Estimation of Phase Noise for QPSK Modulation over AWGN Channels

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Abstract

Every oscillator used in bandpass communication suffers from an instability of their phase (a.k.a. phase noise) that, if left unaddressed, can lead to great degradation of the system performance. In this paper, we tackle the problem of minimising the effect of oscillator phase noise on the coherent detection of a quadrature phase shift keying (QPSK) modulation operating on an Additive White Gaussian Noise (AWGN) channel. The phase noise process is modeled as a Wiener-Levy (random walk) process. Our approach uses maximum likelihood (ML) estimation of phase noise. Thorough analysis and derivation for Decision Directed (DD), Non-Data Aided (NDA), used with and without symbol differential encoding, and pilot based estimators are presented. We compare these estimators with respect to their main features and evaluate their bit error rate (BER) performances throught simulations. Results show that for low signal to noise ratio (SNR) applications, the use of differential encoding along with the proposed DD or NDA estimator yields performances with an SNR penalty below the two dB imposed by the non coherent detection methods, while pilot based estimation using wiener interpolation makes it possible to detect a QPSK modulation with SNR penalty around two dB.

Keywords

QPSK, Phase Noise, Wiener-Levy, ML estimation,AWGN, Wiener Interpolation.

1. Introduction

In any bandpass communication system, Radio frequency (RF) hardware such as oscillators are not ideal. The carrier generated by this device is not ideal and experiences phase instability, or phase noise, mainly due to the presence of thermal noise in the circuitry. Circuits designed for very high carrier frequency, such as carrier generator chains used for 60GHz communication (such as in [1]), are very difficult to design with a very stable frequency source. Moreover, these circuits typically make use of frequency multipliers to reach high carrier frequencies, increasing again the level of phase disturbance [2]. It is therefore of interest to take into account their phase noise characteristics when looking at system issues. In this paper, phase noise is modeled using the Wiener-Levy phase noise model. This model has been widely used to analyse phase noise over the years [3], [4], and looks at the the phase noise process as a random walk process. We estimate and compensate for the effect of phase noise at the receiver using Maximum Likelyhood (ML) methods as covered in [5] and [6].

The paper is organised as follows: First we will detail the considered system setup (section 2.1), second we will present in detail our phase noise model (section 2.2). Then we will present our estimations methods (section 3) and their associated results (section 4) , before concluding.

2. System Setup and Models

2.1. System Description

Figure 1 shows the general block diagram of the considered system in its baseband equivalent (complex lowpass) representation. bits are modulated using a Quadrature Phase Shift Keying (QPSK) to obtain the complex symbols $s_n = e^{j\theta_n}$, where θ_n can take values $m\frac{\pi}{2} + \frac{\pi}{4}$, $m = 1, 2, 3, 4$. The symbols are then multiplied by the phasor e^{ϕ_n}, where ϕ_n is a random variable and accounts for the total phase noise for frequency sources in the system.

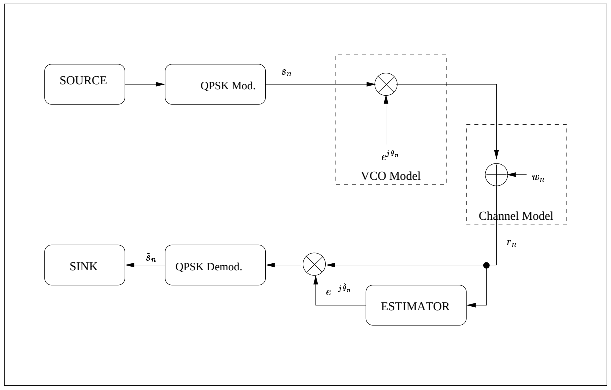

Figure 1: System Setup .

More detailed discussion on ϕ_n can be found in section 2.2. The signal is then passed throught an Additive White Gaussian Noise channel, so that the received signal is

$$r_n = s_n e^{\phi_n} + w_n \tag{1}$$

where w_n is a zero-mean, complex Gaussian random variable with variance N_0. The signal is then passed throught a phase estimator that produce an estimate $\hat{\phi}_n$of the phase noise event. The QPSK demodulator outputs a decision $\tilde{s}_n$ of the transmitted signal based on the observation of the counter-rotated received signal $r_n e^{-j\hat{\phi}_n}$, before the transmitted bits are decoded.

2.2. Phase Noise Model

The power spectrum of an oscillator producing phase noise does not have the arrow shape that we would ideally expect at the carrier frequency. Instead, this, this spectrum is broader and has a Lorentzian shape [7]. For simulation and analysis, the phase noise process ϕ_n, can be modeled as a wiener levy (random walk) process

$$\phi_n = \phi_{n-1} + \Delta_n \tag{2}$$

where Δ_n is refered as the *stepsize* of the walk and is a zero mean Gaussian random variable. Its variance sets the speed of the process and is equal to $\sigma_\Delta^2 = 2\pi BT$. The product BT is refered as the *phase noise rate* and express the relative double-sided bandwidth of the process with respect to the symbol rate. Phase noise is assumed to remain constant between symbols. Figure 2 shows a realisation of the process and its corresponding power spectrum.

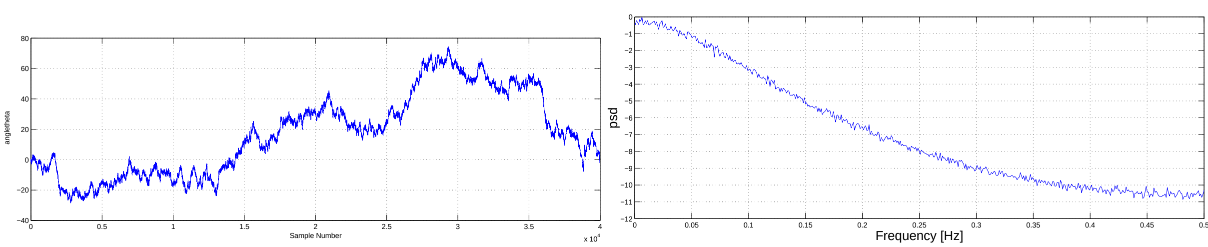

Figure 2: A realisation and Its associated Power Spectrum for a phase noise process with a phase noise rate BT=0.2 .

3. Estimators

3.1. NDA and DD estimators

Prior to the estimation, we perform some transformations to the received symbol as defined in equation 1. We rotate r_n by s_n^* to get rid of the data dependancy (The subject of removal of this dependency is the matter of section 3.2). After rotation, the received symbol becomes

$$\dot{r_n} = e^{j\theta_n} + \dot{w_n} \tag{3}$$

where $\dot{w_n} = w_n s_n^{*}$[1] is a rotated version of the AWGN event, and still has the same statistical properties as w_n.

ML estimators seek to find the estimate of the phasor $e^{j\phi_n}$ that maximise the conditional probability density function $f(\mathbf{r}|\phi_\mathbf{n})$ at a given time n, where $\mathbf{r}$ is a vector of N observed received signal points

$$\mathbf{r} = \Big[\dot{r}_{n-N-1}, ..., \dot{r}_{n-2}, \dot{r}_n\Big]^T \tag{4}$$

from Equation 1 and the definition of the phase noise process in 2 we can express the received signal at time $n-i, i = 1, ..., N$ as

$$\dot{r}_{n-i} = e^{(j\theta_n + \sum_{u=0}^{i-1} \Delta_u)} + \dot{w}_{n-i} \tag{5}$$

Let us assume that the variable $\sum_{u=0}^{i-1} \Delta_u$ has a small value compare to one. Then,

$$e^{j\theta_n + \sum_{u=0}^{i-1} \Delta_u} \approx e^{j\theta_n}(1 + j\sum_{u=0}^{i-1} \Delta_u) \tag{6}$$

Conditionning on the value θ_n that we seek to estimate, $\dot{r_{n-i}}$ is a function of two independant gaussian variables (namely the phase noise step Δ_u and the AWGN process $\dot{w}_{n-i}$), thus the observed vector also has a multivariate gaussian distribution [8].

$$f_{\mathbf{r}|\theta_n}(\mathbf{r}|\theta_n) = \frac{1}{(2\pi)^{\frac{N}{2}} \det \mathbf{C}^{-1}} \exp\left[\frac{1}{2}(\mathbf{r} - \mathbf{m}_r)^H \mathbf{C}^{-1} (\mathbf{r} - \mathbf{m}_r)\right] \tag{7}$$

The mean vector $\mathbf{m}_r$ value at time $n-i$ is $\mathbf{m}_r(i) = \mathbf{E}(\dot{r}_{n-i})$, for all $i = 1, 2, ..., N$. Given that both Δ_u and $\dot{w}_{n-i}$ are zero mean, the mean is $\mathbf{E}(\dot{r}_{n-i}) = e^{j\theta_n}$ and thus the mean vector is

$$\mathbf{m}_r = e^{j\theta_n}\Big[1, ..., 1, 1\Big]^T = e^{\theta_k}\mathbf{1}^T \tag{8}$$

The covariance matrix content is the correlation between points in the vector at offsets i and l, that is,

$$\mathbf{C}(i,l) = \mathbf{E}\Big(\dot{r}_{n-i}^* - \mathbf{E}(\dot{r}_{n-i}))(\dot{r}_{n-l} - \mathbf{E}(\dot{r}_{n-l})\Big) \tag{9}$$

This is reduced as

$$\mathbf{C}(i,l) = \min(i,l)\sigma_\delta^2 + \delta(i-l)\sigma_w^2 \tag{10}$$

The pdf in 7 is maximised when the log-likelihood function $\Lambda = \Big[(\mathbf{r} - \mathbf{m}_r)^H \mathbf{C}^{-1} (\mathbf{r} - \mathbf{m}_r)\Big]$ is minimised. It can be shown that this is solved for the phasor

$$e^{\hat{\theta}_n} = \sum_{u=1}^{N} \alpha_u \dot{r}_{k-u} \tag{11}$$

with the coefficient vector

$$\alpha = \mathbf{1}^T \mathbf{C}^{-1} \tag{12}$$

[1] $()^*$ defines the complexe conjucate

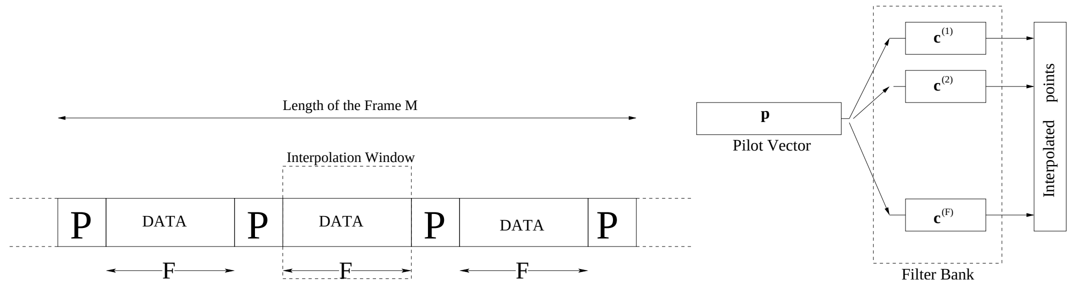

Figure 3: Diagram and Frame Organisation of the interpolator. One packet of F symbols is interpolated from N pilots symbols, with half of the pilots taken from the past symbols and the other half coming from the coming symbols (hence the need of a delay).

3.2. Removal of Data Dependancies

As we have seen, the estimator derived in 11 assumes that the data has been removed from the received signal. At this stage of the receiver, this can be done by Decision Directed (DD) methods or Non Data Aided (NDA) methods. In a DD estimator, a prehemptive decision $\hat{s_n}$ is made before doing the estimation and this decision is used to remove the data dependancies. With a reasonably high SNR, good estimates can be obtained.

The NDA method for QPSK modulation raises the received signal the power of 4 in order to remove the data. The estimator output in this case need to be divided by four and is folded, yielding a phase ambiguity. To resolve the ambiguity, differential encoding is apply prior to transmitting the symbols [8].A thorough coverage of DD and NDA methods is given in [5].

3.3. Pilot-Based Estimation

In a pilot-based transmission, known data symbols, or pilots, are inserted into the data stream in order to recover the phase errors. The algorithm we propose is to use interpolation to allow tracking between pilot symbols. Wiener Interpolation algorithm [6] allows to design banks of linear filter to optimally estimate phasors between pilots. The Interpolator diagram is shown on figure 3.

The interpolator works as follow: we consider one frame of transmitted signal as shown in 3. The F symbols we seek to interpolate are shown in the dashed box. Pilots symbols are inserted in a by chunks of equal size every F symbols, and in the frame we have inserted N/2 Pilot symbols before and after the symbols to interpolate as shown on figure.

We use a bank of F Wiener Filter to produce the interpolated points. Define $\mathbf{x}$ The vector of size M where the frame is stored. The Pilots of the frame are stored into a vector $\mathbf{p}$ and the position of these pilots in the frame are stored in a vector $\mathbf{q}_{\text{pilots}}$. The position of the data in the frame is stored is a vector $\mathbf{q}_{\text{data}}$. We obtain the interpolated phasor at time k by applying the kth linear filter with a coefficient vector $\mathbf{c}_k$ to the vector $\mathbf{p}$.

$$e^{\hat{\theta}_k} = \mathbf{c}_k^T \mathbf{p} \tag{13}$$

The calculation for the coefficient vectors makes use of wiener filter theory detailed in [6]. For such a filter applied to interpolation, the filter coefficient for the ith coefficient of the kth linear filter, are given by

$$c(i)_k = \mathbf{C}^{-1}\mathbf{R}(k) \tag{14}$$

Where $\mathbf{C}$ is the covariance matrix of the observed pilots, and $R(k)$ is the cross correlation between the pilots in the frame and the kth phasor we seek to interpolate. These values are possible to calculate in advance to be stored in the receiver. Specifically the covariance matrix content is

$$\mathbf{C}(u,v) = e^{-\frac{1}{2}|\mathbf{q}(u)_{\text{pilots}} - \mathbf{q}(v)_{\text{pilots}}|\sigma_\delta^2} + \delta(u-v)\sigma_w^2 \tag{15}$$

And the Cross correlation vector for the kth filter bank is set by

$$\mathbf{R}^{(k)}(u) = e^{-\frac{1}{2}|\mathbf{q}(u)_{\text{pilots}} - \mathbf{q}(k)_{\text{data}}|\sigma_\delta^2} \tag{16}$$

4. Results

4.1. DD and NDA Methods

Figure 4 shows an example of a couter rotated constellation for a signal to noise ratio of 10dB. Figure 5 shows the results obtain on QPSK using differential encoding with decision directed and NDA estimation.The results are shown for five phase noise rate and compared to the theoretical probability of error for QPSK and differentially encoded QPSK (DQPSK) The DD algorithm designed failed to work for a QPSK modulation without differential encoding, with DD or NDA estimators. The reason for this is the lack of reliable data symbol at such a low SNR. As suggested by [5] a constellation working at a higher SNR, such as 16QAM would be more suitable.

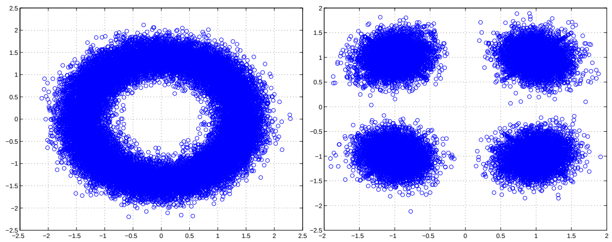

Figure 4: Example of a received constellation before and after estimation of phase noise, for an SNR of 10dB. .

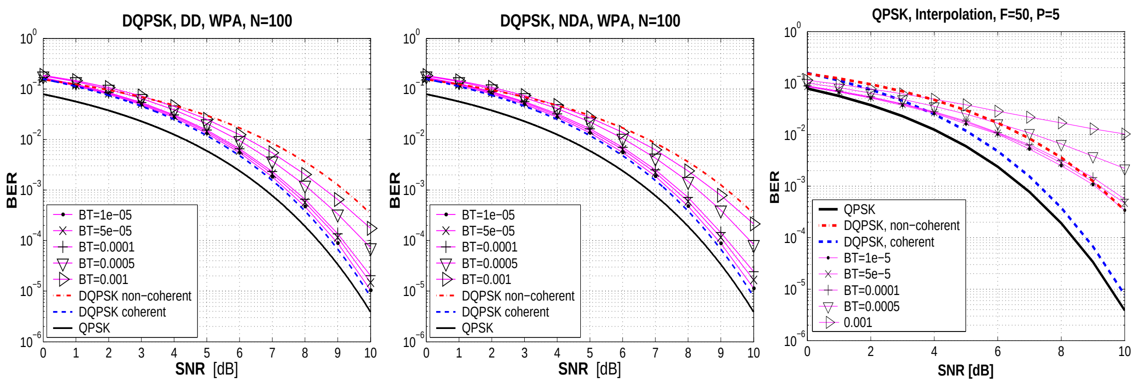

Figure 5: BER results for decision directed (DD), Non Data Aided (NDA) and pilot-based estimators. .

4.2. Interpolation

For (uncoded) QPSK, the results of interpolation shown on figure 5are satisfying. The percentage of pilot inserted was set to keep a small estimation error variance). Another constraint was that the throughput should not drop by more than 10 percent. With these constrains, the estimator performs better than a non coherent detector when phase noise rate does not exceeding $BT = 5.10^{-4}$. The interpolator could perform better by reducing the time between pilots insertion, but the cost in throughput would increase.

5. Conclusions

The results show that using differential encoding, methods employing either NDA estimation or DD estimation perform well with low SNR conditions, yielding an SNR penalty of about 1.7dB for a a case of high phase noise ($BT = 10.^{-3}$). The main limitation of the DD estimator for QPSK (no differential encoding) is that the estimator cannot work at low SNR, because of its sensibility to channel noise-induced errors, specially burst errors. Interpolation appears to be a good option but is limited to phase noise with reasonably small phase noise rate.

6. Acknowledgements

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7. References

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