Studies on Consistency Threshold of *Kansei* in Paired Comparison

Taki Kanda
Bunri University of Hospitality
kanda@bunri-c.ac.jp

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Abstract: Human *Kansei* is very complicated and it is not easy to evaluate it. Methods of paired comparison might be useful for evaluating human *Kansei* because it is generally said that human *Kansei* is nonlinear while methods of paired comparison are considered to be able to deal with nonlinear problems. In paired comparison it is important to confirm consistency of answers of each subject because if a subject's answers are inconsistent, it could not be said that evaluation of the subject's *Kansei* is validity. There are many methods of paired comparison which have been proposed and AHP (Analytic Hierarchy Process), which was proposed by Tomas Saaty as a method to help us determine priority of alternatives to select, uses paired comparison for obtaining weights to criteria and values for evaluation of alternatives with respect to criteria. In AHP consistency of a subjects' answers is judged using Consistency Index referred to as *C.I.* in short. with respect to criteria. In AHP consistency of a subjects' answers is judged using Consistency Index referred to as *C.I.* in short. Threshold of *C.I.* on consistency of subjects' answers is generally 0.1 or 0.15 by empirical judgment but there is no theoretical or experimental ground about it. Other than *C.I.* coefficient of consistency is used for judgment on consistency of a subject's answers in paired comparison. Values of coefficient of consistency are calculated from the number of circular triad which implies inconsistency of a subject's answers. Here it is studied how to determine proper threshold of *C.I.* for judgment on consistency of a subject's answers in paired comparison.

1. Introduction

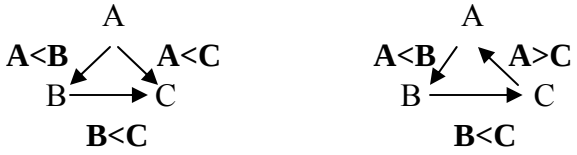
Analytic Hierarchy Process (AHP) is considered to be effective method for evaluating human *Kansei* because it is said that *Kansei* are nonlinear and also has hierarchical structure while the essences of AHP are to make hierarchical structure of the problem dealing with clear setting criteria and alternatives and determine the priority of alternatives using paired comparison. Paired comparison in AHP do not only evaluates human *Kansei* but also judges consistency of the answers of each subject by Consistency Index (*C.I.*). It might be necessary to determine proper threshold of *C.I.* for judgment on consistency of a subjects' answers in paired comparison and hence it is considered to find the proper threshold of *C.I.* perceiving existence of circular triad.

2. Methods to judge consistency

As examples of methods to make judgment on consistency of a subject's answers in paired comparison, coefficient of consistency and Consistency Index (*C.I.*) can be cited. Coefficient of consistency and *C.I.* are obtained based upon the number of circular triad and paired comparison matrix respectively.

3. Coefficient of consistence

Suppose there are three stimuli A, B, C and
A → B implies B is better than A in comparison of A and B. Then the case 1 of Fig.1 determine the order such that 1st → C, 2nd → B, 3rd → A but the case 2 of Figure 1 does not determine the order. The case 2 is called circular triad.



Case 1 Case 2

Figure 1 Circular Triad

Now letting d be the number of circular triad, d is given by

$$d = \frac{1}{6}k(k-1)(k-2) - \frac{1}{2}\sum_{i=1}^{k} a_i(a_i - 1) \tag{1}$$

where k is the number of stimuli and a_i is the number of arrows which go toward outside from each vertex. Letting ζ be coefficient of consistency, ζ is given as follows.
(I) If the number of stimuli k is odd number, we have

$$\zeta = 1 - \frac{24d}{k^3 - 4k}. \tag{2}$$

(II) If the number of stimuli k is even number, we have

$$\zeta = 1 - \frac{24d}{k^3 - k}. \tag{3}$$

For $k \geq 8$, statistical test can be conducted as follows. First, letting f be degrees of freedom, test statistic

$$\lambda_0{}^2 = \frac{8}{k-4}\left[\frac{k(k-1)(k-2)}{24} - d + \frac{1}{2}\right] + f \tag{4}$$

is calculated and letting α and $\lambda(f,\alpha)$ be level of significance and critical value respectively if

$$\lambda_0^{\,2} \geq \lambda(f,\alpha) \tag{5}$$

is satisfied, It is statistically judged that answers of a subject in paired comparison are consistent.

4. Consistency index

In the questionnaire of paired comparison in AHP, subjects choose answers from nine choices and for each choice a value of evaluation are given in comparison of stimuli A and B as shown in Table Ⅰ.

Table Ⅰ: Values for answers in paired comparison of AHP

Choices for answers	A	B
A is extremely important than B.	9	1/9
A is very strongly important than B.	7	1/7
A is strongly important than B.	5	1/5
A is moderately important than B.	3	1/3
A and B are equally Important.	1	1
B is moderately important than A.	1/3	3
B is strongly important than A.	1/5	5
B is very strongly important than A.	1/7	7
B is extremely important than A.	1/9	9

Based upon a subject's answers, in the case where there are n stimuli, paired comparison matrix is made as

$$\mathbf{M} = \begin{bmatrix} m_{11} & m_{12} & \cdots & m_{1n} \\ m_{21} & m_{22} & \cdots & m_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ m_{n1} & m_{n2} & \cdots & m_{nn} \end{bmatrix} \tag{6}$$

where

$$m_{11} = m_{22} = \cdots = m_{nn} = 1, m_{21} = 1/m_{12}, m_{31} = 1/m_{13}, \cdots, m_{n1} = 1/m_{1n}.$$

From the paired comparison matrix, human *Kansei* to each stimuli are evaluated by the eigenvalue method or the geometric mean method and also consistency of answers of each subject is able to be judged.

In order to judge consistency of a subject's answers, Consistency Index, that is referred to as *C.I.* is used. Let $\lambda_{\max}$ be maximum eigenvalue of paired comparison matrix. Then *C.I.* is defined by

$$C.I. = \frac{\lambda_{\max} - n}{n-1} \tag{7}$$

where n is the degree of paired comparison matrix.
Generally, if

$$C.I. \leq 0.1 \quad \text{or} \quad C.I. \leq 0.15 \tag{8}$$

is satisfied, it is empirically judged that a subject's answers are not consistent. Since this judgment is neither theoretical nor experimental, criterion for judgment on consistency of a subject's answers is examined.

5. Judgment on consistency

In order to find proper criterion for judgment on consistency of a subject's answers, the relation between existence of circular triad and values of *C.I.* in the case where the number of stimuli is three is examined.
When the relation among three stimuli does not make circular triad, values of *C.I.* are

$$0.00 \leq C.I. \leq 0.67 . \tag{9}$$

When the relation among three stimuli makes circular triad, values of *C.I.* are

$$0.67 \leq C.I. \leq 3.56 . \tag{10}$$

Therefore if the value of *C.I.* is less than 0.67, there does not exist circular triad.
Now if a subject answers as shown in Figure 2, the order among three stimuli A, B, C is determined such that A is first, B is second, C is third and $C.I. = 0.67$ which is the largest in the case where there does not exist circular triad.

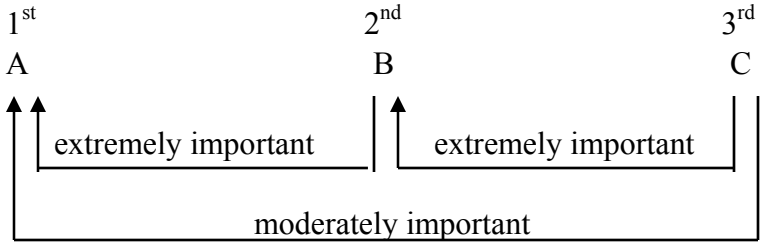


Figure 2: Examples of answers for the maximum of *C.I.*

Table Ⅱ shows the largest values of *C.I.* in the case where there does not exist circular triad when the numbers of stimuli are 3, 4, 5, 6, 7, 8 and 9,

Table Ⅱ: Maximum value of *C.I.* to the number of stimuli

The number of stimuli	Maximum value of *C.I.*
3	0.67
4	0.77
5	0.82
6	0.84
7	0.85
8	0.87
9	0.88

Let x and y be the number of stimuli and the largest value of *C.I.* respectively. Then x and y fit to the quadratic regression equation

$$y = -0.01x^2 + 0.21x + 0.39 \tag{11}$$

where coefficient of determination is 0.995, and also fit to logarithmic regression equation

$$y = 0.41\log x + 0.50 \tag{12}$$

where coefficient of determination is 0.919. In Table Ⅲ estimate by (11) and (12) is shown when the numbers of stimuli are 3, 4, 5, 6, 7, 8 and 9.

For another example of answers if a subject answers as shown in Figure 3, the order among three stimuli A, B, C is also determined such that A is first, B is second, C is third and it is considered that the answers are consistent. In this case $C.I. = 0.28$ and this value is considered to be the threshold of *C.I.* on consistency of a subject's answers in paired comparison.

Table Ⅲ: Estimate for maximum value of *C.I.*

The number of stimuli	Estimate (11)	Estimate(12)
3	0.69	0.70
4	0.75	0.75
5	0.80	0.79
6	0.84	0.83
7	0.85	0.85
8	0.87	0.88
9	0.88	0.90

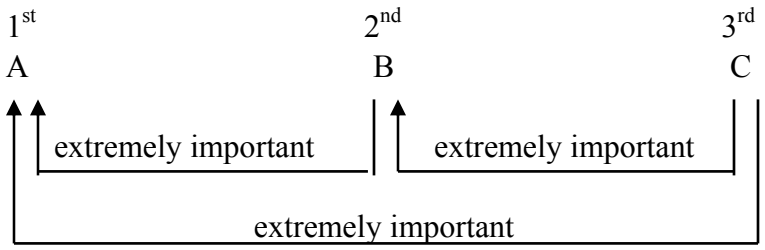


Figure 3: Examples of answers for the maximum of *C.I.*

Table Ⅳ shows the values of *C.I.* in such the case of Fig.3 when the numbers of stimuli are 3, 4, 5, 6, 7, 8 and 9.

Table Ⅳ: Threshold of *C.I.* to the number of stimuli

The number of stimuli	Threshold of *C.I.*
3	0.28
4	0.44
5	0.55
6	0.62
7	0.68
8	0.72
9	0.75

Let x and y be the number of stimuli and the largest value of *C.I.* respectively. Then x and y fit to the quadratic regression equation

$$y = -0.01x^2 + 0.22x - 0.26 \tag{13}$$

where coefficient of determination is 0.998 , and also fit to the logarithmic regression equation

$$y = 0.98\log x - 0.16 \tag{14}$$

where coefficient of determination is 0.992 . In Table Ⅴ estimate by (13) and (14) is shown when the numbers of stimuli are 3, 4, 5, 6, 7, 8 and 9.

Table Ⅴ: Estimate for threshold of *C.I.*

The number of stimuli	Estimate (13)	Estimate(14)
3	0.28	0.31
4	0.44	0.43
5	0.55	0.53
6	0.62	0.60
7	0.68	0.67
8	0.72	0.73
9	0.75	0.78

For the above, it is seen that the largest value of *C.I.* in the case where there does not exist circular triad and the threshold of *C.I.* on consistency are able to be estimated by regression equations.

6. Concluding remarks

Consistency of human *Kansei* in paired comparison was considered taking Consistency Index (*C.I.*) in AHP as the example to express consistency. It might be able to be said that threshold of *C.I.* on consistency of human feelings vary with the number of stimuli and it can be estimated by regression equation.

References

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