h is hydraulic diameter, λ thermal conductance, Re Reynolds number and Pr Prandtl number.

3 Implementation in Modelica

The model of the heat exchanger basically consists of two thermally coupled pipes. So it should be built up by two pipe submodels and intermediate heat-transfer submodel.

3.1 Heat exchanger

As already mentioned in the introduction, many freely available Modelica libraries for modeling thermodynamics systems exist. In our case basic components from the *Modelica_Fluid* library are used and adopted. Here a tube bundle is described by distributed parameters. So a component of a pipe with distributed parameters discretized by finite volume method is taken from the library. For the model of shell a component of a pipe with lumped parameters from the library is used. Thermo-fluid governing equations and properties of the media (Eq. (8)), are realized by a nested component of the *Modelica.Media* library), enabling the avoidance of flow-dynamics equations formulation by hand. Components of the tube bundle and the shell are connected over a wall model, what is a custom made component simulating Eq. (6). The resulting scheme is shown in Fig. 5.

In the Fig. 5 it can be seen that each pipe is consists of three components – pipe is split in two parts and a pressure drop component is placed inbetween. The model of tube bundle is composed from components *TubeDown*, *tubePressureDrop* and *TubeUp* components, while shell comprises *Shell1*, *shellPressureDrop* and *Shell2*. At the outermost edges four connectors are place, namely inlet and outlet for the tube and shell. They represent interface of the heat exchanger component.

3.2 Convective heat transfer

The mentioned components of the pipes from the *Modelica_Fluid* library represent a replaceable nested component of convective heat transfer. However, this component can be replaced by the one including also convective-heat-transfer coefficient from Eq. (10) and

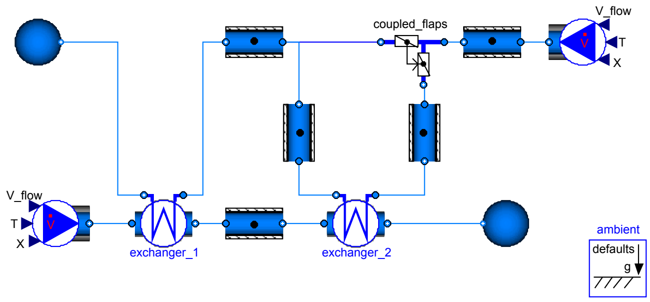

Figure 6: Model of the recuperator's exchangers 1 and 2 with some periphery

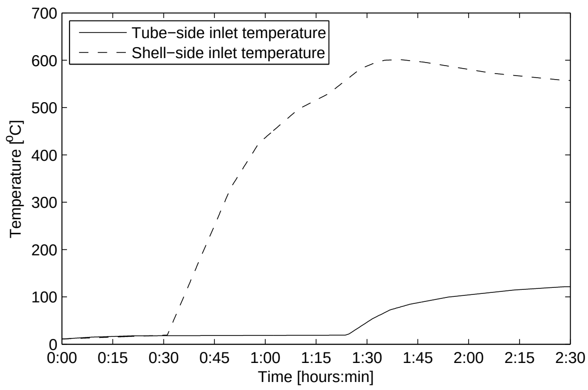

Figure 7: Temperatures of the tube- and shell-side gases at the inlet.

Eq. (11). The code listing of our component for tube's convective heat transfer is the following:

```
model PipeHT_TubeConv
  extends PartialPipeHeatTransfer;
  outer input Medium.BaseProperties[n] medium;
  SI.CoefficientOfHeatTransfer alpha0[n];
  SI.ThermalConductivity lambda[n];
  SI.PrandtlNumber Pr[n];
  outer input SI.ReynoldsNumber Re[n];
equation
  for i in 1:n loop
    lambda[i] = Medium.thermalConductivity(medium[i]);
    Pr[i] = Medium.prandtlNumber(medium[i]);
    alpha[i] = lambda/d_h*0.040*(Re[i]*Pr[i])^0.75;
    thermalPort[i].Q_flow=noEvent(if alpha[i] < 5 then 5
      else alpha[i]*A_h/n*(thermalPort[i].T-T[i]));
  end for;
  thermalPort.Q_Flow=Q_flow
end PipeHT_TubeConv
```

The *PipeHt_TubeConv* is very simple component, because all the needed thermodynamic variables (thermal conductivity and Prandtl number) of the medium in Eq. (10) are computed by *Modelica.Media* library and some other variables (Reynolds number and medium base properties) are computed in embedding pipe model.

3.3 Recuperator

The model of recuperator is decomposed into subsystems, i.e., heat exchangers, pipes and flaps. It means that it is built by connecting components simulating the subsystems.

However, certain difficulties were encountered at the connections of pipes and heat exchangers. An implicit system of nonlinear equations for pressure/flow correlation is namely needed in the connection point [2]. This is the consequence of the discretization of the partial differential equations in the pipe's model. A staggered grid approach is used, leading to half momentum balances between the pipe's boundary and first and last segment (finite volume). The system of nonlinear equations in connection points is especially problematic at the initialization phase due to unsatisfactory knowledge of the pressure drop across the system it is hard to define corresponding initial values. The consequence is that time necessary for simulation severely increases and in some cases a solution can not be found.

Additional difficulties arise from the different cross-areas of the connecting pipes (especially on the tube side – the outlet gases are less dense then the inlet ones and take larger volume, so the outlet pipe have larger diameter than inlet one). This fact causes numeric problems in the kinetic terms of the flow-equations and special care must be taken to handle them appropriately [2]. Due to numerical problems in the junctions of the pipes the building of models from prepared heat-exchanger and pipe (sub)models is still very tedious task.

In Fig. 6 a model of the connected heat exchangers 1 and 2 is shown (illustrated also on the left side of Fig. 1). It represents part of the recuperator. Tubes of both exchangers are connected and the temperature at the outlet of the exchanger 1 is controlled by two coupled flaps which define the portion of the flow passing through heat exchanger 2. The simulation run of the model takes more than 10 hours, while the simulation of the single heat exchanger finishes in a few minutes.

4 Validation

Validation of the heat exchanger model was performed on a real measurements data (validation of the whole recuperator is impossible due to the lack of data). The available measurements were temperatures of the exhaust gases at the entrance into the shell of the heat exchanger and at the exit from the shell, input and output temperatures of the tube gases as well as volume flow through the tube. Unfortunately, volume flow through the shell was not measured. So it was supposed to be constant during the simulation and equal to the nomi-

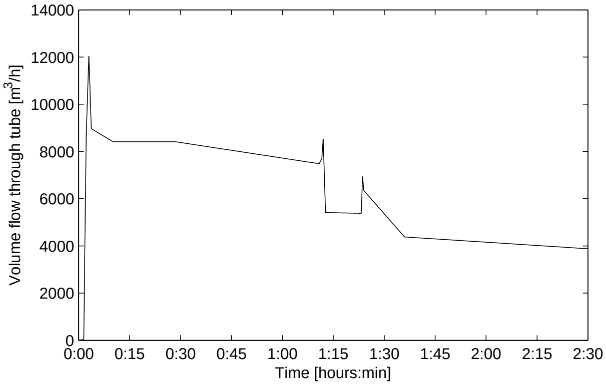

Figure 8: Volume flow through the tube.

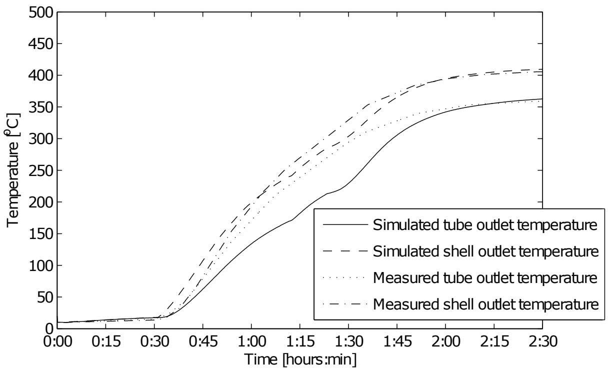

Figure 9: Comparison of the measured and temperatures obtained by simulation of the gases at outlet of tube and shell.

nal one.
The temperature measurements of the entering gases into the heat exchanger are shown in Fig. 7 and measurements of volume flow through tube in Fig. 8. Simulation results, i.e. temperatures of tube and shell gases at outlet, compared with measured ones are shown in Fig. 9.

As it can be seen in Fig. 9, response of the model fits the measured data relatively well. The biggest discrepancy is during the rise time what can be assigned to the missing shell flow measurements. At the end of the simulation, when the shell flow should reach its nominal value, also the difference between measured and calculated response is smaller.

Important property of the model is that it is relatively unaffected by a sharp changes in tube flow as it can be seen in Fig. 8 and Fig. 9. The steep changes of the tube flow cause only little disturbances in the temperatures at the outlet. By experimenting on the model it was found out that it is a consequence of the nonlinear convective heat transfer.

Nevertheless, validation of the model is not yet satisfactory. It should be validated on more measurements and the data should also include measurements of the flow through the shell.

5 Conclusion

As it was shown in the paper, object-oriented acausal modelling approach with support of freely available libraries offers a very rapid development of the complex and highly nonlinear models. In the paper a model of the heat exchanger is shown.

However, in the case of more complex case of connecting heat-exchanger models, many difficulties appear. Solving them makes the model unnecessarily complicated. The numerical problems of the recuperator model thus originate in the components of the *Modelica_Fluid* library and the design of fluid connector due to limitation of Modelica [2]. This has been improved in the latest language-standard change and the *Modelica_Fluid* library was reimplemented.

In our future work we plan to port our model to the newest version of the *Modelica_Fluid* library which will help us to approach our goal of creating a clear and reusable model for control engineers. We will also intend to acquire better measurements from the new plants which will enable more proper validation of the single heat exchanger model as well as the whole recuperator plant.

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