# Statistical methods for FET-model extraction

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***Abstract*— A statistical method for extracting small-signal FET models is presented. The method is derived as a maximum likelihood estimate of the model parameters with respect to measured S-parameters. The method accounts for uncertainties both in measurements and in extracted parasitic elements. An advantage with the proposed method is that the covariance of the estimate is obtained. The covariance is then used to calculate statistical bounds of the extracted parameters.**

**Monte-Carlo simulations are used to verify the statistical bounds of the parameters. Good agreement between theory and experiments is obtained, especially for the parasitic elements.**

## I. Introduction

Detailed prediction of circuit behavior requires accurate device models to be used and active devices usually are the most critical components. At high frequency, field effect transistors (FETs) are most commonly used. FET small-signal equivalent circuit models are not only used for circuit designs, but serves as basis for empirical large-signal models and their parameters are used to evaluate FET processing variations. Proper extraction of small-signal FET transistor model parameters is therefore of fundamental concern. Normally, the direct extraction method [1] is used to determine the FET model parameters. However, despite this wide range of application, very little work has been reported on how to determine the uncertainty in the model parameters obtained.

This work tries to fill this void by applying statistical estimation methods to the FET-model extraction problem. We propose using a maximum likelihood estimate (MLE) of the measured S-parameter data. The proposed method is then verified using extensive Monte-Carlo simulations.

## II. Statistical extraction

The standard small-signal model of a high-frequency FET is shown in Fig. 1(c). The model consists of two parts: a *parasitic* network describing bonding wires, pads and access resistances. The second part is the *intrinsic* network describing the (small-signal response) of the active semiconductor region. The parasitic model elements are independent of external bias whereas the intrinsic elements are considered as being bias dependent.

### A. Parasitic element extraction

To extract the parasitic elements, measurements are taken at two cold bias conditions: one below pinch-off and one under full channel conditions. The FET has very different small-signal responses for these two bias conditions; mainly due to the fact that the channel resistance is varied by more than four orders of magnitude. In Fig. 1(a) and 1(b) small-signal models for these two bias conditions are shown.

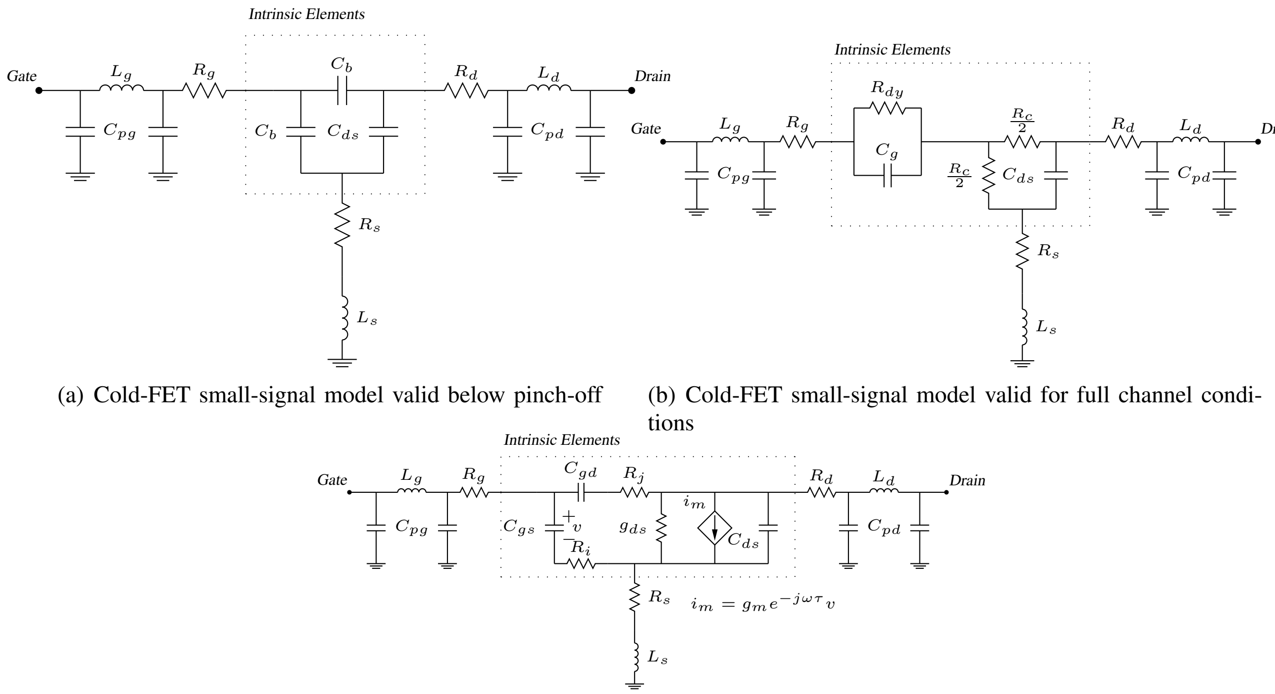
Let $\mathbf{p}$ denote the vector of parasitic model parameters. For each measurement frequency, subscript $n$, a vector error function,$\mathbf{h}(\cdot)$, is defined as

$$\mathbf{h}_n(\mathbf{p}) \quad = \quad \mathbf{s}_n - \mathbf{g}_n(\mathbf{p}) \tag{1}$$

where $\mathbf{s}_n$ and $\mathbf{g}_n$ denotes the measured and modelled S-parameters at the $n$'th frequency point, respectively. Notice that both $\mathbf{s}_n$ and $\mathbf{g}_n$ are vectors e.g. $\mathbf{s}_n = [S_{11}^{po}, S_{12}^{po}, S_{21}^{po}, S_{22}^{po}, S_{11}^{ch}, S_{12}^{ch}, S_{21}^{ch}, S_{22}^{ch}]^T$ and similar for the model function $\mathbf{g}_n$. The superscripts $po$ and $ch$ denote measured S-parameters for pinched and channel conditions, respectively. In the actual implementation only real variables are used, therefore the latter vector is twice as long since each S-parameter is represented by its real and imaginary part. For cold bias points the transistor is reciprocal and the average of the transfer terms $S_{12}$ and $S_{21}$ are used.

Further assume the S-parameter measurement uncertainties to be normal distributed and known by their covariance matrix, $\mathbf{C}_w$. The likelihood function for the error function, lik $\mathbf{h}(\cdot)$, can then be expressed by [2]

$$\text{lik}\,\mathbf{h}(\mathbf{p}) = e^{-\mathbf{h}^T\mathbf{C}_w^{-1}\mathbf{h}/2} \tag{2}$$




(a) Cold-FET small-signal model valid below pinch-off

(b) Cold-FET small-signal model valid for full channel conditions

(c) FET small-signal model

Fig. 1. Small-signal FET-models

For several reasons it is wise to consider the logarithm of the likelihood-function. Maximizing the log likelihood-function with respect to the parasitics $\mathbf{p}$ gives the maximum likelihood estimate (MLE):

$$\hat{\mathbf{p}} = \arg\min_{\mathbf{p}} \mathbf{h}(\mathbf{p})^T \mathbf{C}_w^{-1} \mathbf{h}(\mathbf{p})/2 \tag{3}$$

An approximate covariance matrix of the estimate $\hat{\mathbf{p}}$ is:

$$\mathbf{C}_p \approx \mathbf{H}^{-1} \tag{4}$$

where $\mathbf{H}$ is the Hessian of the negative log likelihood function, whose elements are given by:

$$H_{k,l} = \frac{\partial^2}{\partial p_k \partial p_l} \left( \mathbf{h}(\mathbf{p})^T \mathbf{C}_w^{-1} \mathbf{h}(\mathbf{p})/2 \right) \tag{5}$$

The Hessian should be evaluated at $\mathbf{p} = \hat{\mathbf{p}}$.

### B. Intrinsic parameter extraction

For a MLE of the intrinsic model parameters to be made, the uncertainties in the parasitic elements must also be considered. The likelihood function for both the parasitics and the intrinsic elements are,

$$\text{lik } \mathbf{h}(\mathbf{x}, \mathbf{p}) = e^{-\mathbf{h}^T \mathbf{C}_w^{-1} \mathbf{h}/2} \tag{6}$$

where $\mathbf{h}$ is a vector containing measured S-parameters for an active bias point.

Thus it remains to eliminate the parasitics from (6). This is done by treating them as *nuisance parameters* by methods found in [2]. The parasitic element values are thereby being eliminated from the likelihood-function as follows.

$$\text{lik } \mathbf{h}(\mathbf{x}) = \int_{-\infty}^{\infty} \text{lik } \mathbf{h}(\mathbf{x}, \mathbf{p}) f(\mathbf{p}) \mathrm{d}\mathbf{p} \tag{7}$$

where $f(\mathbf{p})$ denotes the parasitic element probability density function.

$\mathbf{h}(\mathbf{x}, \mathbf{p})$ is linearized with respect to $\mathbf{p}$ around $\mathbf{p} = \mathbf{p}_0$ to facilitate an analytical solution of (7).

$$\mathbf{h}(\mathbf{x}, \mathbf{p}_0 + \Delta\mathbf{p}) \approx \mathbf{h}(\mathbf{x}, \mathbf{p}_0) + \frac{\partial \mathbf{h}(\mathbf{x}, \mathbf{p}_0)}{\partial \mathbf{p}} \Delta\mathbf{p} \tag{8}$$

The parasitic element deviations are assumed to be normal distributed with mean $\mathbf{p}_0$ and covariance matrix $\mathbf{C}_p$. This is used with (6) in (7) to yield:

$$\text{lik } \mathbf{h}(\mathbf{x}) = \int_{-\infty}^{\infty} e^{-(\mathbf{h}_0 + \mathbf{h}'\Delta\mathbf{p})^T \mathbf{C}_w^{-1} (\mathbf{h}_0 + \mathbf{h}'\Delta\mathbf{p})/2} \cdot e^{-\Delta\mathbf{p}^T \mathbf{C}_p^{-1} \Delta\mathbf{p}/2} \mathrm{d}\Delta\mathbf{p} \tag{9}$$

where $\mathbf{h}_0 = \mathbf{h}(\mathbf{x}, \mathbf{p}_0)$ and $\mathbf{h}' = \frac{\partial \mathbf{h}(\mathbf{x}, \mathbf{p}_0)}{\partial \mathbf{p}}$. By collecting the $\Delta\mathbf{p}$ terms in the exponents and completing the squares, the following expression is found for lik $\mathbf{h}(\mathbf{x})$:

$$\text{lik } \mathbf{h}(\mathbf{x}) = e^{-\mathbf{v}(\mathbf{x})/2} \cdot \int_{-\infty}^{\infty} e^{-(\Delta\mathbf{p} - \mathbf{u}(\mathbf{x}))^T \mathbf{A}(\mathbf{x}) (\Delta\mathbf{p} - \mathbf{u}(\mathbf{x}))} \mathrm{d}\Delta\mathbf{p} \tag{10}$$

where

$$\mathbf{A}(\mathbf{x}) = \mathbf{h}'(\mathbf{x})^T \mathbf{C}_w^{-1} \mathbf{h}'(\mathbf{x}) + \mathbf{C}_p^{-1} \quad (11)$$

$$\mathbf{u}(\mathbf{x}) = -\mathbf{A}(\mathbf{x})^{-1} \mathbf{h}'(\mathbf{x})^T \mathbf{C}_w^{-1} \mathbf{h}_0(\mathbf{x}) \quad (12)$$

$$\begin{aligned} v(\mathbf{x}) &= \mathbf{h}_0(\mathbf{x})^T \mathbf{C}_w^{-1} \mathbf{h}_0(\mathbf{x}) - \\ &- \left(\mathbf{h}'(\mathbf{x})^T \mathbf{C}_w^{-1} \mathbf{h}_0(\mathbf{x})\right)^T \\ &\cdot \mathbf{A}(\mathbf{x})^{-1} \mathbf{h}'(\mathbf{x})^T \mathbf{C}_w^{-1} \mathbf{h}_0(\mathbf{x}) \end{aligned} \quad (13)$$

The integral in (10) may be analytically solved:

$$\text{lik}\ \mathbf{h}(\mathbf{x}) = \frac{e^{-v(\mathbf{x})/2}}{\sqrt{|\mathbf{A}(\mathbf{x})|}} \quad (14)$$

where terms independent of $\mathbf{x}$ have been omitted. A maximum likelihood estimate of $\mathbf{x}$ is therefore given by,

$$\begin{aligned} \hat{\mathbf{x}} &= \arg\max_{\mathbf{x}} \frac{e^{-v(\mathbf{x})/2}}{\sqrt{|\mathbf{A}(\mathbf{x})|}} \\ &= \arg\max_{\mathbf{x}} \left(-\frac{v(\mathbf{x})}{2} - \frac{1}{2}\ln|\mathbf{A}(\mathbf{x})|\right) \end{aligned} \quad (15)$$

And as for the parasitics an approximate covariance matrix is found as the inverse hessian of the minus log likelihood function.

## III. Experimental results

The estimators are implemented in MATLAB, to solve the nonlinear minimisation/maximisation problems the function `fminc` are used. To verify the method a Monte-Carlo simulation is performed:

First, a MLE extraction is performed on real world transistor data. The data are on-wafer measurements of an 4x25 micron GaAs-FET. The extracted model parameters are then assumed as an ideal reference and are used to calculate modelled S-parameters. Three sets of modelled S-parameter data are then calculated: *pinched*,*channel* and *active*. Statistical deviations (noise) are then added to these S-parameters according to an empirical measurement uncertainty model. For each noise implementation a new MLE extraction is performed. This procedure is repeated 750 times.

In Table I the mean of all extracted parameter values and the value extracted from the GaAs-FET data are shown. From this table it is clear that the MLE clearly finds the correct values, thus it is an *unbiased* estimate.

In Fig. 2 histograms of extracted parasitics are shown. Imposed on the histograms are fitted normal distributions and estimated normal distributions. The estimated normal distributions are given by the diagonal elements of the extracted covariance matrix $\mathbf{C}_p$. The fit is very good, except for the resistances $R_s$ and $R_d$ which are clearly not normal distributed. The reason for this is that they are strongly correlated to each other as well as to the channel resistance. In this experiment the channel resistance was allowed to vary between 200 and 0.5 Ohms and unfortunately this lower limit was not low enough.

In Fig. 3 histograms of extracted parasitics are shown. Contrary to the parasitics the estimate of the distributions are not very close to the Monte-Carlo simulations. Although the MLE does not predict the distributions perfectly they are still unbiased and the parameter deviations are small. The reasons for the poor fit is not yet fully understood and is subject of investigation.

## IV. Conclusions

A maximal likelihood estimate of FET small-signal model parameters are presented, together with statistical bounds for the parameter variations. The method accounts for uncertainty both in parasitics and measurements.

The results for the parasitic extraction is excellent and thus shows great promise for actual deployment in industry. A big advantage with the proposed method is that it is very flexible, one can easily replace any of the small-signal models with different ones and still obtain accurate estimates. The method could not only be used for FET or transistor modelling but also to passive devices such as SMT-capacitors etc.

## References


[1] G. Dambrine, A. Cappy, F. Heliodore, and E. Playez. A new method for determining the FET small-signal equivalent circuit. *IEEE Transactions on Microwave Theory and Techniques*, 36(7):1151–1159, 1988.

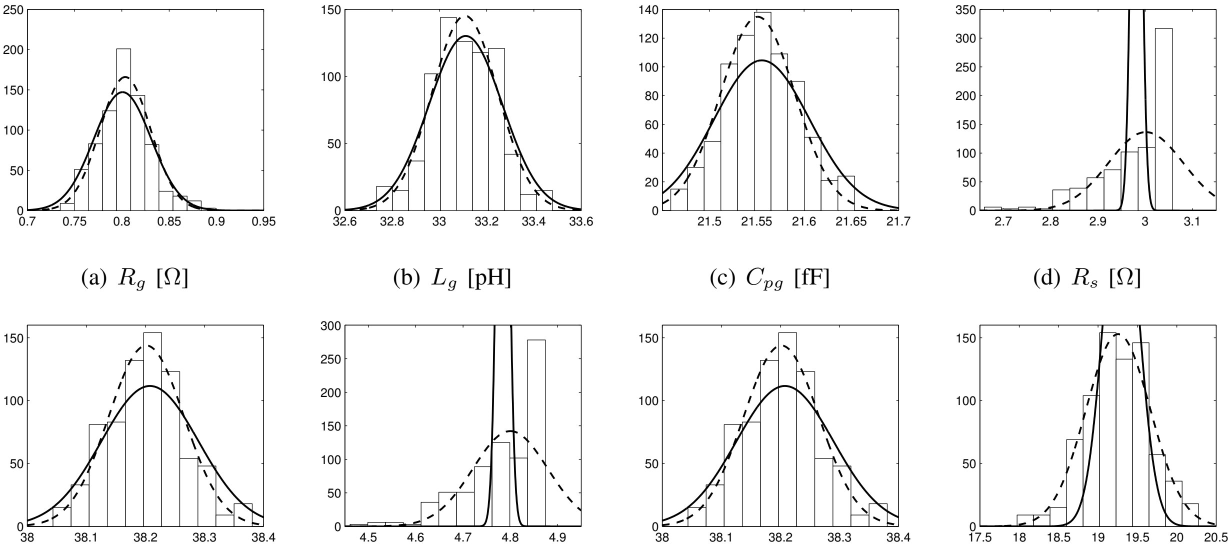
[2] Steven Kay. *Fundamentals of statistical signal processing: Estimation theory*. Prentice Hall, 1993.

TABLE I

EXTRACTED MODEL PARAMETERS

| | $R_g$ [Ω] | $L_g$ [pH] | $C_{pg}$ [fF] | $R_s$ [Ω] | $L_s$ [pH] | $R_d$ [Ω] | $L_d$ [pH] | $C_{pd}$ [fF] |
|---|---|---|---|---|---|---|---|---|
| GaAs-FET | 0.8007 | 33.11 | 21.56 | 2.981 | 5.217 | 4.784 | 38.21 | 19.29 |
| sim | 0.8029 | 33.11 | 21.56 | 2.979 | 5.216 | 4.783 | 38.20 | 19.26 |

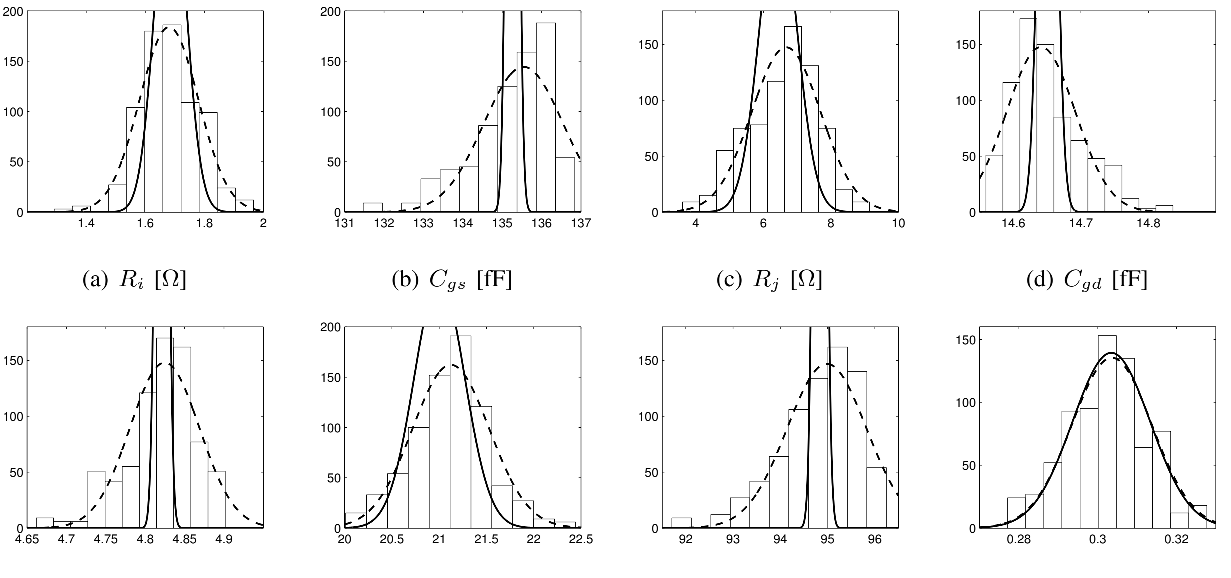
| | $R_i$ [Ω] | $C_{gs}$ [fF] | $R_j$ [Ω] | $C_{gd}$ [fF] | $g_{ds}$ [mS] | $C_{ds}$ [fF] | $g_m$ [mS] | $\tau$ [ps] |
|---|---|---|---|---|---|---|---|---|
| GaAs-FET | 1.685 | 135.3 | 6.439 | 14.65 | 4.821 | 21.00 | 94.84 | 0.3035 |
| sim | 1.684 | 135.2 | 6.563 | 14.65 | 4.820 | 21.11 | 94.82 | 0.3024 |



(a) $R_g$ [Ω] (b) $L_g$ [pH] (c) $C_{pg}$ [fF] (d) $R_s$ [Ω]

(e) $L_s$ [pH] (f) $R_d$ [Ω] (g) $L_d$ [pH] (h) $C_{pd}$ [fF]

Fig. 2. Histograms for extracted parasitics. Dashed lines are fitted normal distributions and solid lines are normal distributions depicted from the MLE covariance matrix.



(a) $R_i$ [Ω] (b) $C_{gs}$ [fF] (c) $R_j$ [Ω] (d) $C_{gd}$ [fF]

(e) $g_{ds}$ [mS] (f) $C_{ds}$ [fF] (g) $g_m$ [mS] (h) $\tau$ [ps]

Fig. 3. Histograms for extracted intrinsics. Dashed lines are fitted normal distributions and solid lines are normal distributions depicted from the MLE covariance matrix.