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Campus Helsingborg

Apply Grid in Design to Bayes' Procedure of Game Theory

Jaw-Sin Su (1) Juei -Yang Su(2)

(1) Associate Professor
Department of Information Management
Chinese Culture University

Email: jawsin@faculty.pccu.edu.tw

55 Hwa Kang RD. Yang Ming Shan
Taipei, Taiwan

(2) Secretary
Department of Accounting
Sususing Co., Ltd.,
P.O.BOX: 2298
Newcastle, 2940, RSA
Email:s250139@ms12.hinet.net

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Introduction

The grid in design (g[k][q]) is used to find the attribute area to data records (Chiang ,2003). It belongs to the algorithm and is applied to the IT field normally (Pfleeger, 1997). The base of grid technique is introduced the combinations of rectangle diagrams or the square diagrams with algorithm concept to complete (Berry ,1997). Normally, it is a two dimensions form (Yia ,2004). The base of a zero-sum game is two players' game and could be solved by game theory (Johnson, ,1976), and is extended to the statistical game (Taylor, 1996) even the Bayes' procedure (Su,1999). How to connect between grid technique and Bayes' procedure and to apply in the practical field is an important problem in the modern business operating. in this paper, the grid in design was introduced to find the solution of Bayes' procedure of zero-sum game

Bayes' Procedure with Grid Model

The statistical game is developed from the zero-sum game by two players, and after we give that first player as the nature and second player as the decision actors then the contents of making decision model are as follows:

The data to give

Conditional probability

$P(z/w)$ or $P_w(Z)$, $\sum P_w(Z)=1$ or $\int P_w(z)\, d(z)=1$

Loss matrix or Loss function

$L(w_i, d(z_j))$, $i=1,2,\ldots,m$; $j=1,2,\ldots,n$

Decision function

$d(z_j)=a_j$,

The solution of as the probability of nature (ξi) to give is

$$EOL(a^j)= \sum (\xi i)\, L(w^i, d(z^j))$$

To find the minimum regret of EOL (expected opportunity loss)

If the probability of nature (ξi) unknown, then the probability of nature (ξ) could be find for decision making, and as we have:

$$EOL(a_j)= EOL(a_k)$$

The Bayes'procedure of game is:

The Bayes' probability is given as follows:

$$\xi(wi/z)= \frac{p(z/wi)\xi(wi)}{\Sigma p(z/\theta)\xi(\theta)})$$

The decision function is the posteriori risk function, and is as follows:

$$\tau^{z}(d)= \frac{\Sigma l(w,d(z))p(z/wi)\xi(wi)}{\Sigma p(z/\theta)\xi(\theta)}$$

To making decision, the minimum risk could be found from the posteriori risk function by strategies.

We could construct any form of the strategies of nature and decision actors by loss matrix ($L(w_i,d(z_j)$, $i\geq 2$ and $j\geq 2$). To get the solution of 2*j or i*2 loss matrix would be easily, but 3*j or i*3 ($i\geq 3$ and $j\geq 3$) loss matrix would be difficult. The grid technique was introduced to solution the 3*3 loss matrix in the paper.

Case Example

The Jan-Son computer system Co., Ltd. in central Taiwan was a net and coffee shop for service to net workers since 2002. Twenty PC sets as the work stations for twenty-four hours service to customers set in the center. The fixed costs of 250 per day set were given. The revenue was come from the times of service. Tree strategies to three times revenue of one PC set were given and it is as follows:

Strategy 1 R1=150t; Strategy 2 R2 =100 + 100t: Strategy 3 R3=500

t; represent a computer working tomes.

The break point of equal to revenue was the t=2 of strategy 3 and 2 and t=4 of strategy 2 and 1, and the diagram is as follows [Figure 1]:

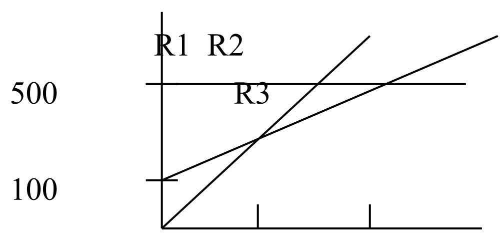

Figure 1: The break point of equal to revenue

The loss matrix

$$L(w_3, d(z_3)) = \begin{pmatrix} 0 & 5 & 35 \\ 5 & 0 & 10 \\ 40 & 20 & 0 \end{pmatrix}$$ [Appendix A]

The decision function

$d(z_1)=a_1,\ z \leq 2$; $d(z_2)=a_2,\ 2 < z \leq 4$; $d(z_3)=a_3, 4 < z$

General situation

Because the probability of nature (ξi) unknown, so the probability of nature (ξ) could be find for decision making as the minimum revenue of strategy finding (Min(EOL(a_i))).

Although we could be introduced the quantitative method to make strategy decision, but it is may be a complex rules (Lan, 2002).

The grid technique was used to cut the right triangle into 231 rhombuses to represent the probabilities of three strategies happen times were given [Appendix B]. From the grid, the rules of making decision for the minimum revenue were as follows [Table I]:

Table I: Probabilities of area

Strategies	$\xi(1)$	$\xi(2)$	$\xi(3)$
1	$\geq$ 0..5	$\leq$ 0.5	$\leq$ 0. 25
2	$\leq$ 0.9	$\geq$ 0.0	$\leq$ 0.75
3	$\leq$ 0.25	$\geq$ 0.3	$\geq$ 0.7

For example, if $\xi(1)=1/3$, $\xi(2)=1/3$, and $\xi(3)=1/3$, then second strategy would be chosen because of the point was drop down on the second strategy and the break even point ($\pi=0$) was t=2.5 from the profit formula (π= R2 -F=(100 + 100t)-250)).
.

Bayes' situation

Additional data were:

The conditional probability function (p(t/u)) was given from the exponential service time distribution and it is as follows:

$p(t/u)=ue^{-ut}$.

u=the average number of customers who can be served per day.

The conditional probability matrix could be gotten, and it is as follows [Table II]:

Table II: The conditional probability matrix

P(z/w)	z1	z2	z2
w=1	0.86	0.12	0.02
2	0.49	0.25	0.26
3	0.39	0.24	0.37

We could make decision to chose the strategy in according to the minimum risk by each posteriori risk function ($\tau^{z}(d)=\frac{\Sigma\, l(w,d(z))p(z/wi)\xi(wi)}{\Sigma\, p(z/\theta)\xi(\theta)}$).

Because the probability of nature (ξi) unknown, so the probability of nature (ξ) could be find for decision making as the minimum revenue of strategy finding (Minτz(d)).

It is may be difficult to get the solution by quantitative method.

The grid technique was used to cut the right triangle into 231 rhombuses to represent the probabilities of three strategies happen times were given [Appendix C]. From the grid, the rules of making decision for the minimum revenue were as follows [Table III]:

Table III: Probabilities of area

Strategies	$\xi(1)$	$\xi(2)$	$\xi(3)$
1	≥ 0.5	≤ 0.5	≤ 0.25
2	≤ 0.45	≥ 0.25	≤ 0.5
3	≤ 0.5	≤ 0.5	≥ 0.5

For example, if $\xi(1)=1/3$, $\xi(2)=1/3$, and $\xi(3)=1/3$, then second strategy would be chosen because of the point was drop down on the second strategy and the break even point was as same as the general situation.

Comparison between two models

The grid technique was used to cut the right triangle into 231 rhombuses to represent the probabilities of three strategies happen times were given.

The base of minimum revenue was used to make decision by both models.

In the general situation, after three computer working times was given, to make decision may be clearly for the probabilities of times that have not considered.

In the Bayes' situation, the conditional probabilities of computer working times and Bayes' probability are considered to make decision. The more information be gotten, the more fine decision could be found.

Extension

The above case was one PC set twenty hours working decision making.

The extensions are that a PC set work times could be set one more sections, for example three times section is from 00:00 -08:00, 08:00 – 16:00, and 16:00 to 24:00 to make decision for different probabilities of each strategy.

All computer work on the online system and the service quality would be promoted.

Conclusions and Discussions

Conclusions

To make decision of general or Bayes'game, the quantitative method could be introduced to get the answer when the probabilities of the nature strategies were given. In the case of the probabilities of the nature strategies unknown, not only the model is more complex, but also it may be difficult to find the results by quantitative method using. We introduced the grid technique to solute the 3X3 zero-sum game of general situation and of Bayes'situation decision kaming. To prove, the model could be used to the practical case, the network computer service system with 3X3 zero-sum game supported by Jan-Son computer system Co., Ltd. in central Taiwan to make decision as the case of our example.Because we have three dimensions, so the right triangle was cut into the numbers of rhombuses to instead to the plane was cut into the numbers of square in our practical case example.

The grid technique can get the solution of complex Bayes' formula of zero-sum game and that the model is practicable.

Discussions

Although any MxN (M, N >1) zero-sum game could be gotten the solution by quantitative method or others, but it is may need to construct the more complex model or to find the more complex rules to get answer.

But any MxN (M, N >1) zero-sum game can be gotten the solution by grid technique and the different types of grid may be developed to the different game situations.

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Appendices

Appendix A: The loss matrix

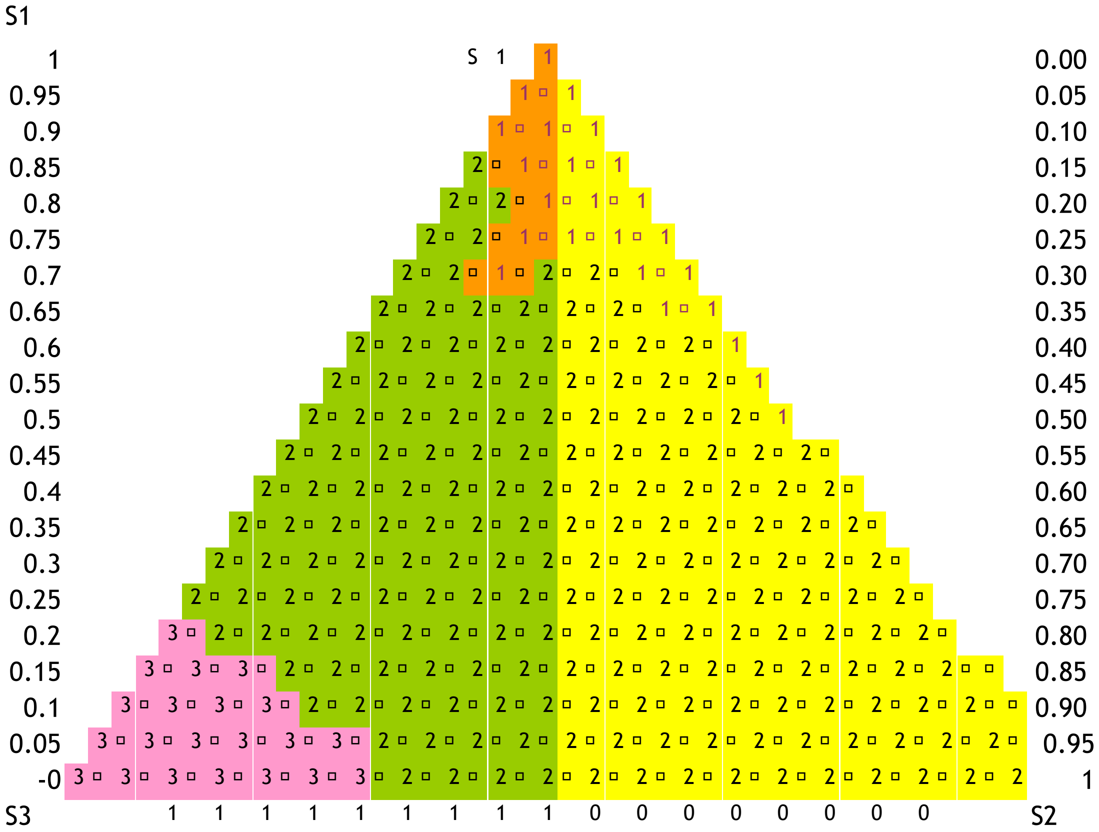
	150t	100+100t	500
1	150	200	500
3	450	400	500
6	900	700	500
	0	50	350
	50	0	100
	400	200	0
	0	5	35
	5	0	10
	40	20	0

Appendix B: The diagram of to cut the right triangle into 231 rhombuses in the general situation.

The data

	t1	t2	t3	s00,05,40	s05,00,10	s40,20,00	Min.	S
1	1.00	0	0.00	0	5	40	0	1
2	0.95	0.00	0.05	2	5.25	38	2	1
3	0.95	0.05	0.00	0.25	4.75	39	0.25	1
4	0.90	0	0.1	4	5.5	36	4	1
5	0.90	0.05	0.05	2.25	5	37	2.25	1
	…							
227	0.00	0.8	0.20	12	2	16	2	2
228	0.00	0.85	0.15	10.25	1.5	17	1.5	2
229	0.00	0.9	0.10	8.5	1	18	1	2
230	0.00	0.95	0.05	6.75	0.5	19	0.5	2
231	0.00	1	0.00	5	0	20	0	2

The right triangle



Appendix C: The diagram of to cut the right triangle into 231 rhombuses in the Bayes' situation.

The data

u	24	8	4
w=24/u	1	3	6
from t	0	2	4
to t	2	4 ~	

w	$p(t_j/u_i)$	$t_1=2/24$ $\exp(-u_i t)$	$t_2=4/24$ $\exp(-u_i t)$	$t_3=24/24$ $\exp(-u_i t)$
1	24	0.1353	0.01832	0
3	8	0.5134	0.2636	0
6	4	0.6065	0.36788	0

p(z/w)	z1	z2	z3
1	0.8647	0.11702	0.0183
3	0.4866	0.24982	0.2636
6	0.3935	0.23865	0.3679

The right triangle